THOMSON'S
PRACTICAL
ARITHMETIC.

Revised and Enlarged.

FREE ACADEMY.

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Day and Thomson's Series.

PRACTICAL ARITHMETIC,

UNITING THE

INDUCTIVE WITH THE SYNTHETIC MODE OF INSTRUCTION.

FOR SCHOOLS AND ACADEMIES.

BY JAMES B. THOMSON, LL. D.

AUTHOR OF MENTAL ARITHMETIC; SLATE AND BLACK-BOARD EXERCISES; ARITHMETICAL ANALYSIS; HIGHER ARITHMETIC; EDITOR OF DAY'S SCHOOL ALGEBRA; LEGENDRE'S GEOMETRY, ETC.

NEW STEREOTYPE EDITION, REVISED AND ENLARGED.

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PREFACE.

It has been well said, that "whoever shortens the road to knowledge, lengthens life." The value of a knowledge of *Arithmetic* is too generally appreciated to require comment. When properly studied, two important ends are attained, viz: *discipline* of mind, and *facility* in the application of numbers to business calculations. Neither of these results can be secured, unless the pupil *thoroughly understands the principle* of every operation he performs. There is no *uncertainty* in the conclusions of mathematics; there should be no *guess-work* in its operations.

What then is the cause of so much *groping* and *fruitless effort* in this department of education? Why this *aimless, mechanical* "ciphering," that is so prevalent in our schools?

Many of these evils, it is believed, arise from the practice of requiring beginners to solve problems *above* their *comprehension*, and to learn abstract rules without *analysing* their *principles*, or *explaining* the *reasons* upon which they are based. Taking his slate and pencil, the pupil sits down to the solution of his problem, but soon finds himself involved in an impenetrable *maze.* He anxiously asks for light, and is directed "to learn the rule." He does this to the letter, but his mind is still in the dark. By *puzzling* and repeated *trials*, he at length finds that certain multiplications and divisions produce the answer in the book; but so far as the *reasons* of the process, and the *principles* of the rule are concerned, he is *totally ignorant.*

It needs no arguments to show that this course is calculated to *dampen the ardor* of a child, and make him a mechanical

cipherer. To require a pupil to *learn* and *understand* a rule before he is permitted to see its principles illustrated by simple practical examples, places him in the condition of the boy, whose mother charged him never to go into the *water* till he had *learned* to swim.

These embarrassments are believed to be as *unnecessary*, as they are *deleterious*. The present work was undertaken, with the hope of contributing something towards their removal. Its plan is the following:

1. The *definitions* are designed to be *simple*, *brief*, and *comprehensive*. If they are not simple, children can not *understand* them; if long, it is difficult to *remember* them; and if not *comprehensive*, they are not *worth remembering*.

2. The pupil is led to a knowledge of the rule by *induction*, a process by which he is taught to reason from *particular* examples to *general principles*. To this end the examples at the commencement of the rules are *practical*, and are *adapted* to illustrate the particular principles under consideration. Every teacher can bear testimony, that children reason upon *practical* questions with far greater *facility* and *accuracy*, than they do upon *abstract* numbers.

3. The separate principles being analyzed and understood, the general rule is then deduced, and arranged in a convenient manner for reference and review; thus combining the advantages both of the *inductive* and *synthetic* modes of instruction.

4. The rules, as far as possible, are constructed in such a manner as to *suggest* the *principles* upon which they depend; and the *reasons* for the various operations are *carefully given*.

5. The work abounds in *examples* for *practice*, which are drawn from the various departments of business and science, and are calculated to call into exercise the different principles of the rules, to wake up thought, and to prepare the learner for the active duties of life.

Their arrangement is *gradual* and *progressive*. At first, the numbers are small and refer to objects with which the pupil is acquainted, in order that he may clearly understand the nature of the question and the reason for every step in its solution. As he becomes familiar with the operation and the principles of the rule, the numbers are larger, and the combinations more *complicated and difficult*.

6. Mental exercises are frequently interspersed through the work, which, if properly attended to, are among the best means to *arrest* and *prevent* habits of mechanical ciphering.

7. In the arrangement of subjects, it has been a cardinal point to follow the *natural order* of the science. No principle is used in the explanation of another, until it has itself been demonstrated or explained. Common fractions, therefore, are placed immediately after division, for two reasons. *First*, they *arise* from division, and are *inseparably connected* with it. *Second*, in Reduction, Compound Addition, &c., it is frequently necessary to use fractions; consequently fractions must be explained before the Compound rules can be understood.

For the same reason, Federal Money, which is based upon the *decimal notation*, is placed after Decimal Fractions. Interest, Insurance, Commission, &c., are also placed after Percentage, upon whose principles they are based.

8. In preparing the tables of Weights and Measures, particular pains have been taken to ascertain those that are in *present use* in our country, and to give the *legal standards* as adopted by the General Government, in 1834.*

9. The subject of *Analysis* is deemed so essential to a thorough knowledge of arithmetic and business calculations, that an entire section is devoted to its development and application. The principles of *Cancellation* are carefully explained, and its important applications pointed out, in their proper places. The Square and Cube Roots are illustrated by geometrical figures and cubical blocks.

10. The work contains much valuable information respecting business transactions and matters of science, not found in other school Arithmetics.

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* Nearly twelve years had elapsed after the Government adopted a *uniform standard* of Weights and Measures, before the publication of the first edition of this work; and yet not a single arithmetic, so far as we know, had given these standards to the public, or even intimated that any thing had been done upon the subject. In the year 1836, Congress directed the Secretary of the Treasury to cause to be delivered to the Governor of each State in the Union, or to such person as he should appoint, a complete set of all the Weights and Measures adopted as standards, for the use of the States respectively; to the end that a *uniform standard* might be established throughout the United States. Since that, many of the States have adopted the same, and it is to be hoped that every State of the Union will promptly unite in the accomplishment of an object so conducive both to individual and public good.

With regard to the present edition, it may be remarked, that the stereotype plates having become so much impaired by use as to render it necessary to renew them, the author has availed himself of the opportunity thus presented, to revise the work, and make such additions as experience and the advancement of this department of instruction, have suggested. By condensing some parts and increasing the number of pages, an opportunity has been afforded for the introduction of much new and valuable matter. Among the improvements are the addition of many new examples, a more comprehensive view of Duties, Equation of Payments, Domestic and Foreign Exchange, Progression, &c. The articles are numbered the same as before, and the new examples are placed after those already given, so that the present may be used with the former editions, if desired.

Such is a brief outline of the present work. It is designed to present the elements of practical arithmetic in a *lucid* and *systematic* manner. It embraces, in a word, all the principles and rules which the business man has occasion to use, and is particularly adapted to precede the study of Algebra and the higher branches of mathematics.

In conclusion, the author begs leave to express his obligations to teachers and the public, for the very flattering manner in which his former works have been received. The circulation has far exceeded his most sanguine expectations, and it is hoped the present edition will be found worthy of continued favor.

J. B. THOMSON.

NEW YORK, March 4th, 1853

*** A KEY to this edition of the "Practical Arithmetic," containing an *Analysis* of the more difficult questions, the results of the several steps in the operation, etc., is published separately for the use of teachers, and private students.

SUGGESTIONS ON TEACHING ARITHMETIC.

I. Qualifications.—The chief qualifications requisite in teaching Arithmetic, as well as other branches, are the following:

1. A thorough knowledge of the subject.

2. A love for the employment.

3. An aptitude to teach. These are *indispensable to success.*

II. Classification.—*Arithmetic*, as well as reading, grammar, &c., should be taught in *classes.*

1. This method saves much time, and thus enables the teacher to devote more attention to *oral illustrations.*

2. The action of mind upon mind, is a *powerful stimulant* to exertion, and can not fail to create a *zest* for the study.

3. The mode of analyzing and reasoning of one scholar, often *suggests new ideas* to others in the class.

4. In the classification, those should be put together who possess as nearly equal capacities and attainments as possible. If any of the class learn quicker than others, they should be allowed to take up an extra study, or be furnished with additional examples to solve, so that the whole class may advance together.

III. Apparatus.—The *Blackboard* and *Numerical Frame* are as indispensable to teachers, as tables and cutlery to housekeepers.

Not a recitation passes without use for the blackboard. When a principle is to be demonstrated, or an operation explained, if done upon the *blackboard*, all can see and understand it at once.*

IV. Recitations.—The *first* object, in conducting a recitation, should be, to secure the *attention* of the class. This is done chiefly by throwing *life* and *variety* into the exercise. Children loathe dullness, while animation and variety are their delight.

3. Every example should be *analyzed*, the "why and the wherefore" of every step in the solution should be required, till the learner becomes perfectly familiar with the process of reasoning.

4. To ascertain whether every pupil has the right answer, it is an excellent method to name a question, then call upon some one to

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* Every one who ciphers, will of course have a slate. Indeed, it is desirable that every scholar in school, even to the very youngest, should be furnished with a small slate, so that when the little fellows have learned their lessons, they may busy themselves in writing and drawing various familiar objects. *Idleness* in school is the parent of *mischief*, and *employment* is the best antidote against *disobedience.*

Geometrical diagrams and *solids* are also highly useful in illustrating many points in arithmetic, and no school should be without them.

give the answer, and before deciding whether it is right or wrong, ask how many in the class agree with it. The answer they give by raising their hand, will show at once how many are right. The explanation of the process may now be made.

Another method is to let the class exchange slates with each other, and when an answer is decided to be right or wrong, let every one mark it accordingly. After the slates are returned to their owners, each one will correct his errors.

V. THOROUGHNESS.—The motto of every teacher should be *thoroughness.* Without it, the great ends of the study are *defeated.*

1. In securing this object, much advantage is derived from *frequent reviews.*

2. Not a recitation should pass without *practical exercises* upon the blackboard or slates, besides those assigned for the lesson.

3. After the class have solved the examples under a rule, they should be required to give an *account* of its principles with the *reason* for each step, either in their own language or that of the author.

4. *Mental Exercises* in arithmetic, are *exceedingly useful* in making ready and accurate arithmeticians, and should be *frequently* practiced.

VI. SELF-RELIANCE.—The *habit* of *self-reliance* in study, is confessedly *invaluable.* Its power is proverbial; I had almost said, *omnipotent.* "Where there is a *will,* there is a *way.*"

1. To acquire this habit, the pupil, like a child learning to walk, must be taught to *depend upon himself.* Hence,

2. When assistance is required, it should be given *indirectly;* not by taking the slate and solving the example for him, but by explaining the *meaning* of it, or illustrating the *principle* on which the operation depends, by supposing a more familiar case. In this way the pupil will be able to solve the question himself, and his eye will sparkle with the consciousness of victory.

3. Finally he must learn to solve examples *independent* of the answer. Without this attainment the pupil receives but little or no *discipline* from the study, and is *unfit* to be trusted with business calculations. What though he comes to the recitation with an occasional wrong answer; it were better to solve one question *understandingly* and *alone,* than to copy a *score* of answers from the book. What would the study of mental arithmetic be worth, if the pupil had the answers before him? What is a young man good for in the *counting-room,* who can not perform arithmetical operations without learning the answer?

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ARITHMETIC.

SECTION I.

ART. 1. ARITHMETIC *is the science of numbers.*

OBS. Arithmetic is sometimes regarded both as a *science* and an *art*; a science, because it explains the relations and properties of numbers; an art, because it shows how to apply them to the practical concerns of life.

2. Any single thing, as a peach, a rose, a book, is called a *unit*, or *one;* if another single thing is put with it, the collection is called *two;* if another still, it is called *three;* if another, *four;* if another, *five*, &c.

The terms, *one*, *two*, *three*, *&c.*, by which we express *how many single things* or *units* are under consideration, are the *names of numbers.* Hence,

3. NUMBER signifies a *unit*, or a *collection of units.*

Numbers are expressed by *words*, by *letters*, and by *figures.*

Note.—The questions on the observations may be omitted, by beginners, till review, if deemed advisable by the Teacher.

NOTATION.

4. NOTATION *is the art of expressing numbers by letters or figures.*

There are two methods of notation in use, the *Roman* and the *Arabic.*

QUESTIONS.—1. What is Arithmetic? 2. What is a single thing called? If another is put with it, what is the collection called? If another, what? What are the terms one, two, three, &c.? 3. What, then, is number? How are numbers expressed? 4. What is Notation? How many methods of notation are in use?

I. ROMAN NOTATION.

5. The *Roman Notation* is the method of expressing numbers by *letters;* and is so called because it was invented by the ancient *Romans.* It employs seven capital letters, viz: I, V, X, L, C, D, M. When standing alone, the letter I denotes *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* M, *one thousand.*

To express the intervening numbers from one to a thousand, or any number larger than a thousand, we resort to repetitions and various combinations of these letters, as seen in the following

TABLE.

I	denotes	one.	XXX	denote	thirty.
II	"	two.	XL	"	forty.
III	"	three.	L	"	fifty.
IV	"	four.	LX	"	sixty.
V	"	five.	LXX	"	seventy.
VI	"	six.	LXXX	"	eighty.
VII	"	seven.	XC	"	ninety.
VIII	"	eight.	C	"	one hundred.
IX	"	nine.	CI	"	one hundred and one.
X	"	ten.	CX	"	one hundred and ten.
XI	"	eleven.	CC	"	two hundred.
XII	"	twelve.	CCC	"	three hundred.
XIII	"	thirteen.	CCCC	"	four hundred.
XIV	"	fourteen.	D	"	five hundred.
XV	"	fifteen.	DC	"	six hundred.
XVI	"	sixteen.	DCC	"	seven hundred.
XVII	"	seventeen.	DCCC	"	eight hundred.
XVIII	"	eighteen.	DCCCC	"	nine hundred.
XIX	"	nineteen.	M	"	one thousand.
XX	"	twenty.	MM	"	two thousand.
XXI	"	twenty-one.	MDCCCXLV,	one thousand eight hundred and forty-five.	
XXII	"	twenty-two, &c.			

QUEST.—5. What is the Roman notation? Why called Roman? How many letters does it employ? What does each of these letters denote when standing alone? How are the intervening numbers from one to a thousand expressed? How express Two? Four? Six? Eight? Nine? Fourteen? Sixteen? Nineteen? Twenty-four? Twenty-eight? What does XL denote? LX? XC? *CX? CI? CIV?* CL? MD?

OBS. 1. The learner will perceive from the Table above, that every time a letter is repeated, its *value* is repeated. Thus I, standing alone, denotes *one;* II, *two ones* or *two*, &c. So X denotes *ten;* XX, *twenty*, &c.

2. When two letters of different value are joined together, if the less is placed before the greater, the value of the greater is *diminished* as many units as the less denotes; if placed after the greater, the value of the greater is *increased* as many units as the less denotes. Thus, V denotes five; but IV denotes only four; and VI, six. So X denotes ten; IX, nine; XI, eleven.

3. A line or bar (—) placed over a letter, increases its value a *thousand times.* Thus V denotes five, $\overline{V}$ denotes five thousand; X, ten; $\overline{X}$, ten thousand.

4. The Roman notation is chiefly used to denote chapters, sections, and other divisions of books and discourses.

In the early periods of this notation, four was written IIII, instead of IV; nine was written VIIII, instead of IX; forty was written XXXX, instead of XL, &c.

A *thousand* was originally written CIↃ, which, in later times, was changed into M; *five hundred* was written IↃ instead of D. Annexing Ↄ to IↃ increased its value ten times. Thus, IↃↃ denoted *five thousand;* IↃↃↃ, *fifty thousand*, &c.

II. ARABIC NOTATION.

6. The *Arabic Notation* is the method of expressing numbers by *figures;* and is so called because it is supposed to have been invented by the *Arabs.* It employs the following *ten characters* or *figures*, viz:

1	2	3	4	5	6	7	8	9	0
one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	naught.

The first nine are called *significant* figures, because each one always expresses a value, or denotes some number. They are also called *digits*, from the Latin word *digitus*, signifying a finger, because the ancients used to count on their fingers.

The last one is called *naught*, because when standing *alone*, it expresses *nothing*, or the *absence* of number. It is also called *cipher* or *zero.*

OBS. It must not be inferred, however, that the cipher is *useless;* for when placed on the right of any of the significant figures, it increases their value. It may therefore be regarded as an *auxiliary* digit, whose office, it will be seen hereafter, is as important as that of any other figure in the system.

QUEST.—*Obs.* What is the effect of repeating a letter? If a letter is placed before another of greater value, what is the effect? If placed after, what? When a letter has a line placed over it, how is its value affected? To what use is the Roman notation chiefly applied? 6. What is the Arabic notation? Why called Arabic? How many characters does it employ? What are the first nine called? Why? What else are they called? Why? What is the last one called? Why? *Obs.* Is the cipher useless? What may it be regarded?

7. It will be seen that *nine* is the greatest number that can be expressed by any *single figure* in the Arabic notation.

All numbers *larger* than nine are expressed by combining together two or more of these ten figures, and assigning different values to them, according to the different places which they occupy. For example, to express *ten*, we place the 0 on the right of the 1, thus 10; to express *eleven*, we use two 1s, thus 11; to express *twelve*, we place the 2 on the right of the 1, thus 12; to express *twenty*, we place the 0 on the right of the 2, thus 20; to express a *hundred*, we place two 0s on the right of the 1, thus 100, &c., as seen in the following

TABLE.

1, one.	28, twenty-eight.	55, fifty-five.
2, two.	29, twenty-nine.	56, fifty-six.
3, three.	30, thirty.	57, fifty-seven.
4, four.	31, thirty-one.	58, fifty-eight.
5, five.	32, thirty-two.	59, fifty-nine.
6, six.	33, thirty-three.	60, sixty.
7, seven.	34, thirty-four.	70, seventy.
8, eight.	35, thirty-five.	80, eighty.
9, nine.	36, thirty-six.	90, ninety.
10, ten.	37, thirty-seven.	91, ninety-one.
11, eleven.	38, thirty-eight.	92, ninety-two.
12, twelve.	39, thirty-nine.	93, ninety-three.
13, thirteen.	40, forty.	94, ninety-four.
14, fourteen.	41, forty-one.	95, ninety-five.
15, fifteen.	42, forty-two.	96, ninety-six.
16, sixteen.	43, forty-three.	97, ninety-seven.
17, seventeen.	44, forty-four.	98, ninety-eight.
18, eighteen.	45, forty-five.	99, ninety-nine.
19, uineteen.	46, forty-six.	100, one hundred.
20, twenty.	47, forty-seven.	200, two hundred.
21, twenty-one.	48, forty-eight.	300, three hundred.
22, twenty-two.	40, forty-nine.	400, four hundred.
23, twenty-three.	50, fifty.	500, five hundred.
24, twenty-four.	51, fifty-one.	600, six hundred.
25, twenty-five.	52, fifty-two.	700, seven hundred.
26, twenty-six.	53, fifty-three.	900, nine hundred.
27, twenty-seven.	54, fifty-four.	1000, one thousand.

QUEST.—7. What is the greatest number that can be expressed by one figure? How are larger numbers expressed? How express ten? Eleven? Twelve? Twenty? What is the greatest number that can be expressed by two figures? How express a hundred? What is the greatest number that can be expressed *by three figures*? How express a thousand?

OBS. 1. The terms *thirteen*, *fourteen*, *fifteen*, &c., are obviously derived from three and ten, four and ten, five and ten, which by contraction become thirteen, fourteen, fifteen, &c., and are therefore significant of the numbers which they denote. The terms *eleven* and *twelve*, are generally regarded as primitive words; at all events, there is no perceptible analogy between them and the numbers which they represent. Had the terms *oneteen* and *twoteen* been adopted in their stead, the names would then have been significant of the numbers one and ten, two and ten; and their etymology would have been similar to that of the succeeding terms.

2. The terms *twenty*, *thirty*, *forty*, &c., were formed from two-tens, three-tens, four-tens, which were contracted into twenty, thirty, forty, &c.

3. The terms *twenty-one*, *twenty-two*, *twenty-three*, &c., are compounded of twenty and one, twenty and two, &c. All the other numbers as far as ninety-nine are formed in a similar manner.

4. The terms *hundred* and *thousand* are primitive words, and bear no analogy to the numbers which they denote. The numbers between a hundred and a thousand are expressed by a repetition of the numbers below a hundred. Thus we say, one hundred and one, one hundred and two, one hundred and three, &c.

8. It will be seen from the preceding table, that *the same figures* standing in different places, express *different values.*

When standing *alone* or in the *right hand* place, the figure 1 denotes a *single thing* or *one*, which is called a unit of the *first order*, or simply a *unit.* So the other digits, 2, 3, 4, 5, &c., standing alone or in the right hand place, denote *single things* or *ones*, which are also called units of the *first order*, or simply *units.*

When standing in the *second* place, (10), the 1 denotes one *ten*, which is called a unit of the *second order*. Now this 1 ten or unit of the *second order*, is equal to *ten ones*, or *ten units* of the first order. That is, when the 1 stands in the second place, it denotes *ten times* as many single things or ones, as when standing in the first, or right hand place. So the other digits, 2, 3, 4, 5, &c., occupying the second place, denote *tens* or units of the *second order*, as 2 tens, 3 tens, 4 tens; and the value of each is *ten times* as much as when it occupies the right hand place.

QUEST.—*Obs.* From what is the term thirteen derived? Fourteen? Sixteen? Eighteen? What is said of the terms eleven and twelve? How are the terms twenty, thirty, &c., formed? What is said of the terms hundred, and thousand? How are the numbers between a hundred and a thousand expressed? 8. Does the same figure always express the same value? What does each of the digits 1, 2, 3, &c., denote, when standing in the right hand place? What are these units or ones called? What do they denote when standing in the second place? What are tens called?

When standing in the *third* place, (100), the 1 denotes one *hundred*, which is called a unit of the *third order*. This 1 hundred or unit of the *third order*, is equal to *ten* tens, or ten units of the *second order;* consequently its value is *ten times* as much as when it stood in the second place. So the other digits standing in the third place, denote *hundreds* or units of the *third order*, as 2 hundreds, 3 hundreds, 4 hundreds, &c., and their value is *ten times* as much as when they occupy the second place.

Again, when standing in the *fourth* place, (1000), the 1 denotes 1 *thousand*, which is called a *unit* of the *fourth order*. This 1 thousand or unit of the *fourth order*, is equal to *ten* hundreds, or ten units of the *third order;* therefore its value is *ten times* as much as when it stood in the third place. The same is true of the other digits. That is,

Ten *units* make one *ten;*

Ten *tens* make one *hundred;*

Ten *hundreds* make one *thousand*, &c. Hence, universally,

9. *Ten* of any *lower* order is equal to *one* in the next *higher* order. It is therefore a general law that,

Numbers increase from right to left by tens, or in a tenfold ratio; consequently each removal of a figure one place towards the left, increases its value ten times.

10. The different values which the same figures express, are called *simple* and *local* values.

The *simple* value of a figure is the number which its name denotes when it stands alone, or in the right hand place.

The *local* value of a figure is the *increased* value which it expresses, when it has other figures placed on its right. Hence,

---

QUEST.—What do they denote when standing in the third place? What are hundreds called? When standing in the fourth place what do they denote? What are thousands called? How many units make one ten? How many tens make a hundred? How many hundreds make a thousand? 9. How many of any lower order make one of the next higher? What is the general law by which numbers increase? What is the effect upon the value of a figure to remove it one place towards the left? 10. What are the different values of the same figure called? What is the simple value of a figure? What the local value?

The local value of a figure depends on its locality, or the place which it occupies in relation to other figures with which it is connected, counting from the right. (Art. 8.)

Obs. 1. The *reason* for assigning different values to the same figures according to the place which they occupy, is to enable us to express large numbers intelligibly, and at the same time with few characters. Otherwise we must have as many different characters as we have different numbers to express, and the labor of learning them would be greater than that of learning the whole English language.

2. The Arabic notation is also called the *decimal* notation, because its orders increase in a tenfold ratio. The term *decimal* is derived from the Latin word *decem*, which signifies ten.

NUMERATION.

11. Numeration *is the art of reading numbers expressed by figures.*

Obs. Numeration bears the same relation to Notation, that *reading* does to *writing*; though often confounded, they are entirely distinct.

The pupil has already become acquainted with the names of numbers, from one to a thousand. He will now easily learn to read and express the higher numbers in common use, from the following scheme, called the

NUMERATION TABLE.

Hundreds of Quadrillions.	Tens of Quadrillions.	*Quadrillions.*	Hundreds of Trillions.	Tens of Trillions.	*Trillions.*	Hundreds of Billions.	Tens of Billions.	*Billions.*	Hundreds of Millions.	Tens of Millions.	*Millions.*	Hundreds of Thousands.	Tens of Thousands.	*Thousands.*	Hundreds.	Tens.	*Units.*
5	6	8,	3	4	2,	9	7	5,	8	9	7,	6	4	5,	4	3	2.
Period VI. Quadrillions.			Period V. Trillions.			Period IV. Billions.			Period III. Millions.			Period II. Thousands.			Period I. Units.		

Quest.—Upon what does the local value of a figure depend? *Obs.* What is the Arabic notation sometimes called? Why? 11. What is Numeration? Repeat the Numeration Table, beginning at the right hand. What occupies the first place on the right? The second place? The third? Fourth? Fifth? Sixth? Seventh? Eighth? Ninth? Tenth, &c.?

12. The different orders of numbers are divided into *periods* of three figures each, *beginning* at the *right hand.*

The *first period* on the right, is called *units' period*, because it is occupied by units, tens of units, and hundreds of units.

The *second* is called *thousands' period*, because it is occupied by thousands, tens of thousands, and hundreds of thousands, as may be seen from the table above.

The figures in the table are read thus: Five hundred and sixty-eight *quadrillions*, three hundred and forty-two *trillions*, nine hundred and seventy-five *billions*, eight hundred and ninety-seven *millions*, six hundred and forty-five *thousand*, four hundred and thirty-two. Hence,

13. To read numbers expressed by figures.

First, point them off into periods of three figures each, counting from the right.

Then, beginning at the left hand, read the figures of every period as though it stood alone, and to the last figure of each, add the name of the period.

OBS. 1. In *pointing off* figures, the learner must be careful to begin at the *right* hand; and in *reading* them, to begin at the *left* hand.

2. Since the figures in the first or right hand period always denote units, the name of the period is not pronounced. Hence, in reading figures, when no period is mentioned, it is always understood to be the right hand, or units' period.

14. The method of dividing numbers into periods of *three* figures, as in the preceding articles, is called the *French* Numeration, because it was invented by the French.

The *English* divide numbers into periods of *six* figures. The French method is the more simple and convenient. It is generally used on the continent of Europe, as well as in America, and has been recently adopted by some English authors.

QUEST.—12. How are the orders of numbers divided? What is the first period called? Why? What is the second called? Why? What is the third called? Why? What is the fourth called? Why? What is the fifth called? Why? 13. How do you read numbers expressed by figures? Do you pronounce the name of the right hand period? When no period is named, what is understood? 14. In the French numeration, how many figures are there in a period? How many in the English method? Which method is preferable?

EXERCISES IN NUMERATION.

Note.—In reading large numbers, beginners should at first point to each figure, and pronounce its name. Thus, beginning at the right hand, he should say, "Units, tens, hundreds, thousands," &c. It will be a profitable exercise for young scholars to write the examples upon their slates or paper, then point them off into periods, and read them.

Read the numbers expressed by the following figures:

Ex. 1.	127	11.	75407	21.	5604700
2.	172	12.	125242	22.	2020105
3.	721	13.	240251	23.	45001003
4.	520	14.	407203	24.	30407045
5.	603	15.	300200	25.	145560800
6.	4506	16.	1255673	26.	8900401
7.	7045	17.	5704086	27.	250708590
8.	8700	18.	207047	28.	803068003
9.	25008	19.	2605401	29.	2175240670
10.	40625	20.	4040680	30.	7240305060

31.	45290100300	36.	13657240129698
32.	160000050000	37.	98609006006906
33.	7005003007	38.	80079401697000
34.	101279200361	39.	167540000000465
35.	1143206000675	40.	504069470300400

EXERCISES IN NOTATION.

15. To express numbers by figures.

BEGIN *at the left hand of the highest period, and write the figures of each period as though it stood alone.*

If any intervening order, or period, is omitted in the given number, write ciphers in its place.

Write the following numbers in figures:

1. Twenty-seven.
2. Seventy-two.
3. One hundred and twenty-five.
4. Three hundred and fifty-two.
5. Two hundred and four. *Ans.* 204.

QUEST.—15. How do you express numbers by figures? If any intervening order, or period is omitted in the given number, how is its place supplied?

6. One thousand, and forty-two.
7. Thirty thousand, nine hundred and seven.
8. Forty-six thousand, and four hundred.
9. Ninety-two thousand, one hundred and eight.
10. Sixty-eight thousand, and seventy.
11. One hundred and twenty-four thousand, six hundred and thirty.
12. Two hundred thousand, one hundred and sixty.
13. Four hundred and five thousand, and forty-five.
14. Three hundred and forty thousand.
15. Nine hundred thousand, seven hundred and twenty.
16. One million, and seven hundred thousand.
17. Thirty-six millions, twenty thousand, one hundred and fifty.
18. One hundred millions, and forty-five.
19. Mercury is thirty-seven millions of miles from the sun.
20. Venus, sixty-nine millions.
21. The Earth, ninety-five millions.
22. Mars, one hundred and forty-five millions.
23. Jupiter, four hundred and ninety-four millions.
24. Saturn, nine hundred and seven millions.
25. Herschel, one billion, eight hundred and ten millions.
26. Seven billions, nine hundred millions, and forty thousand.
27. Sixty billions, seven millions, and four hundred.
28. One hundred and thirteen billions, six hundred and fifty thousand.
29. Four hundred and six billions, eighty millions, and seven hundred.
30. Twenty-five trillions, and ten thousand.
31. Two hundred and six billions, five hundred and sixty thousand, and forty-five.
32. Six hundred millions, seventeen thousand, three hundred and eight.
33. Ninety-seven trillions, sixteen millions, seventy thousand, and thirty.
34. Eight hundred and forty billions, fifty millions, three hundred and one thousand.
35. Three hundred and sixty-five quadrillions, two hundred trillions, six hundred and ninety billions, seven millions, three thousand and six.

SECTION II.

ADDITION.

MENTAL EXERCISES.

ART. **16.** Ex. 1. George bought a slate for 9 cents, a sponge for 6 cents, and a pencil for 1 cent: how many cents did he pay for all?

Solution.—9 cents and 6 cents are 15 cents, and 1 cent more makes 16 cents. He therefore paid 16 cents for all.

ADDITION TABLE.

2 and	3 and	4 and	5 and	6 and	7 and	8 and	9 and
1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10
2 " 4	2 " 5	2 " 6	2 " 7	2 " 8	2 " 9	2 " 10	2 " 11
3 " 5	3 " 6	3 " 7	3 " 8	3 " 9	3 " 10	3 " 11	3 " 12
4 " 6	4 " 7	4 " 8	4 " 9	4 " 10	4 " 11	4 " 12	4 " 13
5 " 7	5 " 8	5 " 9	5 " 10	5 " 11	5 " 12	5 " 13	5 " 14
6 " 8	6 " 9	6 " 10	6 " 11	6 " 12	6 " 13	6 " 14	6 " 15
7 " 9	7 " 10	7 " 11	7 " 12	7 " 13	7 " 14	7 " 15	7 " 16
8 " 10	8 " 11	8 " 12	8 " 13	8 " 14	8 " 15	8 " 16	8 " 17
9 " 11	9 " 12	9 " 13	9 " 14	9 " 15	9 " 16	9 " 17	9 " 18
10 " 12	10 " 13	10 " 14	10 " 15	10 " 16	10 " 17	10 " 18	10 " 19

Note.—It is indispensable to accuracy both in *arithmetic* and *business*, to have the common arithmetical tables *distinctly* and *indelibly* fixed in the mind. Great care should therefore be taken to prevent them from being recited *mechanically*, or from a knowledge of the regular increase of numbers.

11. How many are 12 and 10? 22 and 10? 32 and 10? 42 and 10? 52 and 10? 62 and 10? 72 and 10? 82 and 10? 92 and 10?

12. How many are 24 and 10? 36 and 10? 48 and 10? 53 and 10? 67 and 10? 91 and 10? 86 and 10? 78 and 10? 69 and 10? 97 and 10?

13. How many are 19 and 4? 29 and 4? 39 and 4? 79 and 4? 59 and 4? 89 and 4? 99 and 4? 69 and 4? 49 and 4?

14. How many are 17 and 8? 27 and 8? 47 and 8? 67 and 8? 57 and 8? 97 and 8? 87 and 8?

15. How many are 16 and 7? 26 and 7? 56 and 7? 86 and 7? 76 and 7? 96 and 7?

16. How many are 14 and 6? 24 and 6? 84 and 6? 74 and 6? 54 and 6? 64 and 6? 94 and 6?

17. Add 2 to itself, till the sum is a hundred.

18. Add 3 to itself, till the sum is a hundred and two.

19. Add 5 to itself, till the sum is a hundred and ten.

20. Add 4 to itself, till the sum is a hundred and twelve.

21. Add 10 to itself, till the sum is a hundred and twenty.

22. A man bought a sheep for 3 dollars, a cow for 21 dollars, and a calf for 5 dollars: how much did he pay for the whole?

23. A shopkeeper sold a dress to a lady for 15 dollars, a muff for 10 dollars, and a bonnet for 6 dollars: what was the amount of her bill?

24. A drover bought 16 sheep of one farmer, 9 of another, 10 of another, and 6 of another: how many sheep did he buy of all?

25. Harry gave 31 cents for his arithmetic, 10 cents for a writing-book, 8 cents for a ruler, and 6 cents for a lead pencil: how many cents did he pay for all?

26. What is the sum of 10 and 12 and 5 and 4?

27. William bought a pair of boots for 26 shillings, and a cap for 9 shillings: how much did he pay for both?

28. Susan bought a comb for 17 cents, a purse for 8 cents, and a spool of cotton for 5 cents; how much did she pay for all?

29. A farmer sold a ton of hay for 18 dollars, a cow for 10 dollars, and a cord of wood for 3 dollars: how much did they all amount to?

30. A merchant sold 15 barrels of flour to one man, 5 to another, and 7 to another: how many barrels of flour did he sell?

31. In a certain school there are 60 boys and 30 girls: how many scholars does that school contain?

Analysis.—60 is the same as 6 tens, and 30 the same as 3 tens; now 6 tens and 3 tens are 9 tens, and 9 tens are 90. Therefore the school contains 90 scholars.

32. A mechanic sold a wagon for 30 dollars, and a sleigh for 20 dollars: how much did he get for both?

33. 40 is how many tens? 60? 20? 30? 70? 80? 50? 90? 100?

34. 7 tens and 2 tens are how many? 7 tens and 4 tens?

35. 6 tens are how many? 8 tens? 9 tens? 10 tens? 11 tens? 12 tens? 13 tens? 14 tens? 15 tens? 16 tens? 17 tens? 18 tens? 19 tens? 20 tens?

36. 8 tens and 3 tens are how many? 5 tens and 8 tens? 7 tens and 8 tens? 6 tens and 9 tens? 9 tens and 8 tens? 10 tens and 6 tens?

37. In a certain orchard there are 80 apple-trees, and 40 peach-trees: how many trees does it contain?

38. A traveler rode 90 miles in the cars, and 60 miles in stages: how many miles did he travel?

39. A man gave 60 dollars for his horse, 30 dollars for his harness, and 20 dollars for his cart: how much did he pay for all?

40. A man bought a horse for 98 dollars, and a wagon for 65 dollars: how much did he give for both?

Analysis.—98 is composed of 9 tens and 8 units, and 65 is composed of 6 tens and 5 units. (Art. 7. Obs. 3.) 9 tens and 6 tens are 15 tens, or 1 hundred and 5 tens; 8 units and 5 units are 13 units, or 1 ten and 3 units; now 1 ten added to 5 tens, makes 6 tens or 60, and 3 units are 63, which, added to the hundred, makes 163. He therefore paid 163 dollars.

OBS. In adding large numbers mentally, it is more convenient and expeditious to begin with the highest order.

41–45. How many are 68 and 25? 56 and 23 and 5? 83 and 72 and 4 and 6? 72 and 25 and 10 and 2? 63 and 24 and 12 and 10 and 7?

46. Bought a pound of tea for 60 cents, an ounce of pepper for 8 cents, and a quart of molasses for 10 cents: what does my bill amount to?

47. The price of a geography is 55 cents, and the price of a grammar is 42 cents: what is the cost of both?

48. Paid 7 dollars for a barrel of flour, 17 dollars for a ton of hay, and 30 dollars for a cow: what is the cost of all?

49. In January there are 31 days, and in February 28 days: how many days are there in both months?

50. A man, having three sons, gave 50 dollars to the oldest, 40 dollars to the second, and 30 dollars to the youngest: how many dollars did he give to the three?

EXERCISES FOR THE SLATE.

ART. **17.** In each of the preceding examples it will be observed, we have *two* or *more numbers* given, and from these it is required to find a *single* number, which is equal to the several given numbers *united* together. The operation by which this number is found, is called *Addition.* Hence,

18. ADDITION *is the process of uniting two or more numbers in one sum.*

The *answer*, or number *obtained* by addition, is called the *sum*, or *amount.*

OBS. When all the numbers to be added are of the *same kind*, or *denomination*, as all books, all yards, &c., the operation is called *Simple Addition.*

18. *a.* SIGNS. The relations of numbers, and the operations which are performed with them, are often denoted by certain characters, called *signs.*

19. *The sign of addition* is a *perpendicular cross* (+), called *plus*, and shows that the numbers between which it is placed, are to be added together. Thus, the expression 6+8, signifies that 6 is to be added to 8. It is read, "6 plus 8," or "6 added to 8."

Note.—The term *Plus* is a Latin word, originally signifying *more.* In Arithmetic, it means *added to.*

20. *The sign of equality* is *two horizontal lines* (=), and shows that the numbers between which it is placed, are *equal* to each other. Thus, the expression 5+3=8, denotes that 5 added to 3 are equal to 8. It is read, "5 plus 3 equal 8," or "the sum of 5 plus 3 is equal to 8." So 7+5=8+4=12.

Read the following expressions: 3+4+1=3+5.
17+3+12=15+7+10. 13+6+2+3=7+5+12.
25+6+17+3=26+3+2+20. 36+9+5=24+8+3+15.
65+10+12+20+16+41+7=40+35+15+17+25+39.

QUEST.—18. What is addition? What is the answer called? *Obs.* When the numbers to be added are all of the same denomination, what is the operation called? 18. *a.* How are the relations of numbers and their operations sometimes denoted? 19. What is the sign of addition? What does it show? *Note.* What is the meaning of the term plus? 20. What is the sign of equality? What does it show? How is the expression 5+3=8, read? How, 7+5=8+4=12?

CASE I.—*When the sum of a column does not exceed* 9.

ART. 21. Ex. 1. A man bought 436 pounds of tea, and 253 pounds of coffee: how many pounds of both did he buy?

Suggestion.—Write the numbers under each other, *units* under *units*, *tens* under *tens*, &c., and draw a line beneath them, as in the margin. Then, beginning at the right hand, proceed in the following manner: 3 units and 6 units are 9 units. Set the 9 in units' place under the column added *because it denotes units.* (Art. 8.) Next, 5 tens and 3 tens are 8 tens. Write the 8 in tens' place, *because it denotes tens.* Finally, 2 hundreds and 4 hundreds are 6 hundreds. Write the 6 in hundreds' place, *because it denotes hundreds.* He therefore bought 689 pounds of both.

Operation.

	hund.	tens	units	
	4	3	6	tea
	2	5	3	cof.
Ans.	6	8	9	pounds.

22. In the solution above it is important to observe, that *units* are added to *units*, *tens* to *tens*, &c. Hence, universally,

Figures of the same order must be added to each other.

The *reason* is, that figures of *different* orders express *units* of different *values;* consequently, if added together, the *amount* would neither be of one order nor another. (Art. 8.) Thus, 3 units and 3 tens will neither make *six units*, nor *six tens*, any more than 3 apples and 3 oranges will make 6 apples, or 6 oranges. In like manner, it is plain that 5 tens and 4 hundreds will neither make 9 tens, nor 9 hundreds.

OBS. The object of writing *units* under *units*, *tens* under *tens*, &c., is to prevent mistakes which might occur from adding *different* orders to each other.

Solve the following examples in a similar manner:

2. A butcher purchased two droves of sheep, the first containing 436, and the second 243: how many sheep did both droves contain? *Ans.* 679 sheep.

3. A man found two purses of money, one containing 425 dollars, the other 361 dollars: how many dollars did both purses contain?

QUEST.—21. Explain the solution of the first example from your slate. 22. What orders of figures do you add together? Why not add figures of different orders to each other?

4. A man bought two tracts of wild land, one containing 3261 acres, and the other, 5428 acres: how many acres of land did he buy?

(5.)	(6.)	(7.)	(8.)
45436	420261	3021040	730043000
12321	231204	5630721	268900483

(9.)	(10.)	(11.)	(12.)
221	4212	62022	82202310
345	3120	5103	3060231
422	341	21640	617403

CASE II.—*When the sum of a column exceeds* 9.

ART. **23.** Ex. 13. A man bequeathed 5876 dollars to his oldest child, 4629 dollars to the second, and the balance of his estate, which was 3548 dollars, to the youngest: what was the amount of his property?

Suggestion.—Having set down the numbers, and added the units' column as before, we find the sum is 23. Now 23 units are equal to 2 tens and 3 units, for every *ten* in a *lower* order makes *one* in the next *higher*. (Art. 9.) We therefore set the 3 or right hand figure under the units, and reserving the 2 or left hand figure, add it to the next column. Thus, 2 tens (which were reserved) added to 4 make 6 tens, and 2 are 8, and 7 are 15 tens, which are equal to 1 hundred and 5 tens. Set the 5 or right hand figure under the column added, and add the 1 or left hand figure to the next column as before. Now 1 hundred added to 5 makes 6 hundreds, and 6 are 12, and 8 are 20 hundreds, which are equal to 2 thousands and 0 hundreds. Set the 0 or right hand figure under the column added, and add the 2 to the next column. In like manner we find the sum of the thousands' column is 14; and as this is the last column, we set down the *whole sum*. Therefore the amount of his property was 14053 dollars.

Operation.

5876
4629
3548
14053 *Ans.*

QUEST.—23. Describe the solution of the 13th example. 24. What is meant *by carrying the tens?*

24. The process of reserving the tens or left hand figure, when the sum of a column exceeds 9, and adding it to the next column, is called *carrying tens.*

25. When the sum of a column *exceeds* 9, the *reason* we set the *units*, or *right hand* figure under the column added, is *because* it is *the same order as that column.*

The *reason* for carrying the *tens* or *left-hand* figure to the next column, is *because it is the same order as that column;* and figures of the *same order* must always be added to each other. (Art. 22.)

26. The *principle* of *carrying* may perhaps be better understood by the following illustration. Take the last example, and set the sum of each column in a separate line; then adding these results together as they stand, units to units, tens to tens, &c., the amount is 14053 dollars, which is the same result as in the solution above.

5876	
4629	
3548	
23	units.
13	tens.
19	hunds.
12	thous.
14053	dolls.

Now we have seen that *ten* of any lower order make *one* of the next higher, (Art. 9.) Hence, when the sum of a column exceeds 9, it must contain *one* or *more* units of the next higher order. But it is manifest that these units of the next higher order which are denoted by the left hand figure, must be added to the next column, that the amount of the several given numbers may be expressed in the highest orders, and by a single number, which is the object of the rule. (Arts. 17, 18.)

OBS. 1. The *reason* we carry *one* for every *ten*, instead of one for every *eight*, or *twelve*, or *any other* number, is because numbers increase in a *ten-fold ratio.* If they increased in an *eight-fold* ratio, we should carry one for every *eight;* if in a *twelve-fold* ratio, we should carry one for every *twelve;* and universally, we must carry *one* for that number which it takes of a lower order to make a unit of the next higher.

2. The *reason* for setting down the *whole sum* of the last or left hand column, is because there are no figures in the next order to which the left hand figure can be added. It is, in effect, carrying it to the next column.

3. The object of beginning to add at the right hand, is that we may *carry the tens*, as we proceed in the operation. The result will obviously be the same, whether we carry them as we proceed, or reserve them until afterwards, and then add them to their appropriate orders. The *former* method is the more *convenient* and *expeditious*, and is therefore adopted in practice.

29. From the preceding illustrations and principles, we derive the following

GENERAL RULE FOR ADDITION.

I. *Write the numbers to be added one under another, so that units may stand under units, tens under tens, &c.*

II. *Beginning at the right hand, add each column separately, and if the sum of a column does not exceed* 9, *write it under the column added. But if the sum of a column exceeds* 9, *write the units' figure under the column, and carry the tens to the next column.* (Arts. 24, 25.)

III. *Proceed in this manner through all the orders, and finally set down the whole sum of the last or left hand column.*

PROOF.—*Beginning at the top, add each column downward, and if the second result is the same as the first, the work is supposed to be right.*

OBS. The *reason* for beginning at the top and adding downward, is that the figures may be taken in a different connection from that in which they were added before. The order being reversed, the presumption is, that any mistake which may have been made in the operation, will thus be detected; for it can hardly be supposed that two mistakes exactly equal will occur.

Note 1.—After the pupil understands the *nature* and *reason* of the several steps in the operation, it is advisable for him to learn to drop the intervening words, and pronounce, with rapidity, the results of adding the respective figures. Thus, instead of saying 6 units and 3 units are 9 units, and 4 units are 13 units, &c., he should simply say, *six, nine, thirteen, twenty-one,* &c. (See Ex. 68.)

2. When two or more figures coming together make 10, as 7 and 3, or 2, 3, and 5, it accelerates the process to consider them 10, and add the sum at once.

15. Find the sum of 478 and 987, and prove the operation.

(16.)	(17.)	(18.)	(19.)
4674	67375	84056	405673
6206	87649	5721	720021
4821	6048	41630	869115
8569	452	168	505181

QUEST.—29. How do you write numbers for addition? When the sum of a column does not exceed 9, how do you proceed? When it exceeds 9, how? 25. Why set the units' figure under the column added? Why carry the tens to the next column? 26. *Obs.* Why carry 1 for every 10, instead of 1 for every 8, 12, or any other number? Why do you set down the whole sum of the last column? Does this differ from carrying? 22. *Obs.* Why place units under units, tens under tens, &c. 26. *Obs.* Why begin to add at the right hand or units' column? *How is* addition proved? *Obs.* Why begin at the top and add downward?

20. Find the sum of 256, 763, and 894.
21. Find the sum of 8054, 5730, and 3056.
22. Find the sum of 74502, 83000, and 62581.
23. Find the sum of 68056, 31067, 680, and 200.
24. Find the sum of 50563, 8276, 75009, 31, and 856.
25. Find the sum of 65031, 2900, 35221, and 870.

EXAMPLES FOR PRACTICE.

1. A man bought a quantity of flour for 38 dollars, a ton of hay for 14 dollars, and a firkin of butter for 12 dollars. How much did he give for the whole?

2. A grocer bought three lots of honey; the first contained 322 pounds, the second 215, and the third 429 pounds. How many pounds were there in all?

3. A man being asked his age, answered that it was double the united ages of his three grand-children, the oldest of whom was 18, the second 16, and the third 14 years old. What was his age?

4. A man bought 5 hogsheads of molasses for 238 dollars, 7 hogsheads for 463 dollars, and sold it all so as to gain 275 dollars. How much did he sell it for?

5. A lady purchased materials for 3 dresses; for the first she paid 15 dollars, for the second, 19 dollars, and for the third, 27 dollars, and a cloak for 48 dollars. How much did she pay for them all?

6. A boy bought a cap for 22 shillings, a pair of gloves for 6 shillings, a pair of boots for 36 shillings, and a vest for 42 shillings. How much did he give for the whole?

7. A gentleman owns 3 houses; for the first he receives a rent of 150 dollars, for the second 175, and for the third 225 dollars. What is the sum of all his rents?

8. A shopkeeper commenced business with 1530 dollars; the first year he gained 950 dollars, the second, 867 dollars. How much had he then?

9. A man bought a horse for 87 dollars, a carriage for 275 dollars, a harness for 68 dollars, and his barn cost as much as all the rest. How much did he give for the whole?

10. What number of dollars are there in four purses; the first containing 325 dollars, the second 673, the third 784, and the fourth 596 dollars?

11. A poor man having lost his house by fire, received the following donations: 425 dollars, 315 dollars, 510 dollars, 538 dollars, and 376 dollars. How much did he receive in all?

12. In a certain school there are three departments; the first contains 768 scholars, the second 511, and the third 614. How many scholars attend that school?

13. A merchant, on closing his business for the day, found he had received 423 dollars from one customer, 657 from another, 531 from another, and 205 from various others. How much did he receive that day?

14. A laborer in pursuit of employment, walked 27 miles the first day, 20 the second, 32 the third, 35 the fourth, and 29 the fifth day. How far had he then walked?

15. A man, owning a large farm, gave to one of his sons 112 acres, to another 123, to the third 147, and had 200 acres left. How large was his farm at first?

16. A man bought a cask of oil for 230 dollars, and sold it so as to treble his money. How much did he sell it for?

17. A lad bought a geography for 50 cents, a grammar for 75 cents, an arithmetic for 83 cents, and a slate for 30 cents. How much did he give for them all?

18. A gentleman purchased carpets for 638 dollars, chairs for 236 dollars, bureaus for 315 dollars, and tables for 212 dollars. What did his bill amount to?

19. A merchant had 4 notes; one for 157 dollars, another for 368, another for 576, and another for 1687 dollars. What was the whole amount of his notes?

20. A gentleman bought a cloak for 56 dollars, a coat for 25 dollars, a vest for 9 dollars, a hat for 7 dollars, and a wardrobe for 167 dollars. What did he give for the whole?

21. A fashionable lady purchased a cashmere shawl for 469 dollars, a watch for 237 dollars, a pocket handkerchief for 87 dollars, and a bonnet for 53 dollars. What was the amount of her bill?

22. A farmer had 375 sheep and 168 lambs in one pasture, in another 379 sheep and 197 lambs. How many sheep had he? How many lambs? How many sheep and lambs together?

23. Four men entered into partnership; one furnished 2878

dollars, another 1784 dollars, a third 1265 dollars, and the fourth 894 dollars. What was the amount of their stock?

24. A man sold three house lots; for one he received 975 dollars, for another 763 dollars, and for the third 586 dollars. What did the whole amount to?

25. A gentleman purchased a store for 4500 dollars, and paid 75 dollars for repairs, and 150 dollars for having it enlarged. For how much must he sell it, in order to gain 175 dollars?

26. A gentleman paid 375 dollars for one piece of cloth, 467 dollars for another, 254 dollars for another, and 348 dollars for another. How much did he pay for all?

27. A certain orchard contains 256 apple-trees, 119 peach-trees, 83 plum-trees, and 45 cherry-trees. How many trees are there in the orchard?

28. The distance from New York to Albany is 150 miles, from Albany to Utica 93 miles, from Utica to Rochester 158 miles, and from Rochester to Buffalo 75 miles. How far is it from New York to Buffalo?

29. A man being asked his age, said he was 17 years old when he left the academy, he spent 4 years in college, 3 years in a law school, practiced law 15 years, was a member of congress 18 years, and it was 16 years since he retired from business. How old was he?

30. A shopkeeper having a note due, paid 184 dollars at one time, at another 268 dollars, at another 379 dollars, at another 467 dollars, and there were 350 dollars still unpaid. What was the amount of his note?

31. A gentleman owns a house and lot worth 10800 dollars, a store worth 5450 dollars, a house-lot worth 3700 dollars, and has 15000 dollars in personal property. What is the whole amount of his property?

32. A man left his estate to his wife, his three sons, and two daughters; to his wife he gave 10350 dollars, to his sons 5450 dollars apiece, and his daughters 3500 dollars apiece. How large was his estate?

33. A merchant, on looking over his accounts, found he owed one man 750 dollars, another 648, another 597, another 486, another 379, and another 287 dollars. What was the amount of his debts?

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; sixty millions, sixty thousand and

34. A man bought a span of horses for 275 dollars, a carriage for 150 dollars, and a harness for 87 dollars. How much did he give for the whole?

35. A man bought 268 bushels of wheat for 287 dollars, 187 bushels of corn for 98 dollars, and 156 bushels of oats for 128 dollars. How many bushels of grain did he buy; and how much did he give for the whole?

36. A man wishing to stock his farm, paid 197 dollars for a span of horses, 86 dollars for a yoke of oxen, 175 dollars for cows, and 169 dollars for sheep. How much did he give for the whole?

37. A butcher sold to one customer 157 pounds of meat, to another 159, to another 149, to another 97, and to another 68 pounds. How much did he sell to all?

38. A carpenter received 879 dollars for one job, for another 786, for another 693, for another 587, for another 476, and for another 368 dollars. How much did he receive in all?

39. A grocer bought 375 dollars worth of sugar, 287 dollars worth of molasses, 168 dollars worth of tea, 158 dollars worth of coffee, and 137 dollars worth of spices. What was the amount of his bill?

40. A merchant bought calico to the amount of 568 dollars, silks to the amount of 479 dollars, and broadcloths to the amount of 784 dollars. He sold them so as to gain 134 dollars on the calico, 178 dollars on the silks, and 242 dollars on the broadcloths. How much did he sell them for; and what was the amount of his gains?

41. A merchant pays 560 dollars a year for store rent, 386 dollars to one clerk, 267 to another, and 369 dollars for various other expenses. What does it cost him a year to carry on his business?

42. A man receives 568 dollars rent for one store, 479 for another, and 276 for another. How much does he receive for them all?

43. The distance from Boston to Springfield is 98 miles, from Springfield to Pittsfield is 53 miles, from Pittsfield to Albany is 49 miles, from Albany to Auburn is 173 miles, and from Auburn to Buffalo is 152 miles. How far is it from Boston to Buffalo?

44. A man bought a quantity of oil for 2649 dollars, and

candles for 1367 dollars; he afterwards sold them so as to gain 568 dollars on the oil, and 346 dollars on the candles. How much did he receive for the whole?

45. In 1840, the state of Maine contained 501793 inhabitants; New Hampshire, 284574; Vermont, 291948; Massachusetts, 737699; Connecticut, 309978; and Rhode Island, 108830. What was the population of New England?

46. In 1840, the state of New York contained 2428921 inhabitants; New Jersey, 373306; Pennsylvania, 1724033; and Delaware, 78085. What was the population of the Middle States?

47. In 1840, the state of Maryland contained 470019 inhabitants; Virginia, 1239797; North Carolina, 753419; South Carolina, 594398; Georgia, 691392; Alabama, 590756; Mississippi, 375651; and Louisiana, 352411. What was the population of the Southern States?

48. In 1840, the state of Tennessee contained 829210 inhabitants; Kentucky, 779828; Ohio, 1519467; Michigan, 212267; Indiana, 685866; Illinois, 476183; Missouri, 383702; and Arkansas, 97574. What was the population of the Western States?

49. In 1840, the territory of Florida contained 54477 inhabitants; Wisconsin, 30945; Iowa, 43112; and the District of Columbia, 43712; on board vessels of war, 6100. What was the population of the Territories and naval service of the United States?

50. What was the whole population of the United States in 1840?

51. What is the sum of four thousand and twenty-three; twenty-five thousand two hundred and one; two thousand and seven?

52. What is the sum of sixty thousand and two hundred; ten thousand and six; four hundred and ten; twenty-six thousand and seventeen?

53. What is the sum of six hundred and one thousand and seven; fifty thousand and two hundred; four hundred thousand and twenty-five; one thousand and one?

54. What is the sum of two millions, two thousand and twenty; thirty thousand and eighty; four hundred and five thousand and two hundred; sixty millions, sixty thousand and sixty?

55. What is the sum of five billions, ten millions and forty-five; eight millions, eight thousand and eight; two billions, four hundred and thirty millions, two hundred thousand and four hundred?

56. A man paid 2243 dollars for a house, 825 dollars for a barn, and for his farm as much as for his house and barn together: how much did he pay for his farm; and how much for all?

57. A man having 7 children gave a farm to each worth 2378 dollars: what was the value of all their farms?

58. A man bequeathed 6275 dollars apiece to his three children, and to his wife the balance of his property, which was equal to the amount he gave all his children: what was the value of his estate?

59. Sir Isaac Newton was born in the year 1642, and died in his eighty-fifth year: in what year did he die?

60. Four men, A, B, C, and D, built a school-house; A gave 1500 dollars, B 1750 dollars, C 1975, and D gave the land, which was worth as much as A and B gave: what was the whole cost of the house?

61. The world was created 4004 years before the Christian era: how many years from the creation to the present time?

62. In a common year, one month contains 28 days, four contain 30 days each, and seven 31 days each: how many days are there in a common year?

63. If a man lays up 26 dollars a month, how much will he lay up in a year?

64. How many strokes does a common clock strike in 24 hours?

65. If a person receives a present of 2345 dollars every birthday, how much will he have received when he is 21 years old?

66. In 1850, the population of the Eastern or New England States was 2,727,397; of the Middle States 5,990,270; of the Southern States 6,689,985; of the Western States 7,757,065; and of the Territories 92,255: what was the whole population of the United States in 1850?

67. America contains 62,500,000 inhabitants; Europe contains 260,250,000; Asia contains 450,000,000; Africa contains *57,000,000*, and Oceanica contains 20,250,000. What is the *whole population* of the world?

(68.)	(69.)	(70.)	(71.)	(72.)
5265	9543	3456	32789	53469
6376	8432	7890	43673	74578
2467	7341	1764	54785	63689
8579	6532	2653	63692	54578
9632	7683	3764	78901	43689
1105	8204	4675	93689	34578
5284	9438	6586	72543	45683
4371	5784	5697	62028	96798
3465	4632	7488	73456	83540
2352	6213	8590	32543	91302
1443	5462	9608	36721	13467
4359	2571	1589	82109	52369
8202	3062	2697	76542	45468
2718	7856	4576	63897	34382
8654	8345	3465	56432	27563
2548	3251	2354	10973	28824
8234	6479	1243	53864	32536
9283	5037	8765	38762	71839
8796	4659	6789	34687	64587

(73.)	(74.)	(75.)	(76.)
34567	543219	25305	361305
89012	087654	9021	48689
34567	321234	430268	437
89023	567890	423	7235007
45678	987654	16587	600438
90876	321023	307379	6470
86754	456789	79486	8407208
32549	827546	68076	45729
58763	324832	374	7085
49238	569754	576600	837282
32446	321506	30579	7641
48392	783762	743261	3146708
67438	458532	97045	834123
53927	387658	268	61520
48384	433683	84386	5704306
59876	594832	467578	600051
43203	728534	29183	7638
28354	653428	870	263
72863	347259	629324	8569765

SECTION III.

SUBTRACTION.

MENTAL EXERCISES.

ART. **30.** Ex. 1. Henry having 7 peaches, gave 4 of them to his sister: how many had he left?

Solution.—4 peaches taken from 7 peaches, leave 3 peaches. Therefore, if Henry had 7 peaches, and should give away 4 of them, he would have 3 left.

SUBTRACTION TABLE.

2 from			3 from			4 from			5 from			6 from			7 from			8 from			9 from		
2	leaves	0	3	lea.	0	4	lea.	0	5	lea.	0	6	lea.	0	7	lea.	0	8	lea.	0	9	lea.	0
3	"	1	4	"	1	5	"	1	6	"	1	7	"	1	8	"	1	9	"	1	10	"	1
4	"	2	5	"	2	6	"	2	7	"	2	8	"	2	9	"	2	10	"	2	11	"	2
5	"	3	6	"	3	7	"	3	8	"	3	9	"	3	10	"	3	11	"	3	12	"	3
6	"	4	7	"	4	8	"	4	9	"	4	10	"	4	11	"	4	12	"	4	13	"	4
7	"	5	8	"	5	9	"	5	10	"	5	11	"	5	12	"	5	13	"	5	14	"	5
8	"	6	9	"	6	10	"	6	11	"	6	12	"	6	13	"	6	14	"	6	15	"	6
9	"	7	10	"	7	11	"	7	12	"	7	13	"	7	14	"	7	15	"	7	16	"	7
10	"	8	11	"	8	12	"	8	13	"	8	14	"	8	15	"	8	16	"	8	17	"	8
11	"	9	12	"	9	13	"	9	14	"	9	15	"	9	16	"	9	17	"	9	18	"	9
12	"	10	13	"	10	14	"	10	15	"	10	16	"	10	17	"	10	18	"	10	19	"	10

OBS. This Table is the reverse of Addition Table. Hence, if the pupil has thoroughly learned that, it will cost him but little time or trouble to learn this. (See observations under Addition Table.)

11. 4 from 7 leaves how many? 4 from 9? 4 from 12? 4 from 8? 4 from 11? 4 from 13?

12. 6 from 8 leaves how many? 6 from 10? 6 from 13? 6 from 11? 6 from 15? 6 from 12? 6 from 16?

13. 7 from 9 leaves how many? 7 from 11? 7 from 14? 7 from 15? 7 from 16? 7 from 13? 7 from 17?

14. 8 from 11? 8 from 13? 8 from 16? 8 from 12? 8 from 15? 8 from 17? 8 from 14? 8 from 18?

15. 9 from 12? 9 from 14? 9 from 11? 9 from 13? 9 from 17? 9 from 15? 9 from 18? 9 from 19?

16. 2 from 4 leaves how many? 2 from 14? 2 from 24? 2 from 34? 2 from 44? 2 from 54? 2 from 64? 2 from 74? 2 from 84? 2 from 94?

17. 3 from 6? 3 from 16? 3 from 26? 3 from 36? 3 from 46? 3 from 56? 3 from 66? 3 from 76? 3 from 86? 3 from 96?

18. 4 from 9? 4 from 29? 4 from 39? 4 from 49? 4 from 59? 4 from 69? 4 from 79? 4 from 89? 4 from 99?

19. 6 from 15? 6 from 25? 6 from 35? 6 from 45? 6 from 55? 6 from 65? 6 from 75? 6 from 95?

20. 8 from 14? 8 from 24? 8 from 34? 8 from 44? 8 from 54? 8 from 64? 8 from 74? 8 from 84? 8 from 94?

21. A gentleman bought a coat for 15 dollars, and a hat for 6 dollars: how much more did his coat cost than his hat?

22. A farmer having sold 6 cords of wood for 18 dollars, took a barrel of flour at 6 dollars towards his pay, and the rest in cash: how much money did he receive?

23. A lady bought a shawl for 15 dollars, and handed the shopkeeper a 20 dollar bill: how much change ought she to receive back?

24. A man having 25 watermelons in his garden, some wicked boys stole 9 of them: how many had he left?

25. James is 14 years old, and his sister is 19: what is the difference in their ages?

26. A merchant had a piece of calico which contained 33 yards; on measuring the remnant he finds he has but 7 yards left: how many yards has he sold?

27. A hogshead of cider contains 63 gallons: after drawing out 9 gallons, how many will be left?

28. Henry had 48 silver dollars, and gave 9 to the orphan asylum: how many dollars did he have left?

29. A man bought a piece of cloth containing 39 yards, and sold 6 yards of it: how many yards had he left?

30. George gave 75 cents for a pair of skates, and sold them for 9 cents less than he gave: how much did he get for his skates?

31. William had 67 cents; he spent 5 for chestnuts and 2 for apples: how many cents had he left?

32. A man sold a load of wood for 18 shillings; he laid out 4 shillings for tea and 6 for sugar: how many shillings had he to carry home?

33. Sarah having 85 cents, gave 10 cents to the Sabbath School Society, 8 to the Bible Society, and spent 6 for candy: how many cents had she left?

34. If I pay 27 dollars for a cow and sell it for 18 dollars, how much do I lose by the bargain?

35. Richard had 45 marbles; he lost 7 and gave away 5: how many had he left?

36. A man having 56 dollars in his pocket, bought a hat for 5 dollars, a coat for 10, and a pair of boots for 4: how much money had he left?

37. If I owed a merchant 50 dollars and should pay him 20 dollars, how many dollars should I then owe him?

Suggestion.—It is advisable for beginners to analyze the numbers in this question, as in Ex. 31, p. 24, and then take 2 tens from 5 tens.

38. A farmer having 80 sheep, sold all but 30: how many did he sell?

39. A man having 90 acres of land, gave 50 acres to his son: how many acres had he left?

40. George having 70 cents, spent 30: how many had he left?

41. In a certain orchard there are 100 trees; 60 of them are apple-trees, and the rest are peach-trees: how many peach-trees are there?

42. A grocer bought 150 eggs, and afterwards found that 20 of them were broken: how many sound ones were there?

43. In the Centre School there are 150 scholars, 60 of whom are girls: how many boys are there?

44. A man bought a horse for 90 dollars, and sold it immediately for 130 dollars: how much did he make by his bargain?

45. A man owing me 200 dollars, turned me out a horse worth 80 dollars, and is to pay the balance in cash: how much money must he pay me?

46. A boy going to market with 80 cents, bought 20 cents worth of cheese, and 30 cents worth of butter: how much change had he left?

47–49. 35 from 42 leaves how many? 63 from 75? 26 from 40? 35 from 45? 65 from 85? 82 from 94?

50. 8 from 17 leaves how many? 13 from 26? 6 from 25? 8 from 94? 5 from 68? 17 from 34? 7 from 43? 6 from 72? 9 from 75? 7 from 86?

EXERCISES FOR THE SLATE.

ART. **31.** Each of the foregoing examples in this section contains *two numbers*, and the object is to find the *difference* between them. The operation by which these and all similar examples are solved, is called *Subtraction.* Hence,

32. SUBTRACTION *is the process of finding the difference between two numbers.*

The *answer*, or number *obtained* by subtraction, is called the *difference* or *remainder.*

OBS. 1. The number to be subtracted is often called the *subtrahend*, and the number from which it is subtracted, the *minuend.* These terms, however, are calculated to embarrass, rather than assist the learner, and are properly falling into disuse.

2. Subtraction, it will be perceived, is the *reverse* of addition. Addition unites two or more numbers into one single number; subtraction, on the other hand, separates a number into two parts.

3. When the given numbers are of the *same kind*, or *denomination*, the operation is called *Simple Subtraction.* (Art. 18. Obs.)

33. *The sign of subtraction* is a *horizontal line* (—), called *minus*, and shows that the number placed *after* it, is to be subtracted from the one *before* it. Thus, the expression 8—5, signifies that 5 is to be subtracted from 8; and is read, "8 minus 5," or "8 less 5."

Note.—The term *minus*, is a Latin word which signifies *less.*

CASE I.—*When each figure in the lower number is smaller than the figure above it.*

ART. **34.** Ex. 1. A man gave 475 dollars for a span of horses, and 352 dollars for a carriage: how much more did he pay for his horses than for his carriage?

Operation.

4	7	5	Horses.
3	5	2	Carriage.
1	2	3	Rem.

Suggestion.—Write the *less* number under the *greater*, units under units, tens under tens, &c., and draw a line beneath them as in the margin. Then, beginning at the right hand, proceed thus: 2 units from 5 units

QUEST.—32. What is subtraction? What is the answer called? *Obs.* What is the number to be subtracted sometimes called? That from which it is subtracted? Of what is subtraction the reverse? When the given numbers are of the same denomination, what is the operation called? 33. What is the sign of subtraction? What does it show? How is the expression 8—5 read? *Note.* What is the meaning of the term minus? 34. Explain the solution of the first example from your slate.

leave 3 units; write the 3 in units' place, under the figure subtracted, *because it denotes units.* Next, 5 tens from 7 tens leave 2 tens; set the 2 in tens' place, *because it denotes tens.* Finally, 3 hundreds from 4 hundreds leave 1 hundred; write the 1 in hundreds' place, *because it denotes hundreds.* He therefore paid 123 dollars more for his horses, than for his carriage.

34. *a.* It is important to observe in the preceding solution, that *units* are subtracted from *units*, *tens* from *tens*, &c. Hence, universally,

Figures of the same order only, can be subtracted from each other.

The *reason* of this is, that figures of different orders express *units* of *different* values; consequently, if a figure of one order is taken from a figure of a different order, the remainder will neither be one order nor the other. (Art. 22.) Thus, 2 *units* from 7 *tens* will neither leave 5 *units* nor 5 *tens*, any more than 2 cents from 7 dimes will leave 5 *cents*, or 5 *dimes*.

OBS. The *less* number is written *under* the *greater*, simply for convenience in subtracting; and *units* are placed under *units*, *tens* under *tens*, &c., to avoid mistakes which might occur from taking *different* orders from each other.

Solve the following examples in a similar way.

2. A merchant bought 268 barrels of flour; and on examination, found that only 123 barrels were fit for use: how many were damaged? *Ans.* 145 barrels.

3. A traveler having 576 dollars, was robbed of 344 dollars: how many dollars had he left?

4. What is the difference between 648 and 235?

	(5.)	(6.)	(7.)	(8.)
From	876 dolls.	759 feet	4567 pounds	8643 quarts
Take	523 dolls.	341 feet.	1235 pounds.	5412 quarts.

	(9.)	(10.)	(11.)	(12.)
From	68476	765274	563181	3286732
Take	36124	152140	32040	135011

QUEST.—34. *a.* What orders of figures do you subtract from each other? Why not subtract figures of different orders from each other?

CASE II.—*When a figure in the lower number is larger than that above it.*

35. In the preceding examples, each figure in the lower number is *smaller* than the figure above it. But it often happens that a figure in the lower number is *larger* than that above it, and consequently cannot be taken from it.

13. What is the difference between 9042 and 5837?

Operation.

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9042
5837
3205 Ans.
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Suggestion.—Having written the less number under the greater, begin at the right hand as before. Now, as we cannot take 7 units from 2 units, we add 10 *units* to the 2, which make 12 units: then 7 from 12 leaves 5. To counterbalance the 10 *units* added to the upper number, we add 1 *ten* to the next figure in the lower number, before we subtract. Thus, 1 ten and 3 tens are 4 tens, and 4 from 4 leaves 0. Again, 8 hundreds cannot be taken from 0 hundreds; we therefore add 10 *hundreds* to the 0, and it becomes 10 hundreds: then 8 from 10 leaves 2. Finally, to counterbalance the 10 hundreds added to the upper number, we add 1 *thousand* to the 5 thousands, which makes 6 thousands. Now 6 from 9 leaves 3. The answer, therefore, is 3205.

OBS. Instead of actually adding 10 to the upper figure, and then taking the lower figure from this sum, it is often more convenient to subtract the lower figure from 10, and to the remainder, add the upper figure. Thus, in the example above, we may simply say, 7 from 10 leaves 3, and 2 are 5, &c.

36. The process of adding 10 to the upper figure, *when it is less than the figure below it, is called borrowing ten.*

N.B. When we *borrow* 10, we must be careful to add 1 to the next figure in the lower number. The object of this is, to *counterbalance* the 10 added to the upper number.

OBS. 1. This method of *borrowing* depends on the self-evident principle, that if any two numbers are *equally* increased, their difference will not be *altered.*

That the two given numbers are equally increased by this process, is evident from the fact that the 1 added to the lower number, is the next higher order than the 10 added to the upper number, and is therefore equal to it. (Art. 8.)

2. The *reason* we borrow 10, instead of 8, 12, or any other number, is because numbers increase in a *tenfold* ratio. (Art. 9.) If they increased in an *eightfold* ratio, we should borrow 8; if in a *twelvefold* ratio, we should borrow 12, and universally, we borrow as many as it takes of a *lower* order to make *one* of the *next higher.*

QUEST.—35. Explain the solution of the thirteenth example.

3. The *reason* we begin to subtract at the right hand, is because when we have occasion to *borrow*, it is necessary to *pay* before the next figure is subtracted.

	(15.)	(16.)	(17.)	(18.)
From	574	78562	645630	70430256
Take	326	24380	520723	4326107

SECOND METHOD OF BORROWING.

37. Required to find the difference between 75 and 48 ?

Analytic Solution.—Since 8 units cannot be taken from 5 units, we must borrow 10. Now, taking 1 ten from 7 tens, and uniting it with the 5 units, the upper number becomes 6 tens or 60 plus 15 units. Again, separating the lower number into the units and tens of which it is composed, it becomes 4 tens or 40 plus 8 units. Now 8 units from 15 units leave 7 units. Next, since the 7 tens have lent 1 ten there are but 6 tens left, and 4 tens from 6 tens leave 2 tens. The remainder is 2 tens (20)+7 units, or 27. That is,

Operation.

75=6 tens (60)+15 units.
48=4 tens (40)+ 8 units.
Rem. 2 tens (20)+ 7, or 27.

38. When a figure in the lower number is larger than that above it; take a *unit* from the next higher order in the upper number, and add it to the upper figure; then subtracting the lower figure from this sum, diminish the next upper figure by 1, and proceed as before.

OBS. This method of borrowing does not affect the difference between the two numbers. This is manifest from the fact that it does not change the *value* of the upper number, but simply *transposes* a *part* of one of its orders to another, which can no more *increase* or *diminish* it, than it can increase or diminish the amount of money a man has, if he takes a part from one pocket and puts it into another.

19. A man having 6042 acres of land, sold 2364 acres: how many acres did he have left? *Ans.* 3678.
20. From 8796 subtract 2675.
21. From 6210896 subtract 3456809.
22. From 1000000 subtract 67583.
23. From 7834501 subtract 1000000.
24. From 68436907 subtract 59476012.
25. From 8006754231 subtract 7975663417.

40. From the preceding illustrations and principles, we derive the following

GENERAL RULE FOR SUBTRACTION.

I. *Write the less number under the greater, so that units may stand under units, tens under tens, &c.*

II. *Beginning at the right hand, subtract each figure in the lower number from the figure above it, and set the remainder under the figure subtracted.* (Art. 34.)

III. *When a figure in the lower number is larger than that above it, add* 10 *to the upper figure; then subtract as before, and add* 1 *to the next figure in the lower number.* (Arts. 35, 36.)

PROOF.—*Add the remainder to the smaller number; and if the sum is equal to the larger number, the work is right.*

OBS. This method of proof depends upon the obvious principle, that if the *difference* between two numbers be added to the *less*, the *sum* must be equal to the *greater*.

Note.—As soon as the learner becomes familiar with the process of subtraction, he should drop the intervening words as in addition, and simply pronounce the results. Thus, in the ninth example, instead of saying 4 from 6 leaves 2: and 2 from 7 leaves 5, &c., he should say *two, five, three, &c.*, setting down each result while pronouncing it.

EXAMPLES FOR PRACTICE.

1. A man bought a piece of cloth containing 237 yards, and sold 124 yards of it. How much had he left?

2. A merchant had on hand a quantity of flour, for which he asked 245 dollars; but for ready money he made a deduction of 24 dollars. How much did he receive for his flour?

3. In a certain academy there were 357 scholars, 168 of whom were young ladies. How many gentlemen were there?

4. A farmer raised 4879 bushels of wheat, and sold 3876 bushels. How much had he left?

QUEST.—40. How do you write numbers for subtraction? How proceed when each figure in the lower number is smaller than that above it? How, when a figure in the lower line is larger than that above it? 36. What is meant by borrowing ten? When you borrow 10, why do you add 1 to the next figure in the lower number? *Obs.* Upon what principle does this method of borrowing depend? How does it appear that the two numbers are equally increased? Why do you borrow 10 instead of 8, 12, or any other number? 38. What is the second method of borrowing? *Obs.* How does it appear that this method does not affect the difference between the two given numbers? 34. *Obs.* Why write the less number under the greater? Why place units under units, tens under tens, &c.? 36. *Obs.* Why begin to subtract at the right hand?

5. A man purchased a farm for 4687 dollars, but was obliged to sell it for 896 dollars less than he gave for it. How much did he sell it for?

6. A merchant bought 2268 dollars worth of goods, which, in consequence of getting damaged, he sold for 848 dollars less than cost. How much did he sell them for?

7. A merchant sold a lot of silks for 561 dollars, which was 179 dollars more than the cost. How much did they cost?

8. A man bought an estate for 8796 dollars, and sold it again for 9875 dollars. How much did he gain by his bargain?

9. A farmer raised 1389 bushels of wheat one year, and 1763 the next. How much more did he raise the second year than the first?

10. A man bought a house and lot for 5687 dollars. The house was worth 3698 dollars: how much was the lot worth?

11. Suppose a man's income is 3268 dollars a year, and his expenses are 2789 dollars. How much can he save in a year?

12. The United States declared their independence in 1776: how many years is it since?

13. Two brothers commenced business at the same time; one gained 3678 dollars in five years, the other gained 2387 dollars. How much more did one gain than the other?

14. The distance from Boston to Springfield is 98 miles, and from Boston to Pittsfield it is 151 miles. How far is it from Springfield to Pittsfield?

15. From New York to Utica it is 243 miles, from New York to Albany 150 miles. How far is it from Albany to Utica?

16. America was discovered by Columbus in 1492: how many years is it since?

17. Dr. Franklin died in 1790, and was 84 years old when he died: in what year was he born?

18. General Washington was born in 1732, and died in 1799: how old was he when he died?

19. The first settlement in New England was made at Plymouth, in the year 1620: how many years is it since?

20. A ship having a cargo valued at 100000 dollars, was overtaken by a storm, and 27680 dollars worth of goods were thrown overboard. How much of the cargo was saved?

21. The population of Massachusetts in 1840, was 737699, and that of Connecticut was 309978. How many more inhabitants were there in Massachusetts than in Connecticut?

22. In 1840, the population of Massachusetts was 737699, and in 1820 it was 523287. How much did the population increase during this period?

23. In 1840, the population of the state of New York was 2428921, and in 1820 it was 1372812. How much did the population increase during that period?

24. In 1840, the population of the New England States was 2234822, and that of the State of New York was 2428921. How many more inhabitants were there in the State of New York than in New England?

25. In 1800, the population of the United States was 5305925, and in 1840 it was 17069453. How much did it increase in forty years?

26. A farmer having 389 acres of land, sold to one man 126 acres, and to another 168. How many acres had he left?

27. A gentleman having 1768 dollars deposited in the bank, gave a check for 175 dollars to one man, to another for 238 dollars, and to another for 369 dollars. How much remained on deposit?

28. A man bought a horse for 87 dollars, a carriage for 75 dollars, and a harness for 16 dollars, and sold them all together for 200 dollars. How much did he gain by the bargain?

29. A man bought a quantity of sugar for 25 dollars, a quantity of molasses for 27 dollars, and a quantity of raisins for 29 dollars, for which he paid a hundred dollar bill. How much change ought he to receive back?

30. An orchard contained 120 apple-trees, 47 peach-trees, and 28 pear-trees. Of the apple-trees 26 were cut down for a railroad to pass through, 18 of the peach-trees died, and 5 of the pear-trees were blown down: How many trees were left in the orchard?

31. A gentleman had 2700 dollars which he wished to distribute among his three sons. To the eldest he gave 825 dollars, to the second 785 dollars, and the remainder to the youngest. How much did the youngest son receive?

32. A man owing 5648 dollars, paid at one time 536 dollars, at another 378 dollars, and at another 896 dollars. How much did he then owe?

33. A man having 7689 dollars, invested 689 dollars in railroad stock, 500 dollars in a woolen factory, and 1250 dollars in bank stock. How much had he left?

34. A man bought a quantity of oil for 1763 dollars, and a lot of candles for 598 dollars. He afterwards sold them both for 2684 dollars. How much did he gain by the bargain?

35. A man owning 3789 acres of land, gave to one son 869 acres, and to another 987 acres. How much had he left?

36. A ship of war sailing with 650 men, lost in one battle 29 men, in another 37, and by sickness 19 more. How many were still living?

37. A merchant owes one man 2684 dollars, another 1786 dollars, another 987 dollars. The whole amount of his property is 4684 dollars. How much more does he owe than he is worth?

38. A man bought three farms: for the first he gave 4673 dollars, for the second 5674 dollars, and for the third 9287 dollars. He sold them all for 37687 dollars. How much did he gain by the bargain?

39. A man bought 86 dollars worth of wheat, 48 dollars worth of butter, and a fine horse worth 148 dollars. He gave his note for 128 dollars, and paid the rest in cash. How much money did he pay?

40. A gentleman left a fortune of 18864 dollars, to be divided between his two sons and one daughter; to one son he gave 6389 dollars, to the other 6984 dollars. How much did the daughter receive?

41. A man owing 8648 dollars, paid at one time 486, at another 684, at another 729 dollars. How much did he still owe?

42. Suppose a man gains by one speculation 867 dollars, by another 687; another time he gains 563 dollars, and then loses 479; still another time he gains 435 dollars, and loses 378. How much more has he gained than lost?

43. A man borrowed of a friend 684 dollars at one time, 786 at another, 874 at another, and 976 at another. He has paid 568 dollars. How much does he still owe?

44. If a man's income is 4586 dollars a year, and he spends 384 dollars for clothing, 568 for house rent, 784 for provisions, 568 for servants, and 369 dollars for traveling, how much will he have left at the end of the year?

45. A merchant bought a quantity of sugar for 8978 dollars, paid 374 dollars freight, and then sold it for 9684 dollars. How much did he gain by the trade?

46. A merchant had in his storehouse 6384 bushels of wheat, 3752 bushels of corn, 4564 bushels of oats, and 1384 bushels of rye: it was broken open and 3564 bushels of grain taken out. How many bushels remained?

47. A man bought a quantity of beef for 5493 dollars, a quantity of coffee for 261 dollars, and a quantity of sugar for 157 dollars; in exchange he gave 3687 dollars worth of flour, 568 dollars worth of oats, and 165 dollars worth of potatoes. How much did he then owe?

48. A gentleman has real estate valued at 3879 dollars, and personal property amounting to 9857 dollars. He owes one man 1350 dollars, and another 2687 dollars. How much would he have left if he should pay his debts?

49. A man having property worth 30000 dollars, lost a store by fire worth 5000 dollars, and goods to the amount of 3578 dollars. How much had he left?

50. A man died leaving an estate of 175000 dollars. He gave to his wife 25000 dollars, to his three sons 32000 apiece, to his two daughters, 23000 dollars each, and the rest he gave to a literary institution. How much did the institution receive?

51. From twenty-five thousand and twenty-five, take 28 hundred.

52. From 16 millions, 16 thousand and 16, take 16 hundred.

53. What is the difference between 185 billions, and 185 millions?

54. How many times can 563 be subtracted from 2815 before the latter will be exhausted?

55. What number must be added to 3645 to make it 630712?

56. Washington was born in 1732, and the independence of the United States was declared in 1776: how old was Washington at the time of the Declaration?

57. How many years from the Declaration of Independence to 1853?

58. If you subtract 681 seven times from 4965, what will be the last remainder?

59. The Pilgrim Fathers landed at Plymouth in 1620: how many years to the present time?

60. How many years since the discovery of America, which was in 1492?

61. A man bought a house for 1400 dollars, and agreed to pay 175 dollars at the end of each year: how many years will it take him to pay for it?

62. What number must be added to 836 to make 2323?

63. What number increased by 1131, will be 3576?

64. What number taken from 2791, will leave 1643?

65. What number taken from 4827, will leave 2491?

66. What is the difference between 265+423−117+236, and 165+361−97?

67. What is the difference between 576−208+1645−321, and 403−256+814−195?

68. The difference of two numbers is 1237, and the greater is 6031: what is the less number?

69. The greater of two numbers is 3107, and the difference is 1920: what is the smaller number?

70. What number increased by 20703, will become 37249?

71. What number taken from 7209407, will leave 999999?

72. What number is that, from which if you take 42371, the remainder will be 19289 less 176?

73. What number is that, from which if you take 18268, the remainder will be 26017—17312?

74. What number is that, from which if 27239 be taken, the remainder will be 9897—3076?

75. A man having 75000 dollars, gave 8265 to one institution, 15687 to another, to a third as much as to the other two. How much had he left?

76. A says to B, I have 2675 sheep; B replies, I have 763 less than you; C adds, I have as many as both lacking 105. How many sheep had B and C?

77. The sum of 3 numbers is 23257; the first is 9277, the second is 1283 less than the first: what is the third number?

SECTION IV.

MULTIPLICATION.

MENTAL EXERCISES.

Art. 41. Ex. 1. What will 3 pencils cost, at 4 cents apiece?

Analysis.—Since 1 pencil costs 4 cents, 3 pencils will cost 3 times 4 cents; and 3 times 4 cents are 12 cents. Therefore 3 pencils, at 4 cents apiece, will cost 12 cents.

MULTIPLICATION TABLE.

2 times	3 times	4 times	5 times	6 times	7 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7
2 " 4	2 " 6	2 " 8	2 " 10	2 " 12	2 " 14
3 " 6	3 " 9	3 " 12	3 " 15	3 " 18	3 " 21
4 " 8	4 " 12	4 " 16	4 " 20	4 " 24	4 " 28
5 " 10	5 " 15	5 " 20	5 " 25	5 " 30	5 " 35
6 " 12	6 " 18	6 " 24	6 " 30	6 " 36	6 " 42
7 " 14	7 " 21	7 " 28	7 " 35	7 " 42	7 " 49
8 " 16	8 " 24	8 " 32	8 " 40	8 " 48	8 " 56
9 " 18	9 " 27	9 " 36	9 " 45	9 " 54	9 " 63
10 " 20	10 " 30	10 " 40	10 " 50	10 " 60	10 " 70
11 " 22	11 " 33	11 " 44	11 " 55	11 " 66	11 " 77
12 " 24	12 " 36	12 " 48	12 " 60	12 " 72	12 " 84

8 times	9 times	10 times	11 times	12 times
1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

Obs. The pupil will find assistance in learning this table, by observing the following particulars.

1. The several results of multiplying by 10 are formed by simply adding a

cipher to the figure that is to be multiplied. Thus, 10 times 2 are 20; 10 times 3 are 30; 10 times 4 are 40, &c.

2. The results of multiplying by 5, terminate in 5 and 0, alternately. Thus, 5 times 1 are 5; 5 times 2 are 10; 5 times 3 are 15, &c.

3. The first nine results of multiplying by 11 are formed by repeating the figure to be multiplied. Thus, 11 times 2 are 22; 11 times 3 are 33, &c.

4. In the successive results of multiplying by 9, the right hand figure regularly decreases by 1, and the left hand figure regularly increases by 1. Thus, 9 times 2 are 18; 9 times 3 are 27; 9 times 4 are 36, &c.

13. At 2 dollars a cord, what will 12 cords of wood cost? 10 cords? 9 cords? 8 cords? 7 cords? 6 cords? 5 cords?

14. In one yard there are 3 feet: how many feet in 12 yards? in 11 yards? 10 yards? 9 yards? 8 yards? 7 yards?

15. In 1 gallon there are 4 quarts: how many quarts in 12 gallons? In 11 gallons? 10 gallons? 9 gallons? 8 gallons?

16. If you buy 5 marbles for a cent, how many can you buy for 12 cents? For 11 cents? 10 cents? 9 cents? 8 cents?

17. In New England a dollar contains 6 shillings: how many shillings do 12 dollars contain? 11 dolls.? 10 dolls.? 9 dolls.? 8 dolls.? 7 dolls.? 6 dolls.? 5 dolls.?

19. In New York a dollar contains 8 shillings: how many shillings do 12 dollars contain? 11 dolls.? 10 dolls.? 9 dolls.? 8 dolls.? 7 dolls.? 6 dolls.? 5 dolls.?

20. At 9 cents a quart, what will 12 quarts of blackberries cost? 11 quarts? 10 quarts? 9 quarts? 8 quarts? 7 quarts?

21. What will be the cost of 12 yards of silk, at 10 shillings per yard? of 11 yards? 10 yards? 9 yards? 8 yards? 7 yards?

22. What cost 8 cords of wood, at 5 dollars per cord?

23. If 7 yards of cloth make a cloak, how many yards will it take to make 8 cloaks?

24. What cost 9 pounds of ginger, at 8 cents a pound?

25. At 12 dollars apiece, what will 10 cows cost?

26. What cost 10 barrels of cider, at 9 shillings a barrel?

27. What will 11 pair of shoes cost, at 10 shillings a pair?

28. If 8 men can do a job of work in 9 days, how long will it take 1 man to do it?

29. If a barrel of beer will last 7 persons 8 weeks, how long will it last 1 person?

30. How much will 3 cows cost, at 24 dollars apiece?

Analysis.—If 1 cow costs 24 dollars, 3 cows will cost 3 times 24 dollars. But 24 is composed of 2 tens and 4 units. Now *3 times 2 tens* are 6 tens, or 60; and 3 times 4 units are 12

units, which added to 60, make 72. Therefore 3 cows, at 24 dollars apiece, will cost 72 dollars.

Note.—When the number to be multiplied is large, it is more convenient and expeditious in mental calculations, to multiply the highest order first; then the next lower order, and add the products as we proceed in the operation.

31. What cost 5 tons of hay, at 13 dollars per ton?

32. What cost 4 hogsheads of molasses, at 15 dollars per hogshead?

33. How much can a man earn in 6 months, at 15 dollars per month?

34. A butcher bought 6 sheep, at 17 shillings apiece: how many shillings did they come to?

35. If a scholar performs 18 examples in one day, how many can he perform in 5 days?

36. In 1 pound there are 16 ounces: how many ounces are there in 8 pounds?

37. How far will a man walk in 5 days, if he walks 20 miles per day?

38. If 19 men can build a barn in 4 days, how long would it take one man to do it?

39. If a shoemaker packs 16 pair of boots in 1 box, how many pair can he pack in 7 boxes?

40. If 1 acre of land produces 23 bushels of wheat, how many bushels will 8 acres produce?

41. A merchant bought 4 pieces of silk, each piece having 24 yards: how many yards did they all contain?

42. What will 6 sleighs cost, at 25 dollars apiece?

43. What cost 7 reading books, at 42 cents apiece?

44. In 1 guinea there are 21 shillings: how many shillings are there in 5 guineas?

45. In 1 hogshead there are 63 gallons: how many gallons are there in 4 hogsheads?

46. What cost 32 pounds of sugar, at 8 cents per pound?

47. What cost 85 reams of paper, at 3 dollars per ream?

48. What cost 90 hats, at 6 dollars apiece?

49. In 1 week there are 7 days: how many days are there in 70 weeks?

50. In 1 hour there are 60 minutes: how many minutes are there in 9 hours?

EXERCISES FOR THE SLATE.

42. It will be noticed that each of the preceding examples contains *two* numbers, and it is required to find *how much one* of them will *amount* to, when repeated or taken as many times as there are *units* in the other. The operation by which we obtain this result, is called *Multiplication.* Hence,

43. *Multiplication is the process of finding the amount of a number repeated or added to itself, a given number of times.*

The number to be *repeated* or *multiplied*, is called the *multiplicand.*

The number by which we *multiply*, is called the *multiplier*, and shows how many times the multiplicand is to be *repeated* or *taken.*

The *answer*, or number *produced* by multiplication, is called the *product.* Thus, when we say 6 times 12 are 72, 12 is the multiplicand, 6 the multiplier, and 72 the product.

OBS. When the multiplicand denotes things of *one kind, or denomination only*, the operation is called *Simple Multiplication.*

44. The multiplier and multiplicand taken together, are often called *factors*, because they make or produce the product.

Note.—The term *factor*, is a Latin word which signifies an *agent*, a *doer*, or *producer.*

45. Multiplying by 1, is taking the multiplicand *once:* thus, 4 multiplied by 1=4.

Multiplying by 2, is taking the multiplicand *twice:* thus, 2 times 4, or 4+4=8.

Multiplying by 3, is taking the multiplicand *three times:* thus 3 times 4, or 4+4+4=12, &c. Hence,

Multiplying by any whole number, is taking the multiplicand as many times, as there are units in the multiplier.

QUEST.—43. What is multiplication? What is the number to be multiplied called? What the number by which we multiply? What does the multiplier show? What is the answer called? When we say, 6 times 12 are 72, which is the multiplicand? Which the multiplier? Which the product? When we say, 6 times 9 are 54, what is the 6 called? The 9? The 54? *Obs.* When the multiplicand denotes things of one denomination only, what is the operation called? 44. What are the multiplicand and multiplier taken together called? Why? *Note.* What does the term factor signify? 45. What is it to multiply *by 1? By 2? By 3?* What is it to multiply by any whole number?

Note.—The application of this principle to *fractional* multipliers, will be illustrated under fractions.

OBS. 1. From the definition of multiplication, it is manifest that the *product* is the *same kind* or *denomination* as the multiplicand; for *repeating* a number or quantity does not *alter* its nature. Thus, if we repeat pounds, they are still pounds; if we repeat yards, they are still yards; if we repeat an abstract number, that is, a number which does not refer to any particular object, it is still an abstract number.

2. Every *multiplier* is to be considered an *abstract number*. In familiar language it is sometimes said, that the price multiplied by the *weight* will give the value of an article; and it is often asked how much 25 cents multiplied by 25 cents will produce. But these are abbreviated expressions, and are liable to convey an erroneous idea, or rather no idea at all. If taken literally, they are absurd; for multiplication is *repeating* a number or quantity a certain *number of times*. Now to say that the price is repeated as many times as the given quantity is *heavy*, or that 25 cents are repeated 25 *cents times*, is nonsense. But we can multiply the price of 1 pound by a *number* equal to the number of pounds in the *weight* of the given article, and the product will be the value of the article. We can also multiply 5 cents by the *number* 5; that is, repeat 5 cents 5 times, and the product is 25 cents. Construed in this manner, the multiplier becomes an abstract number, and the expressions have a consistent meaning.

46. *The sign of multiplication* is an *oblique cross* (×), and shows that the numbers between which it is placed, are to be multiplied together. Thus, the expression 9 × 6, signifies that 9 and 6 are to be multiplied together, and is read, "9 multiplied by 6," or simply "9 into 6."

47. *The product of any two numbers will be the same, whichever factor is taken for the multiplier.*

To illustrate this point; suppose there is a certain orchard which contains 4 rows of trees, and each row has 6 trees. Let the number of rows be represented by the number of horizontal rows of stars in the margin, and the number of trees in each row by the number of stars in a row. Now it is evident, that the whole number of trees in the orchard is equal either to the number of stars in a *horizontal* row taken *four times*, or to the number of stars in a *perpendicular* row taken *six times*; that is, equal to 6 × 4, or 4 × 6.

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48. *Multiplication* is similar in principle to *Addition*, and may be performed by it. For instance, to find how much 3 times 4 cents are, as in the first example, take 4 cents 3 times, and add them together. Thus, 4 cents + 4 cents + 4 cents,

QUEST.—*Obs.* What kind or denomination is the product? How does this appear? What must every multiplier be considered? 46. What is the sign of multiplication? What does it show? How is the expression 9×6 read?

are 12 cents. By multiplication the result is reached by a single step. Thus, 3 times 4 cents are 12 cents. Hence, it may be said,

Multiplication is a short method of performing repeated additions of a number to itself.

CASE I.— *When the multiplier contains but one figure.*

ART. **49.** Ex. 1. What will 3 house-lots cost, at 562 dollars apiece?

Operation.

562	Multiplicand,
3	Multiplier.
1686	Product.

Suggestion.—If 1 lot costs 562 dollars, 3 lots will cost 3 times as much. Now, having written the less number under the greater, we begin at the right hand, and multiply each figure of the multiplicand by the multiplier, setting down the result and carrying as in addition. Thus, 3 times 2 (units) are 6 (units), or simply say, 3 times 2 are 6. Set the 6 in units' place under the figure multiplied. Next, 3 times 6 are 18. Set the 8 or right hand figure under the figure multiplied, and carry the 1 or left hand figure to the next product, *as in addition.* Finally, 3 times 5 are 15, and 1 to carry makes 16, which we set down in full.

OBS. 1. It is immaterial as to the result as we have seen, which of the given numbers is taken for the multiplier. (Art. 47.) But it is generally more convenient and therefore customary to place the larger for the multiplicand and the smaller for the multiplier. Thus it is easier to multiply 254672381 by 7, than it is to multiply 7 by 254672381, but the product will be the same.

2. The learner should *analyze* every example solved upon his slate, and be able to give the *reasoning* in full, as in mental solutions.

Solve the following examples in a similar manner:

2. What will 7 horses cost, at 120 dollars apiece?

Ans. 840 dolls.

3. What is the product of 312 multiplied by 3?

4. What is the product of 121 multiplied by 4?

5. In 1 mile there are 320 rods: how many rods are there in 3 miles?

6. If a man travels 110 miles in 1 day, how far can he travel in 8 days?

QUEST.—49. Explain the solution of the first example upon your slate.

	7.	8.	9.	10.
Multiplicand,	3032	22120	101101	3012302
Multiplier,	3	4	5	6

11. What will 6 stage-coaches cost, at 783 dollars apiece?
12. What cost 83 pounds of opium, at 8 dollars per pound?
13. At 9 shillings per day, how much can a man earn in 213 days?
14. If 1 sofa costs 78 dollars, how much will 8 sofas cost?
15. What cost 879 barrels of flour, at 7 dollars a barrel?
16. At 8 shillings apiece, what will 650 lambs come to?

	17.	18.	19.	20.
Multiply	8006	76030	10906	4608790
By	5	8	7	9

50. The *reason* for *carrying the tens* in multiplication, is the same as in addition, and may be illustrated in a similar manner. (Art 26.)

Take, for instance, the 11th example, and set the product of each figure in a separate line. Thus,

783	Or, separate the multiplicand into the
6	orders of which it is composed,
18 *Prod.* of units.	Thus, $783=700+80+3$.
48 " tens.	Now $700\times 6=4200$ hund.
42 " hunds.	$80\times 6=\ \ 480$ tens.
4698 whole *Prod.*	$3\times 6=\ \ \ 18$ units.

Finally, adding these results together, we have 4698 *Prod.*

OBS. The *reason* we begin to multiply at the right hand of the multiplicand, is that we may carry the tens, as we proceed in the operation.

51. From the preceding illustrations, it will be seen, that units multiplied by units produce units; tens into units produce tens; hundreds into units produce hundreds, &c. Hence,

When the multiplier is units, the product is the same order as the figure multiplied. And, conversely,

When the figure multiplied is units, the product is the same order as the figure by which we multiply; for the product is the *same*, whichever factor is taken as the multiplier.

CASE II.—*When the multiplier contains more than one figure.*

ART. **52.** Ex. 21. What cost 26 horses, at 143 dolls. apiece?

Suggestion.—Beginning with the units, we multiply the multiplicand by each figure of the multiplier separately, and placing the first figure of each partial product under the figure by which we are multiplying, add the two results together. Thus, 6 times 3 are 18; setting down the 8, and carrying the 1, proceed as before. 6 times 4 are 24, and 1 (to carry) makes 25. 6 times 1 are 6, and 2 (to carry) are 8. Next, multiplying by the 2 tens, 20 times 3 units are 60 units, which are equal to 6 tens; or we may simply say, 2 times 3 are 6. Now, as the 6 denotes tens, (Art. 51,) we write it in tens' place in the product; that is, under the figure 2 by which we are multiplying. Again, 20 times 4 tens are 80 tens, equal to 800; or simply say, 2 times 4 are 8: and since the 8 denotes hundreds, it must be set on the left of the 6 in hundreds' place. Next, 20 times 1 hundred are 20 hundred, or 2000; or simply say, 2 times 1 are 2: and since the 2 denotes thousands, set it in thousands' place on the left of the last figure in the product.

Operation.

```
 143 Multiplicand.
  26 Multiplier.
 ---
 858 cost of 6 h.
286     "   20 h.
----
3718    "   26 h.
```

Finally, adding these two results together as they stand, units to units, tens to tens, &c., we have 3718 dollars, which is the whole product required.

OBS. 1. The several products of the multiplicand into the separate figures of the multiplier, are called *partial* products.

2. When the multiplier contains more than *one* figure, the *reason* we multiply by its figures separately, is because when large, it is not convenient to multiply by the *whole* at once.

3. The *object* of placing the first figure of each partial product under the figure by which we are multiplying, is because it is the *same* order as that figure; and if the *same* orders are placed under each other, we are less liable to fall into mistakes in adding the several partial products together. (Art. 51.)

4. The several *partial* products are added together for the obvious purpose of finding the *whole* product, or answer required.

22. What is the product of 958 multiplied by 607?

QUEST.—51. What do units multiplied into units produce? Tens into units? Of what order is the product universally, when the multiplier is units? 52. Explain the solution of the twenty-first example.

Suggestion.—Proceed as when the multiplier contains but *two* figures. After multiplying by the 7 units, we next multiply by the 6 hundreds, for the reason that there are no tens in the multiplier, and place the first figure of this partial product directly under the figure 6 by which we are multiplying.

Operation.

```
   958
   607
  ----
  6706
 5748
------
581506 Ans.
```

54. From the preceding illustrations and principles, we derive the following

GENERAL RULE FOR MULTIPLICATION.

I. *Write the multiplier under the multiplicand, units under units, tens under tens, &c.*

II. *When the multiplier contains but one figure, begin with the units, and multiply each figure of the multiplicand by the multiplier, setting down the result and carrying as in addition.*

III. *If the multiplier contains more than one figure, multiply each figure of the multiplicand by each figure of the multiplier separately, and write the first figure of each partial product under the figure by which you are multiplying.* (Art. 52.)

Finally, add the several partial products together, and the sum will be the whole product, or answer required.

PROOF.—*Multiply the multiplier by the multiplicand, and if the second result is the same as the first, the work is right.*

OBS. 1. This method of proof depends upon the principle, that the product of any two numbers is the same, whichever is taken for the multiplier. (Art. 47.)

2. A *second method of proof* is, to multiply the multiplicand by the multiplier *diminished* by 1; to this partial product *add* the multiplicand, and if the *sum* is equal to the *first result*, the work is right.

QUEST.—54. How do you write numbers for multiplication? When the multiplier contains but one figure, how do you proceed? When the multiplier contains more than one figure, how proceed? 52. *Obs.* What is meant by partial products? Why multiply by the figures separately? Why place the first figure of each partial product under the figure by which you multiply? 51. How does it appear, that the first figure of each partial product is the same order as the figure by which you are multiplying? Why add the several partial products together? 50. *Obs.* Why begin to multiply at the right hand of the multiplicand? 49. *Obs.* Does it make any difference with the result, which of the given numbers is taken for the multiplier? Which is usually taken? Why? 54. How is multiplication proved?

3. When the multiplier is small, we may add the multiplicand to itself as many times as there are units in the multiplier, and if the sum is equal to the product, the work is right. Thus 78×3=234. *Proof.*—78+78+78=234, which is the same as the product.

4. Multiplication may also be proved by *division*, and *by casting out the nines*; but neither of these methods can be explained here without anticipating principles belonging to division, with which the learner is supposed as yet to be unacquainted.

Note.—As soon as the pupil becomes familiar with the operation of multiplying, it will accelerate the process to drop the intervening words and simply pronounce the results as in addition. Thus, in multiplying 8543 by 6, instead of saying, "6 times 3 are 18: set down the 8 and carry the 1. 6 times 4 are 24, and 1 to carry makes 25," &c.; he should say, *eighteen*, (6×3), *twenty-five* (4×6 +1), *thirty-two*, (5×6+2), &c., setting down the units and carrying the tens while pronouncing each result.

23. Find the product of 63 multiplied by 36, and prove the operation.

Operation.		*Proof.*	
63	Multiplicand.	36	
36	Multiplier.	63	
378		108	
189		216	
2268	*Product.*	2268	*Product.*

24. Find the product of 87 multiplied by 45, and prove the operation.

25. Find the product of 256 multiplied by 75, and prove the operation.

26. Find the product of 278 multiplied by 83, and prove the operation.

27. Find the product of 347 multiplied by 256, and prove the operation.

28. Find the product of 569 multiplied by 308, and prove the operation.

29. Find the product of 67025 multiplied by 4005, and prove the operation.

30. Find the product of 841072 multiplied by 603, and prove the operation.

31. Find the product of 930005 multiplied by 6004, and prove the operation.

32. Find the product of 803044 multiplied by 7008, and prove the operation.

33. Find the product of 9845678 multiplied by 80006.

EXAMPLES FOR PRACTICE.

1. What will 465 hats cost, at 6 dollars apiece?

2. What will 638 sheep cost, at 4 dollars a head?

3. What will 1360 pieces of cloth cost, at 27 dollars a yard?

4. What cost 169 chairs, at 24 shillings apiece?

5. What cost 279 barrels of salt, at 19 shillings a barrel?

6. At 32 dollars a suit, how much will it cost to furnish 1161 soldiers with a suit of clothes apiece?

7. What cost 1565 acres of land, at 27 dollars per acre?

8. What cost 758 baskets of peaches, at 15 shillings a basket?

9. What cost 25650 pounds of opium, at 16 dollars a pound?

10. How much can a man earn in 12 months, at 35 dollars per month?

11. What will 23 loads of hay come to, at 18 dollars a load?

12. What will 45 cows come to, at 21 dollars apiece?

13. What will 56 hogsheads of molasses cost, at 32 dollars a hogshead?

14. What cost 128 firkins of butter, at 13 dollars a firkin?

15. What cost 97 kegs of tobacco, at 26 dollars per keg?

16. What cost 110 barrels of pork, at 19 dollars per barrel?

17. How much will 235 sheep come to, at 21 shillings a head?

18. How many bushels of corn will grow on 83 acres, at the average rate of 37 bushels to an acre?

19. In one bushel there are 32 quarts: how many quarts are there in 92 bushels?

20. What will 463 cattle come to, at 48 dollars per head?

21. How much will 78 thousand of boards cost, at 19 dollars per thousand?

22. What cost 243 chests of tea, at 37 dollars per chest?

23. A man bought 168 horses, at 63 dollars apiece: what did they come to?

24. What cost 256 barrels of beef, at 16 dollars a barrel?

25. If 376 men can build a fortification in 95 days, how long would it take 1 man to build it?

26. Allowing 365 days to a year, how many days has a man lived who is 45 years old?

27. If a garrison consume 725 pounds of beef in one day, how many pounds will they consume in 125 days?

28. How many pounds will the same garrison consume in 243 days?

29. How far will a ship sail in 365 days, at 215 miles per day?

30. What costs 678 tons of iron, at 115 dollars per ton?

31. 4623 × 2185.

32. 50301 × 3076.

33. 68023 × 4231.

34. 84209 × 72032.

35. 940325 × 52363.

36. 452305 × 6321.

37. 503042 × 80423.

38. 843891 × 98756.

39. 7003402 × 603001.

40. 98005075 × 9003007.

41. Multiply two thousand and seven by one thousand and four.

42. Multiply four thousand and forty by two thousand one hundred and three.

43. Multiply forty thousand, four hundred and four by ten thousand and ten.

44. Multiply one hundred and five thousand and seven by sixty thousand, four hundred and three.

45. Multiply five millions, two hundred and six by seventy thousand two hundred and five.

46. A man bought a drove of 560 sheep, at 13 shillings a head; it cost him 68 shillings to send them to market, and they brought him 17 shillings apiece: how much did he make on them?

47. A drover bought 360 head of cattle and 96 horses; he afterwards sold the former at a profit of 19 dollars a head, and the latter at a loss of 23 dollars a head: did he gain or lose by the operation, and how much?

48. A grocer bought 185 barrels of flour at 36 shillings a barrel, and 117 barrels at 41 shillings: he then sold the whole at 39 shillings. What was the result of his speculation?

49. In music, two minims are equal to a semibreve; two crotchets to a minim; two quavers to a crotchet; two semiquavers to a quaver; and two demi-semiquavers to a semiquaver: how many demi-semiquavers are equal to 259 semibreves?

50. Two persons start from the same place, and travel in the same direction; one at the rate of 33 miles per day, and the other at the rate of 37 miles per day: how far apart will they be at the end of 256 days?

CONTRACTIONS IN MULTIPLICATION.

ART. **55.** A *composite* number is one which is produced by *multiplying two or more factors together.* Thus, 14 is produced by multiplying 7 by 2; 15, by multiplying 5 by 3; 35, by 7 into 5, and are therefore composite numbers.

OBS. 1. The factors, which produce a composite number, are called its *component parts.* [illegible]hus, the factors 2 and 7, 5 and 3, 7 and 5, are the component parts of 14, 15, [illegible]d 35.

2. [illegible] of finding the factors which produce a composite number, is cal[illegible] *[illegible]e number into factors.*

1. [illegible]lve 4, 6, 9, and 10 into their factors.
2. [illegible]t are the factors of 15? 21? 22? and 25?
3. [illegible] are the factors of 33? 35? 14? 77? 49? 55?

56. [illegible]mbers may be resolved into *more* than *two* factors, [illegible] *different sets* of factors. Thus, the factors of 2[illegible] 2, 2 and 2; or 4, 3 and 2; or 6, 2 and 2; or 8 and 3; o[illegible]nd 4; or 12 and 2.

4. What are the different factors and sets of factors of 8? 12? 16? 18? 20?
5. What are the different factors and sets of factors of 30? 32? [illegible]6? 40?
6. Resolve 50, 54, 56, 60, 63, 64, 72, 81, 84, 96, into factors.

56. *a.* We have seen that the product of *two* numbers is the same, whichever factor is taken for the multiplier. (Art. 47.) In like manner it may be shown, that the product of any *three* or *more* factors is the same, in whatever order they are multiplied. For, the *product* of *two* factors may be considered as *one number*, and this may be taken either for the multiplicand, or the multiplier. Again, the *product* of *three* factors may be considered as *one number*, and be taken for the multiplicand, or the multiplier, &c. Thus, $24=3\times2\times2\times2=6\times2\times2=12\times2=6\times4=4\times2\times3=8\times3$.

QUEST.—55. What is a composite number? *Obs.* What are the factors which produce a composite number, sometimes called? What is meant by resolving a number into factors? 56. Are numbers ever composed of more than two factors? 56. *a.* When three or more factors are multiplied together, does it make any difference in the result, what order they are taken? How does this appear?

CASE I.—*Multiplying by a composite number.*

Ex. 1. What will 14 hats cost, at 8 dollars apiece?

Suggestion.—14 is a composite number, the factors of which are 7 and 2. Now since 14 is equal to twice 7, it is manifest that 14 hats will cost twice as much as 7 hats. We therefore first find the cost of 7 hats, then multiplying by 2, will give the cost of 14 hats. In other words, we first multiply by the factor 7, and that product by 2. Hence,

Operation.

$$\begin{array}{rr} & 8 \\ & 7 \\ \hline \text{7 hats} & 56 \text{ dolls.} \\ & 2 \\ \hline \text{14 hats} & 112 \text{ dolls.} \end{array}$$

57. To multiply by a composite number.

Multiply the multiplicand by one of the factors of the multiplier, and this product by another, and so on till you have multiplied by all the factors. The last product will be the answer.

OBS. 1. This method of contraction is based upon the principle, that the *product* of two or more factors is the *same*, in whatever order they are multiplied.

2. The *factors* into which a number may be *resolved*, must not be confounded with the *parts* into which it may be *separated.* (Art. 37.) *Factors* must be *multiplied* together to reproduce the given number, while *parts* must be *added* together to reproduce it. Thus, 56 may be resolved into two *factors*, 8 and 7; it may be separated into two *parts*, 5 tens or 50, and 6 units. Now, 8 multiplied by 7=56, and 50 added to 6=56.

2. What will 27 horses cost, at 85 dollars apiece?

3. What will 24 wagons cost, at 37 dollars apiece?

4. What will 36 cows cost, at 19 dollars a head?

5. What cost 45 acres of land, at 110 dollars per acre?

6. At 36 shillings per week, how much will it cost a person to board 52 weeks?

7. If a man travels at the rate of 42 miles a day, how far can he travel in 205 days?

8. At the rate of 56 bushels per acre, how much corn can be raised on 460 acres of land?

9. What cost 672 yards of cloth, at 24 shillings per yard?

10. What cost 1265 yoke of oxen, at 72 dollars per yoke?

QUEST.—57. When the multiplier is a composite number, how do you proceed? *Obs.* Upon what principle does this contraction depend? What is the difference between the factors into which a number may be resolved, and the parts into which it may be separated?

CASE II.—*Multiplying by* 10, 100, 1000, &c.

58. It is a fundamental principle of notation, that each removal of a figure one place towards the left, increases its value *ten times;* (Art. 9;) consequently, annexing a *cipher* to a number will increase its value *ten times*, or *multiply* it by 10; annexing *two* ciphers will increase its value a *hundred times*, or multiply it by 100; annexing *three* ciphers will multiply it by 1000, &c.

Thus, annexing a cipher to 12, it becomes 120, and is the same as 12×10. Annexing *two* ciphers to 12, it becomes 1200, and is the same as 12×100; annexing *three* ciphers, it becomes 12000, and is the same as 12×1000 &c. Hence,

12	12	12
10	100	1000
120	1200	12000

59. To multiply by 10, 100, 1000, &c.

Annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and the number thus formed will be the product required.

Note.—To *annex* means to place *after*, or at the *right hand*.

OBS. The *reason* of this contraction is evident from the fact, that every cipher we annex to the multiplicand removes it one place towards the left, and therefore increases its value 10 times, or multiplies it by 10. (Art. 9.)

11. What will 10 drums of figs weigh, at 28 pounds a drum?

12. How many pages are there in 100 books, each book having 352 pages?

13. Multiply 476 by 1000.

14. Multiply 53486 by 10000.

15. Multiply 12046708 by 100000.

16. Multiply 26900785 by 1000000.

17. Multiply 89063457 by 10000000.

18. Multiply 9460305068 by 100000.

19. Multiply 78312065073 by 10000.

QUEST.—58. What is the effect on the value of a figure to remove it one place towards the left? Two places? Three places? 59. How do you multiply by 10, 100, 1000, &c.? *Note.* What is the meaning of the term annex? *Obs.* How does it appear that this contraction gives the true answer?

CASE III.—*Ciphers on the right of both factors.*

60. Any number with ciphers on its right, is obviously a composite number; the significant figure or figures, being one factor, and 1 with the given ciphers annexed to it, the other factor. Thus 20 may be resolved into the factors, 2 and 10.

20. What will 200 acres of land cost, at 32 dollars per acre?

Suggestion.—In this example, the *multiplier* (200) has ciphers on its right, and is therefore a composite number. Hence, we first multiply by the factor 2; then, by the other factor, 100, by annexing two ciphers to this product.

Operation.

```
      32
     200
Ans. 6400 dolls.
```

21. What is the product of 17000 into 23?

Suggestion.—In this example, the *multiplicand* (17000) has ciphers on its right; consequently, it is a composite number, the factors of which are 17 and 1000. But the product of two or more numbers is the same in whatever order they are multiplied; (Art. 56.*a.*;) consequently, multiplying the factor 17 by 23, and this product by 1000, will give the same result as 17000 × 23.

Operation.

```
       17000
          23
          51
         34
Ans.  391000
```

22. What is the product of 29000 into 1300?

Suggestion.—This example combines the principles of the last two; that is, the *multiplier* and *multiplicand both* have ciphers on the right; consequently, they are both composite numbers. We therefore first multiply the significant figures together, viz: the factors 29 and 13; then multiply this product into the product of the other two factors, (1000 × 100 = 100000), by annexing as many ciphers, as there are on the right of the multiplier and multiplicand. Hence,

Operation.

```
       29000
        1300
         87
        29
Prod. 37700000
```

61. When the *multiplier*, or *multiplicand*, or *both*, have ciphers on the right.

Multiply the significant figures together, and to the product annex as many ciphers as there are on the right of both factors.

OBS. This Case combines the principles of the two preceding Cases; for every

number with one or more ciphers on its right, we have just seen, is a composite number, and one of its factors is always 10, 100, 1000, &c. (Art. 60.)

23. 1920 × 2000.
24. 4376 × 2500.
25. 50634 × 41000.
26. 630125 × 620000.
27. 12000 × 31.
28. 370000 × 32.
29. 8120000 × 46.
30. 56300000 × 64.
31. 62800000 × 890.
32. 54000000 × 700.
33. 43000000 × 600.
34. 563800 × 7200.
35. 1230000 × 12000.
36. 310200 × 20000.
37. 2065000 × 810000.
38. 2100000 × 510000.

CASE IV.—*Multiplying by 9, 99, 999, &c.*

39. What will 99 acres of land cost, at 127 dollars per acre?

Analysis.—Since 1 acre costs 127 dollars, 100 acres will cost 100 times 127 dollars. Now, to multiply by 100, we annex two ciphers to the multiplicand, and it becomes 12700 dollars.

Operation.

12700	cost of	100	A.
127	"	1	"
12573	"	99	"

But we wished to find the cost of 99 acres only. Now 99 is 1 less than 100; therefore, if we subtract the price of 1 acre from the price of 100, it will give the price of 99 acres. Hence,

62. To multiply by 9, 99, 999, or any number of 9s,

Annex as many ciphers to the multiplicand as there are 9s in the multiplier; from the result subtract the given multiplicand, and the remainder will be the answer required.

Note.—The *reason* of this method is obvious from the fact, that annexing as many ciphers to the multiplicand as there are 9s in the multiplier, multiplies it by 100, it repeats it *once more* than is required; consequently, subtracting the multiplicand from the number thus produced, must give the true answer.

40. Multiply 26895 by 99. *Ans.* 2662605.
41. Multiply 78463 by 99.
42. Multiply 84607 by 999.
43. Multiply 678468 by 999.
44. Multiply 734567 by 999.
45. Multiply 846673 by 999.
46. 7894568 × 9999.
47. 8748278 × 99999.
48. 9068207 × 99999.
49. 8760004 × 999999.
50. 9999999 × 999999.

QUEST.—61. How do you proceed when the multiplier, or multiplicand, or both have ciphers on the right? Obs. Upon what principle does this contraction depend? 62. How do you multiply by 9, 99, &c.?

SECTION V.

DIVISION.

MENTAL EXERCISES.

ART. 63. EX. 1. How many boxes of oranges, at 3 dollars a box, can be bought for 12 dollars?

Analysis.—If 3 dollars will buy 1 box of oranges, 12 dollars will buy as many boxes as 3 is contained times in 12. Now 3 is contained in 12, 4 times. Therefore 12 dollars will buy 4 boxes of oranges, at 3 dollars a box.

DIVISION TABLE.

2 in 2, once	3 in 3, once	4 in 4, once	5 in 5, once	6 in 6, once	7 in 7, once	8 in 8, once	9 in 9, once
4, 2	6, 2	8, 2	10, 2	12, 2	14, 2	16, 2	18, 2
6, 3	9, 3	12, 3	15, 3	18, 3	21, 3	24, 3	27, 3
8, 4	12, 4	16, 4	20, 4	24, 4	28, 4	32, 4	36, 4
10, 5	15, 5	20, 5	25, 5	30, 5	35, 5	40, 5	45, 5
12, 6	18, 6	24, 6	30, 6	36, 6	42, 6	48, 6	54, 6
14, 7	21, 7	28, 7	35, 7	42, 7	49, 7	56, 7	63, 7
16, 8	24, 8	32, 8	40, 8	48, 8	56, 8	64, 8	72, 8
18, 9	27, 9	36, 9	45, 9	54, 9	63, 9	72, 9	81, 9

11. How many pair of boots, at 2 dollars a pair, can be bought for 24 dollars? For 22? 20? 18? 16? 14? 12? 10?

12. How many barrels of cider, at 3 dollars a barrel, can you buy for 36 dollars? For 33? 27? 24? 21? 18? 15? 12?

13. How many quarts of milk, at 4 cents a quart, can you buy for 48 cents? For 44? 40? 36? 32? 28? 24? 20? 16?

14. At 5 cents an ounce, how many ounces of wafers can you buy for 60 cents? For 55? 50? 45? 40? 35? 30? 25?

15. At 6 shillings a pair, how many pair of gloves can be bought for 60 shillings? For 54? 48? 42? 36? 30? 24? 18?

16. How many pounds of butter, at 7 cents a pound, can be purchased for 63 cents? 56? 49? 42? 35? 28? 21? 14?

17. How many cloaks will 72 yards of cloth make, allowing *8 yards to a cloak*? How many 64? 56? 48? 40? 32? 24?

18. How many cows, at 9 dollars apiece, can be bought for 81 dollars? For 72? 63? 54? 45? 36? 27? 18? 9?

19. How many times is 4 contained in 36? 48? 40?

20. How many times is 8 contained in 40? 56? 48? 64? 72?

21. In 25, how many times 4, and how many over?

Ans. 6 times and 1 over.

22. In 34, how many times 5, and how many over? In 43? 45? 37? 28? 39?

23. In 20, how many times 3, and how many over? How many times 4? 2? 10? 6?

24. In 24, how many times 7, and how many over? 6? 5? 9? 12? 2?

25. In 36, how many times 6? 7? 3? 8? 12? 5? 9?

26. In 32, how many times 6? 4? 3? 16?

27. How many hats, at 6 dollars apiece, can be bought for 60 dollars?

28. How many tons of hay, at 9 dollars per ton, can you buy for 81 dollars?

29. If you travel 7 miles an hour, how long will it take to travel 70 miles?

30. If you pay 10 cents apiece for slates, how many can you buy for 95 cents, and how many cents over?

31. George bought 24 oranges, which he divided equally among his 3 brothers: how many did each receive?

Analysis.—Since 24 oranges were divided equally among 3 boys, each must have received 1 orange as often as 3 oranges are contained in 24 oranges. Now 3 is contained in 24, 8 times. Therefore, each brother received 8 oranges.

32. Henry has 15 apples which he wishes to divide equally among 3 of his companions: how many can he give to each?

33. A gentleman sent 20 peaches to be divided equally among 4 boys: how many did each boy receive?

34. A dairy-woman having 30 pounds of butter, wishes to pack it in 5 boxes, so that each box shall have an equal number of pounds: how many pounds must she put in each box?

35. I have 21 acres of land, which I wish to fence into 7 equal lots: how many acres must I put into each lot?

36. A boy having 28 marbles, wished to divide them into 4 equal piles: how many must he put in a pile?

37. I have 40 peach-trees, which I wish to set out in 5 equal rows: how many must I set in a row?

38. There were 45 scholars in a certain school, and the teacher divided them into 5 equal classes: how many did he put in a class?

39. If 50 dollars were divided equally among 10 men, how many dollars would each man receive?

40. A company of 8 boys buying a boat for 32 dollars, agreed to share the expense equally: how much must each one pay?

41. In a certain orchard there are 54 apple trees, and 6 trees in each row: how many rows are there in the orchard?

42. If 63 quills are divided equally among 7 pupils, how many will each receive?

43. If you divide 36 into 4 equal parts, how many will there be in a part?

44. If you divide 56 into 8 equal parts, how many will each part contain?

45. If you divide 48 into 6 equal parts, how many will each part contain?

46. A gentleman distributed 40 dollars equally among 8 beggars: how many dollars did he give to each?

47. A company of 6 boys found a pocket-book, and on returning it to its owner, he handed them 60 dollars to be shared equally among them: what was each one's share?

48. A merchant received 72 dollars for 6 coats of equal value: how much was that apiece?

49. A man paid 81 cents for the use of a horse and buggy to ride 9 miles: how much was that a mile?

50. If you divide 90 dollars into 10 equal parts, how many dollars will there be in each part?

51. George laid out 30 cents for apples, at the rate of 6 for 3 cents: how many apples did he buy?

52. A man had a piece of work which 5 men could do in 12 days: how long would it take 6 men to do the same work? How long would it take 12 men?

53. A man bought 48 sheep, and paid at the rate of 12 dollars for 4 sheep: what did they cost him?

54. A lad bought 12 oranges at 8 cents apiece, and sold them for 12 cents more than he gave for them: how much did he *get apiece*; and how much did he make on an orange?

EXERCISES FOR THE SLATE.

63. *a.* In each of the above examples, there are *two given numbers*, and the solution consists in finding *how many times* one of them is contained in the other. The operation by which we obtain this result, is called *Division*. Hence,

64. DIVISION *is the process of finding how many times one given number is contained in another.*

The number to be *divided*, is called the *dividend*.

The number by which we *divide*, is called the *divisor*.

The *answer*, or number *obtained* by division, is called the *quotient*, and shows how *many times* the divisor is contained in the dividend. Hence, it may be said,

65. *Division is finding a quotient, which multiplied into the divisor, will produce the dividend.*

NOTE.—The term *quotient*, is derived from the Latin word *quoties*, which signifies *how often*, or *how many times*.

66. The number which is sometimes *left after* division, is called the *remainder*. Thus, when we say 4 is contained in 25, 6 times and 1 over, 4 is the divisor, 25 the dividend, 6 the quotient, and 1 the remainder.

OBS. 1. The remainder is always *less* than the divisor; for if it were equal to, or greater than the divisor, the divisor could be contained *once more* in the dividend.

2. The remainder is also of the same denomination as the dividend; for it is a part of it.

3. Division, it will be seen, is applied to *two distinct objects:*

First, to find *how many times* one given number is contained in another;

Second, to divide a given number into *equal parts* in order to ascertain the *value* or *magnitude* of those parts.

The first thirty examples in this section, are instances of the former; the next twenty, of the latter. The mode of operation, however, is the same in both cases.

QUEST.—64. What is division? What is the number to be divided called? The number by which we divide? What is the answer called? What does the quotient show? *Note.* What is the meaning of the term quotient? 65. What then may division be said to be? 66. What is the number called which is sometimes left after division? When we say 4 is in 25, 6 times and 1 over, what is the 4 called? The 25? The 6? The 1? When we say 6 is in 45, 7 times and 3 over, which is the divisor? The dividend? The quotient? The remainder? *Obs.* Is the remainder greater or less than the divisor? Why? Of what denomination is it? Why?

66. *a.* It will be perceived that division is *similar* in principle to subtraction, and may be performed by it. For instance, to find how many times 3 is contained in 12, subtract 3 (the divisor) continually from 12 (the dividend) until the latter is exhausted; then counting these repeated subtractions, we shall have the true quotient. Thus, 3 from 12 leaves 9; 3 from 9 leaves 6; 3 from 6 leaves 3; 3 from 3 leaves 0. Now by counting, we find that 3 has been taken from 12, 4 times; consequently, 3 is contained in 12, 4 times.

By division, this result is reached by a single step. Thus, 3 is contained in 12, 4 times. Hence,

Division is sometimes defined to be a short way of performing repeated subtractions of the same number.

OBS. 1. It will help the pupil to understand how much division *abbreviates* subtraction, by considering how long the process would be to find how many times 5 is contained in 6892345 by repeated subtractions.

2. It will also be observed that division is the reverse of multiplication. Multiplication is the *repeated addition* of the same number; division is the *repeated subtraction* of the same number. (Art. 48.) The *product* of the one answers to the *dividend* of the other: but the latter is always *given*, while the former is *required*.

3. When the dividend denotes things *of one kind, or denomination only*, the operation is called *Simple Division*.

67. *The sign of division is a horizontal line between two dots* (÷), and shows that the number *before* it is to be divided by the number *after* it. Thus, the expression $24 \div 6$, signifies that 24 is to be divided by 6, and is read, "24 divided by 6."

Division is also expressed by placing the divisor *under* the dividend with a short line between them. Thus, the expression $\frac{35}{7}$, shows that 35 is to be divided by 7. It is read, "35 divided by 7," or "35 sevenths," and is equivalent to $35 \div 7$.

Read the following expressions: $54 \div 9 + 15 = 5 \times 3 + 6$.
$29 + \frac{15}{3} = 40 \div 5 \times 4$. $48 \div 6 = 63 - 15 \div 12 \times 2$.

QUEST.—66. *a.* To what rule is division similar in principle? In what other way is division sometimes defined? *Obs.* Of what is division the reverse? How does this appear? When the dividend denotes things of one denomination only, what is the operation called? 67. What is the sign of division? What does it *show?* In what other way is division expressed?

SHORT DIVISION.

68. SHORT DIVISION *is the process of dividing, when the operation is carried on in the mind, and the quotient only is set down.*

Ex. 1. How many barrels of cider, at 2 dollars a barrel, can you buy for 648 dollars?

Operation.	
Divisor.	Dividend.
2)	6 4 8
Quot.	3 2 4 b.

Suggestion.—Having written the divisor on the left of the dividend with a curve line between them, we begin at the left hand, and divide each figure by the divisor, setting the result under the figure divided. Thus, 2 is contained in 6, 3 times. Now as the 6 denotes hundreds, the 3 must also be hundreds; we therefore write it in hundreds' place, under the figure divided. Next, 2 is contained in 4, 2 times; and since the 4 is tens, the 2 is also tens, and must be written in tens' place. Finally, 2 is in 8, 4 times; and as the 8 is units, the 4 must also be units, and be written in units' place. *Ans.* 324 barrels.

69. When the divisor is not contained in the *first* figure of the dividend, we must find how many times it is contained in the *first two figures*, or the *fewest* that will contain it.

2. Divide 637 by 7. *Ans.* 91.

3. Divide 56 by 8. 4. Divide 49 by 7.

5. Divide 54 by 9. 6. Divide 72 by 8.

7. How many hats, at 2 dollars apiece, can be bought for 468 dollars? *Ans.* 234 hats.

8. How many sheep, at 3 dollars a head, will 369 dollars buy?

9. A man wishes to divide 248 acres of land equally between his two sons: how many acres will each receive?

10. How many times is 4 contained in 488?

11. A farmer bought 96 dollars worth of dry goods, and agreed to pay in wood at 3 dollars a cord: how many cords did it take to pay his bill? *Ans.* 32 cords.

12. How many yards in 969 feet, allowing 3 feet to a yard?

13. Divide 69936 by 3. 14. Divide 48848 by 4.

15. Divide 55555 by 5. 16. Divide 2486286 by 2.

QUEST.—68. What is Short Division? Explain the solution of the first example.

70. After dividing any figure of the dividend, if there is a *remainder*, prefix it mentally to the next figure of the dividend, and then divide this number as before.

If the divisor is *not contained* in any figure of the dividend, place a *cipher* in the quotient, and prefixing this figure to the next one in the dividend, proceed as before.

Note.—To *prefix* means to place *before*, or at the *left hand*.

OBS. 1. The learner will observe, in *division* we begin at the *left hand*, instead of the *right*, as in Addition, Subtraction, and Multiplication. The *reason* is, that in dividing a *higher order*, there is frequently a remainder which must be united with the next *lower order*, before the division can be performed.

2. The *reason* for placing a cipher in the quotient, when the divisor is not contained in a figure of the dividend, is to preserve the *true local value* of the several quotient figures.

3. The divisor is placed on the *left* of the dividend, and the quotient *under* it, merely for the sake of convenience. When division is represented by the sign (÷), the divisor is placed on the *right* of the dividend; and when represented in the form of a fraction, the divisor is placed under the dividend.

17. How many hats, at 3 dollars apiece, can be bought for 8421 dollars?

Suggestion.—Dividing 8 by 3, there is 2 remainder. This we prefix mentally to the next figure of the dividend. Now, 3 is in 24, 8 times. Again, 3 is not contained in 2, the next figure of the dividend; we therefore place a cipher in the quotient, and prefixing the 2 to the 1, divide as before.

Operation.
3)8421
Ans. 2807 hats.

71. When there is a *remainder*, after dividing the *last figure* of the dividend, it should be written over the divisor and *annexed* to the quotient.

18. A teacher having 125 apples, wishes to divide them equally among 4 pupils: how many can he give to each?

Suggestion.—After giving them 31 apiece, it will be seen that there is one remainder, or 1 apple left, which is not divided. Now it is plain that the whole dividend must be divided, in order to render the division complete. But 4 is not contained in 1; hence the division must be represented by writing the 4 under the 1, thus $\frac{1}{4}$, and in order to complete the quotient, the $\frac{1}{4}$ must be annexed to the 12. (Art. 67.) The true quotient, therefore, is $12\frac{1}{4}$, which is read, "twelve and *one fourth*."

Operation.
4)125
31—1 rem.
Ans. $31\frac{1}{4}$ apples.

72. From the preceding illustrations and principles, we derive the following

RULE FOR SHORT DIVISION.

I. *Write the divisor on the left of the dividend, with a curve line between them.*

Beginning at the left hand, divide each figure of the dividend by the divisor, and place each quotient figure under the figure divided. (Art. 68.)

II. *When there is a remainder after dividing any figure, prefix it to the next figure of the dividend, and divide this number as before. If the divisor is not contained in any figure of the dividend, place a cipher in the quotient, and prefix this figure to the next one of the dividend, as if it were a remainder.*

III. *When there is a remainder after dividing the last figure, write it over the divisor and annex it to the quotient.* (Art. 71.)

PROOF.—*Multiply the divisor by the quotient, to the product add the remainder, and if the result is equal to the dividend, the work is right.* (Art. 65.)

19. Divide 387475 by 6, and prove the operation.

Solution.		*Proof.*	$64579 \times 6 = 387474$
	6)387475	Add the rem.	1
Quot.	64579 and 1 *rem.*	The result is,	387475

OBS. 1. The reason of this method of proof, may be seen from the fact that the quotient shows how many times the divisor is contained in the dividend; consequently, if the divisor is repeated or taken as many times as there are units in the quotient, it must produce the dividend. (Art. 64.)

2. Division may also be proved by subtracting the remainder, if any, from the dividend, then dividing the result by the quotient.

Note.—As soon as the learner becomes familiar with the process of dividing by Short Division, he should drop the intervening words as in the preceding

QUEST.—72. How do you write numbers for division? How proceed in Short Division? When there is a remainder after dividing a figure, what do you do with it? If the divisor is not contained in any figure of the dividend, how proceed? *Note.* What is the meaning of the term, prefix? When there is a remainder after dividing the last figure of the dividend, what must be done with it? 70. *Obs.* Why place the divisor on the left of the dividend and the quotient under it? Why begin to divide at the left hand? Why place a cipher in the quotient, when the divisor is not contained in a figure of the dividend? How is division proved? *Obs.* What other way of proving division is mentioned?

rules, simply pronouncing the quotient figures, and setting them down at once. Thus, in the example above instead of saying, 6 is contained in 38, 6 times and 2 over; 6 is contained in 27, 4 times and 3 over, &c., he should learn to say, *six, four, five, seven*, &c., setting down each quotient figure while pronouncing it.

20. Divide 255 by 5.
21. Divide 1248 by 4.
22. Divide 24693 by 3.
23. Divide 4266 by 6.
24. Divide 35555 by 5.
25. Divide 5677 by 7.
26. Divide 64888 by 8.
27. Divide 8199 by 9.

28. A man bought 741 acres of land, which he divided equally among his 3 sons: how many acres did each receive?

29. If a man travel at the rate of 5 miles an hour, how long will it take him to travel 845 miles? *Ans.* 169 hours.

30. If 192 pounds of flour were equally divided among 4 persons, how many pounds would each receive?

31. Divide 45690 by 6.
32. Divide 52584 by 8.
33. Divide 81670 by 5.
34. Divide 28296 by 9.

35. When flour is 6 dollars a barrel, how much can be bought for 642 dollars?

36. Divide 36060 by 6.
37. Divide 49000 by 7.
38. Divide 45900 by 9.
39. Divide 568000 by 8.

40. Allowing 5 yards of cloth for a suit of clothes, how many suits can be made from 1525 yards? *Ans.* 305 suits.

41. A company of 3 men agree to pay a bill of 321 dollars: how many dollars must each man pay?

42. Divide 14350 by 7.
43. Divide 30420 by 6.
44. Divide 25105 by 5.
45. Divide 643240 by 8.

46. A merchant wishes to divide 549 oranges equally among 4 boys: how many must he give to each?

47. A shoemaker has 672 pair of boots, which he wishes to pack in 6 boxes: how many pair can he put into a box?

48. A baker wishes to lay out 756 dollars in flour: how much can he buy, when the price is 5 dollars a barrel?

49. How many yearlings, at 9 dollars a head, can be bought for 468 dollars?

50. How many acres of land, at 6 dollars an acre, can I buy for 978 dollars?

51. Divide 5468053 by 7.
52. Divide 4672304 by 8.
53. Divide 6000000 by 9.
54. Divide 7003041 by 6.
55. Divide 7034916 by 10.
56. Divide 8097603 by 11.
57. Divide 8306734 by 12.
58. Divide 9603405 by 12.

LONG DIVISION.

ART. 73. *Long Division is the process of dividing, when the result of each step in the operation is set down.*

74. *Long Division is the same in principle as Short Division.* The only difference between them is, that in the former, the *result* of *each step* in the operation is set down; while in the latter, the process of dividing is carried on in the mind, and the *quotient only* is set down. (Art. 68.)

Ex. 1. Divide 1504 by 4, using Long Division.

Operation.

```
Divi. Divid. Quot.
  4)1504(376
    12
    ──
     30
     28
     ──
      24
      24
```

Suggestion.—Having set down the numbers as in Short Division, we first find how many times the divisor 4, is contained in 15, the fewest figures on the left of the dividend that will contain it, (4 is in 15, 3 times,) and place the quotient figure on the *right of the dividend* with a curve line between them. Next, we multiply the divisor by the quotient figure, (3 times 4 are 12,) and write the product under the figures divided. We then subtract this product from the figures divided. (12 from 15 leaves 3.) Finally, we bring down the next figure of the dividend, and placing it on the right of the remainder, divide this number as above. (4 is in 30, 7 times.) Place the 7 on the right of the last quotient figure, then multiply, subtract, and proceed to find the next figure of the quotient as before.

75. From the preceding operation, the learner will perceive, there are *four steps* in Long Division: 1st. Find *how many times* the divisor is contained, &c.; 2d. *Multiply;* 3d. *Subtract;* 4th. *Bring down.*

Note.—To prevent mistakes, it is advisable to put a dot under each figure of the dividend, when it is *brought down.*

2. Divide 578 by 2, and prove the operation. *Ans.* 289.
3. Divide 7560 by 5. *Ans.* 1512.
4. Divide 126332 by 4. *Ans.* 31583.
5. How many times is 6 contained in 763251?

QUEST.—73. What is Long Division? 74. What is the difference between Long and Short Division? 75. How many steps are there in Long Division?

6. How many times is 6 contained in 4026942?
7. How many times is 8 contained in 2612488?
8. How many times is 5 contained in 1682840?
9. How many times is 7 contained in 45063284?
10. How many times is 9 contained in 650981507?
11. Divide 2234 by 21.

Operation.

```
21)2234(106 8/21.
   21
   ---
    134
    126
    ---
      8 rem.
```

Suggestion.—Having subtracted the 21, and brought down the next figure of the dividend, we have 13 to be divided by 21. But 21 is not contained in 13; we therefore put a *cipher* in the quotient, and bring down the next figure. Then 21 is in 134, 6 times, and 8 rem. Write the 8 over the divisor, and annex it to the quotient. Hence,

76. After the first quotient figure is obtained, for *each figure of the dividend which is brought down*, either a *significant figure*, or a *cipher* must be put in the *quotient*.

77. From the preceding illustrations and principles we derive the following

RULE FOR LONG DIVISION.

I. *Beginning on the left of the dividend, find how many times the divisor is contained in the fewest figures that will contain it, and place the quotient figure on the right of the dividend with a curve line between them.*

II. *Multiply the divisor by this figure and subtract the product from the figures divided; to the right of the remainder bring down the next figure of the dividend, and divide this number as before. Proceed in this manner till all the figures of the dividend are divided.*

III. *When there is a remainder after dividing the last figure, write it over the divisor, and annex it to the quotient, as in short division.* (Art. 71.)

OBS. 1. To find the first quotient figure when the divisor is *large*, the learner should consider how many times the *first* figure of the divisor is contained in the

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QUEST.—77. What is the rule for Long Division? If there is a remainder after dividing the last figure of the dividend, what must be done with it? *Obs.* When the divisor is large, how do you find the first quotient figure?

first, or *first two figures of the dividend*, making due allowance for *carrying* from the product of the other figures of the divisor when multiplied into the quotient figure. The other quotient figures may be found in a similar manner.

2. After a figure is brought down, if the divisor is not contained in the number thus formed, place a *cipher* in the quotient, and bring down the next figure. If it is not contained in this number, bring down another, and so on; remembering for every figure brought down, either a cipher or significant figure must be placed in the quotient. (Art. 76.)

3. If the product of the divisor into the figure placed in the quotient, is *larger* than that part of the dividend under consideration, the quotient figure must be *diminished*.

4. If the *remainder*, after subtracting the product of the divisor into the figure placed in the quotient, is *equal* to, or *greater* than the divisor, the quotient figure must be *increased*. (Art. 66, Obs. 1.)

12. Divide 1035 by 45. *Ans.* 23.
13. Divide 5378 by 25. *Ans.* 215$\frac{3}{25}$.
14. Divide 7840 by 22.
15. Divide 59690 by 45.
16. Divide 81229 by 67.
17. Divide 98425 by 81.
18. Divide 18029 by 121. *Ans.* 149.
19. How many times is 93 contained in 100469?
20. How many times is 156 contained in 140672?

PROOF OF MULTIPLICATION BY DIVISION.

77. *a*. *Divide the product by one of the factors, and if the quotient thus arising is equal to the other factor, the work is right.*

OBS. The *reason* of this proof is obvious from the fact that it is simply *reversing* the operation by which the product is formed, and must, therefore lead us back to the number with which we started. That is, the multiplier and multiplicand, are the factors which produced the product; consequently, if the product is divided by one of them, the quotient must be the other.

EXAMPLES FOR PRACTICE.

1. If a man travel at the rate of 16 miles an hour, how long will it take him to travel 384 miles?

2. How many yards of broadcloth, at 18 dollars a yard, can be bought for 648 dollars?

3. A farmer bought a lot of cattle, at 22 dollars per head, and paid 946 dollars for them: how many did he buy?

4. How many tons of Liverpool coal, at 21 dollars a ton, can be bought for 2268 dollars?

QUEST.—After a figure is brought down, suppose the divisor is not contained in the number thus formed, how proceed? 77. *a*. How is multiplication proved by division? *Obs.* Explain the reason of this method of proof?

5. At 18 dollars a month, how long will it take a man to earn 234 dollars?

6. In one day there are 24 hours; how many days are there in 480 hours?

7. A man traveled 315 miles in 21 hours: how many miles did he travel per hour?

8. At 16 dollars a ton, how many tons of hay can be bought for 176 dollars?

9. How many casks of wine, at 25 dollars a cask, can be bought for 275 dollars?

10. The ship George Washington was 25 days in crossing the Atlantic Ocean, a distance of 3000 miles. How many miles did the ship sail per day?

11. The steamer Great Western crossed it in 15 days. How many miles did she sail per day?

12. The steamer Caledonia crossed it in 12 days. How many miles did she sail per day?

13. If a man can earn 35 dollars a month, how long will it take him to earn 420 dollars?

14. If 63 gallons make a hogshead, how many hogsheads will 1260 gallons make?

15. If a ship can sail 264 miles per day, how far can she sail in an hour?

16. How many times 36 in 508, and how many over?

17. How many times 45 in 1890, and how many over?

18. How many times 22 in 865, and how many over?

19. How many times 34 in 2460, and how many over?

20. How many times 26 in 15296, and how many over?

21. How many times 125 in 3061, and how many over?

22. How many times 231 in 1861, and how many over?

23. In 16 times 256, how many times 18?

24. In 36 times 157, how many times 21?

25. In 45 times 2251, how many times 86?

26. In 19 times 186, how many times 75?

27. In 63 times 102, how many times 87?

28. In 78 times 276, how many times 130?

29. In 115 times 321, how many times 95?

30. In 144 times 137, how many times 312?

31. 48670÷419. 33. 589067÷868.

32. 60342÷728. 34. 790671÷978.

35. 448210 ÷ 2145.
36. 203426 ÷ 2456.
37. 3507689 ÷ 4703.
38. 8061072 ÷ 6841.
39. 4562341 ÷ 38026.
40. 9605307 ÷ 41782.
41. 14058230 ÷ 53072.
42. 20380596 ÷ 70321.
43. 80000000 ÷ 860034.
44. 72567845 ÷ 900023.
45. 400340051 ÷ 600724.
46. 270315982 ÷ 321672.
47. 178000467 ÷ 4023412.
48. 2000007121 ÷ 56021348.
49. 62438724161 ÷ 623187912.
50. 887838215678 ÷ 4123418978.

CONTRACTIONS IN DIVISION.

CASE I.—*Dividing by a composite number.*

Ex. 1. A man divided 168 oranges equally among his 14 grandchildren who belonged to 2 families, each family containing 7 children: how many oranges did each child receive?

Operation.

$$\begin{array}{r} 2)168 \\ \hline 7)84 \\ \hline 12 \end{array} \quad \textit{Ans.}$$

Suggestion.—The divisor 14, is a composite number, and its factors 2 and 7, correspond with the number of families and the number of children in each family. We first find how many each family received, by dividing by the factor 2; then how many each child received, by dividing by the factor 7. Therefore each child received 12 oranges. Hence,

78. To divide by a composite number.

I. *Divide the dividend by one of the factors of the divisor, then divide the quotient thus obtained by another factor, and so on till all the factors are employed. The last quotient will be the answer.*

OBS. 1. This contraction depends upon the principle, that if the divisor is resolved into two factors, and the dividend is divided by one of them, the result will be as *many times too large*, as there are units in the other factor; and so on of any number of factors. To correct this result, we divide it by the other factor or factors. Thus, dividing by 2, in the example above, the result is 7 times too large; for the given divisor is 14, which is 7 times 2. Now, the *seventh* part of the *half* is obviously the same as the *fourteenth* part of the whole.

2. This contraction is the *reverse* of that in multiplication. The result will be the same, in whatever order the factors are taken. (Art. 57.)

2. Divide 455 by 35; 585 by 15; 609 by 21; 846 by 18; 1971 by 27; 1127 by 49. *Ans.* 13; 39; 29; 47; 73; 23.

QUEST.—78. How proceed when the divisor is a composite number? *Obs.* Upon what principle does this contraction depend?

3. A teacher having 24 scholars, wishes to distribute 216 pears among them equally: how many can he give to each?

78. *a.* To find the *true* remainder when the divisor is a composite number.

If the divisor is resolved into but two factors, multiply the last remainder by the first divisor, to the product add the first remainder, and the result will be the true remainder.

When more than two factors are employed, multiply each remainder by all the divisors preceding the one from which it arose; to the sum of their products, add the first remainder, and the result will be the true remainder.

OBS. The *object* of multiplying each remainder by all the divisors preceding the one from which it arose, is to find how many units of the same value as those in the given dividend, each contains. To the sum of these products, we then add the first remainder, in order to find the *whole* or *true* remainder.

Thus, dividing by 4 in the solution below, every unit of the quotient manifestly contains 4 of the units in the given dividend, and every unit that remains of it will contain the same; (Art. 65, Obs.;) therefore the second remainder must be multiplied by 4 in order to find the units it contains of the given dividend. Again, dividing by 3, every unit of this quotient will contain 3 of the preceding units, or 12 of the first; therefore, what remains of it, that is, the third remainder, must be multiplied by 12, or by its factors 3 and 4, which are the preceding divisors.

In like manner multiplying any remainder by all the divisors preceding that from which it arose, will show how many of the units in the given dividend it contains, and the sum of all the products added to the first remainder, will be the true remainder.

4. A dairyman having 479 quarts of milk, wished to know how many cans, holding 60 quarts apiece, he could fill, and how many quarts he would have left.

Solution.

4)479	*True remainder.*		
3)119—3 rem.	The first remainder is 3, that is	3 qts.
5)39—2 rem.	The second " is 2, and 2×4	$=$ 8 "
7—4 rem.	The third " is 4, and $4 \times 3 \times 4 =$	48 "
	Hence, the true remainder is	59 qts.

He could, therefore, fill 7 cans, and have 59 quarts left.

5. Divide 453 by 64, using its factors 4, 4, and 4.

6. Divide 237 by 72, using its factors 3, 3, and 8.

QUEST.—78. *a.* How find the true remainder, when the divisor is resolved into but two factors? How, when more than two factors are employed? *Obs.* *Why multiply each* remainder by all the preceding divisors?

CASE II.—*Dividing by* 10, 100, 1000, *&c.*

79. It has been shown that *annexing* a cipher to a number, *increases* its value *ten times*, or *multiplies* it by 10. (Art. 58.)

Reversing this process; that is, *removing* a cipher from the right hand of a number, will evidently *diminish* its value *ten times*, or *divide* it by 10; for each figure in the number is thus restored to its original place, and consequently to its original value. Thus, removing the cipher from 120, it becomes 12, which is the same as 120÷10.

In the same manner, it may be shown, that removing *two* ciphers from the right of a number, divides it by 100; removing *three* divides it by 1000, &c. Hence,

80. To divide by 10, 100, 1000, &c.

Cut off as many figures from the right hand of the dividend as there are ciphers in the divisor. The remaining figures of the dividend will be the quotient, and those cut off the remainder.

7. How many times is 100 contained in 12632?

Suggestion.—As there are two ciphers in the divisor, we cut off two figures on the right of the dividend. The figures which remain in the dividend, (126), are the quotient, and the figures cut off, (32), are the remainder. The answer is 126, and 32 remainder.

Operation.

1|00)126|32

Ans. 126, and 32 rem.

8. In one dime there are 10 cents: how many dimes are there in 100 cents? In 250 cents? In 380 cents? In 25000 cents?

9. In one dollar there are 100 cents: how many dollars are there in 6500 cents? In 76500 cents? In 452000 cents? In 15000000 cents?

10. Divide 675000 by 10000. *Ans.* 67 and 5000 rem.

11. Divide 44860791 by 1000000. Divide 586670517 by 10000000.

12. Divide 82367180809 by 10000000. Divide 963456789121 by 100000000.

QUEST.—79. What is the effect of annexing a cipher to a number? What is the effect of removing a cipher from the right of a number? 80. How divide by 10, 100, 1000, &c.? 79. Explain why this process gives the quotient.

CASE III.—*Ciphers on the right of the divisor.*

13. How many acres of land, at 20 dollars per acre, can you buy for 645 dollars?

Suggestion.—The divisor 20, is a composite number, the factors of which are 2 and 10. (Art. 55.) We may, therefore, divide first by the factor 10, by cutting off the right hand figure of the dividend; then divide the remaining figures of the dividend by 2, the other factor of the divisor, and the result will be the quotient. Hence,

Operation.

2|0)64|5

Ans. 32–5 rem.

81. When there are ciphers on the *right* of the divisor.

Cut off the ciphers from the divisor, also cut off as many figures from the right of the dividend. Then divide the remaining figures in the dividend by those remaining in the divisor, and the result will be the quotient.

Finally, annex the figures cut off from the dividend to the remainder, and the number thus formed will be the true remainder.

OBS. This contraction is based on the principles of the two preceding cases. For, when the divisor has ciphers on its right, it is a composite number, the significant figures being one of its factors, and 1 with the given ciphers annexed to it, the other. (Art. 60.)

Cutting off the ciphers from the right of the dividend, divides it by 1 with the given ciphers annexed.

14. How many horses, at 80 dollars apiece, can you buy for 640 dollars? *Ans.* 8 h.

15. How many barrels will 6800 pounds of beef make, allowing 200 pounds to the barrel?

16. How many regiments of 4000 each, can be formed from 340000 soldiers?

17. Divide 148000 by 2100. 18. Divide 4814670 by 24000.

19. Divide 2371 by 24, using the factors 2, 3, and 4.

20. Divide 6019 by 96, using the factors 2, 2, 3, and 8.

21. Divide 7673 by 48, using the factors 2, 2, 3, and 4.

22. Divide 93051 by 72, using the factors, 2, 3, 3, and 4.

23. Divide 305379 by 144, using the factors 3, 4, 2, and 6.

24. Divide 8738513 by 1728, using the factors 3, 3, 4, 6, and 8.

QUEST.—81. When there are ciphers on the right of the divisor, how proceed? What is to be done with figures cut off from the dividend? *Obs.* Upon what principles is this contraction based? How does this appear? What is the effect of cutting off the figures from the right of the dividend?

EXERCISES IN THE FUNDAMENTAL RULES.

81.a. The four preceding rules, *Addition*, *Subtraction*, *Multiplication*, and *Division*, are called the FUNDAMENTAL RULES of Arithmetic, because they are the *foundation* or *basis* of all arithmetical calculations.

Ex. 1. If from the sum of 1563 and 2570, you subtract 3278, then multiply the remainder by 387, and divide the product by 273, what will be the quotient?

2. If you divide 3456 by 216, multiply the quotient by 279, from the product subtract 308, and to the remainder add 28256, what will be the amount?

3. If the product of 256 into 576, is divided by 192, the quotient increased by 25071, and the sum diminished by 18203, what will be the remainder?

4. If the difference between 8256 and 9408, is divided by 96, the quotient increased by 246045, the sum diminished by 73418, and the remainder multiplied by 3056, what will be the product?

5. Two men start from the same place at the same time, one travels 93 miles a day, and the other 115 miles a day: how many miles will each have traveled, and how far apart will they be at the end of 17 days?

6. A merchant bought 68 bales of goods, each bale contained 34 pieces, and each piece 29 yards: how many yards did he buy?

7. A man sold 155 acres of land at 34 dollars per acre, and took in payment for it, 19 horses at 65 dollars apiece, and 15 cows at 17 dollars apiece: how much was still due him?

8. What number besides 137 will exactly divide 11371?

9. If the quotient is 275, the divisor 383, and the remainder 49, what is the dividend?

10. If the dividend is 2756, the quotient 184, and the remainder 180, what is the divisor?

11. What must 5376 be multiplied by, to make 6521088?

12. How many times can 437 be subtracted from 18791?

QUEST.—81.a. Name the fundamental rules of arithmetic? Why are these called fundamental rules?

13. If a man's salary is 6170 dollars a year, and he spends 7 dollars a day, how much can he lay up?

14. In a single city, there are 2170 dollars spent daily for cigars: how many free schools will this support, at the cost of 1085 dollars each per annum?

15. A man bought 467 acres of land, at 16 dollars per acre, and sold it for 9640 dollars: how much did he get per acre; and how much did he gain, or lose by his bargain?

16. A merchant bought 67 yards of broadcloth at 42 shillings a yard; 89 yards of silk, at 14 shillings a yard; 20 dozen silk hose, at 68 shillings per dozen; and 58 dozen gloves, at 48 shillings per dozen: what was the amount of his bill?

17. A man bought 568 horses, at 65 dollars apiece, and sold them so as to make 860 dollars: how much did he get apiece for them?

18. A miller bought 650 bushels of wheat, at 7 shillings a bushel, 56 bushels of which being worthless he threw away: for how much must he sell the rest per bushel, to make up his loss?

19. Which is worth the most, 860 cows at 90 dollars apiece, or 856 horses at 75 dollars apiece?

20. A owns 1866 acres of wild land, which is 6 times as much as B owns, and B owns twice as much as C: how much land do B and C own; and how much do all own?

21. A drover bought one lot of 75 oxen for 3750 dollars, and another lot of 115 for 8050 dollars: what did he pay for the whole; how much apiece for each lot; and what must he sell them at apiece, that he may neither make nor lose by the operation?

22. The smaller of two numbers is contained 14 times in 252, and the greater number is 49 times the smaller: what are the numbers?

23. A single pound of cotton has been spun into a thread 76 miles long, and a pound of wool into a thread 95 miles long: how many pounds of each would be required to spin threads which will reach round the world, whose circumference is 25000 miles?

24. If 1600 steam engines can do the work of 2 millions *496 thousand* men, to how many men is 1 engine equivalent?

25. If the difference between 69275 and 45263, is multiplied by the sum of 103+28+268, what will be the product?

26. If the sum of 14350 and 7845, is divided by 965, and the quotient multiplied by 386, and the product diminished by 761, what will the remainder be?

27. If the sum of 250 and 178, is multiplied by their difference, and the product divided by 45, what will be the quotient?

28. How many men will it take to do as much work in 1 day, as 268 men can do in 134 days?

29. How many men would it require to do the same work in 16 days?

30. Four men, A, B, C, and D, bought a ship together for 16256 dollars; A paid 4756 dollars, B paid 763 dollars more than A, and C 256 dollars less than B: how much did D pay?

31. A drover bought 538 sheep at 23 shillings a head, and 321 at 18 shillings; he afterwards sold 276 at 27 shillings to one customer, and 186 at 31 shillings to another: how many had he left; and how much did they stand him in apiece?

32. A man bought a drove of oxen for 18180 dollars, and after selling 84 of them at 51 dollars apiece, the rest stood him in 45 dollars apiece: how many did he buy?

33. What is the difference between 9318702858 divided by 1987, and 46481 multiplied by 986?

34. A drover having brought 861 head of cattle to market, which cost him 48 dollars a head, sold 83 to one butcher at 67 dollars apiece, and 96 to another at 56 dollars: how many had he left; and what did they stand him in apiece?

35. The ship America of Boston, sailed 56 hours at the rate of 11 miles per hour, when she encountered a storm of 16 hours duration which drove her back at the rate of 14 miles per hour: how far from port was she at the expiration of the 72 hours?

36. A thief fled from New York, at the rate of 85 miles a day; 5 days after an officer started in pursuit of him at the rate of 130 miles a day: how far from the thief was the officer at the end of 8 days from the time the latter started?

37. A is worth 1265 dollars, B is worth 4 times as much as A, and 183 dollars, and C is worth three times as much as A and B lacking 2348 dollars: how much are B and C worth respectively; and how much are they all worth?

GENERAL PRINCIPLES IN DIVISION.

82. From the nature of division, it is evident that the *value* of the *quotient* depends both on the *divisor* and the *dividend*.

OBS. 1. If the divisor is *equal* to the dividend, the quotient is 1.
2. If the divisor is *greater* than the dividend, the quotient is *less* than 1.
3. If the divisor is *less* than the dividend, the quotient is *greater* than 1.
4. If the divisor is 1, the quotient is *equal to* the dividend.
5. If the divisor is *greater* than 1, the quotient is *less* than the dividend.
6. If the divisor is *less* than [illegible] quotient is *greater* than the dividend.

83. If a given divisor is contained in a given dividend a certain number of times, the same divisor will obviously be contained,

In *double* that dividend, *twice* as many times;

In *three times* that dividend, *thrice* as many times. Hence,

If the divisor remains the same, multiplying the dividend by any number, is in effect multiplying the quotient by that number.

Thus, 4 is contained in 12, 3 times; in two times 12 or 24, 4 is contained 6 times; (i. e. twice 3 times;) in 3 times 12 or 36, 4 is contained 9 times; (i. e. thrice 3 times;) &c.

84. Again, if a given divisor is contained in a given dividend a certain number of times, the same divisor is contained,

In *half* that dividend, *half* as many times;

In a *third* of that dividend, a *third* as many times, &c. Hence,

If the divisor remains the same, dividing the dividend by any number, is in effect dividing the quotient by that number.

Thus, 4 is contained in 24, 6 times; in 24÷2 or 12, (half of 24,) 4 is contained 3 times; (i. e. half of 6 times;) in 24÷3 or 8, (a third of 24,) 4 is contained 2 times; (i. e. a third of 6 times;) &c.

85. If a given divisor is contained in a given dividend a certain number of times, then, in the same dividend,

Twice that divisor is contained only *half* as many times;

Three times that divisor, a *third* as many times, &c. Hence,

If the dividend remains the same, multiplying the divisor by any number, is in effect, dividing the quotient by that number.

QUEST.—82. Upon what does the value of the quotient depend? 83. If the divisor remains the same, what is the effect of multiplying the dividend? 84. What is the effect of dividing the dividend?

Thus, 2 is contained in 12, 6 times; 2 times 2 or 4, is contained in 12, 3 times; (i. e. half of 6 times;) 3 times 2 or 6, is contained in 12, 2 times; (i. e. a third of 6 times;) &c.

86. If a given divisor is contained in a given dividend a certain number of times, then, in the same dividend,

Half that divisor is contained *twice* as many times;

A *third* of that divisor, *three times* as many times, &c. Hence,

If the dividend remains the same, dividing the divisor by any number, is in effect multiplying the quotient by that number.

Thus, 6 is contained in 24, 4 times: $6 \div 2$ or 3, (half of 6,) is contained in 24, 8 times; (i. e. twice 4 times;) $6 \div 3$ or 2, (a third of 6,) is contained in 24, 12 times; (i. e. three times 4 times;) &c.

87. From the preceding articles, it is evident that any given divisor is contained in any given dividend, just as many times, as *twice* that divisor is contained in *twice* that dividend; *three times* that divisor in *three times* that dividend, &c.

Conversely, any given divisor is contained in any given dividend just as many times, as *half* that divisor is contained in *half* that dividend; a *third* of that divisor, in a *third* of that dividend, &c. Hence,

88. *If the divisor and dividend are both multiplied, or both divided by the same number, the quotient will not be altered.*

Thus, 4 is contained in 12, 3 times;
2 times 4 is contained in 2 times 12, 3 times;
3 times 4 is contained in 3 times 12, 3 times, &c.

Again, 6 is contained in 24, 4 times:
$6 \div 2$ is contained in $24 \div 2$, 4 times;
$6 \div 3$ is contained in $24 \div 3$, 4 times, &c.

88.*a*. *If any given number is multiplied and the product divided by the same number, its value will not be altered.* Thus, $12 \times 5 = 60$; and $60 \div 5 = 12$, the given number.

QUEST.—85. If the dividend remains the same, what is the effect of multiplying the divisor? 86. What is the effect of dividing the divisor? 88. What is the effect, if the divisor and dividend are both multiplied or both divided by the same number? 88.*a*. What is the effect of multiplying and dividing any given number by the same number?

ARITHMETICAL TERMS.

ART. 89. Numbers are divided into two classes, *abstract* and *concrete*.

DEF. 1. *Abstract* numbers are numbers used without application to any object; as *two*, *three*, *four*, *five*, &c.

2. *Concrete* numbers are numbers applied to some particular object; as *two* peaches, *three* pounds, &c.

3. Numbers are also divided into *prime* and *composite*.

4. A *prime* number is one which *cannot* be produced by multiplying any two or more numbers together; or which *cannot* be exactly divided by any *whole* number, except a *unit* and *itself*. Thus, 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, &c., are prime numbers.

OBS. 1. The *least* divisor of every number is a prime number.

2. One number is said to be *prime to another*, when a *unit* is the only number by which both can be divided without a remainder.

3. The number of prime numbers is unlimited. All below a hundred are given above. The pupil can easily point out others.

5. A *composite* number is one which is produced by multiplying two or more *factors* together. (Art. 55.)

OBS. 1. Both *prime* and *composite* numbers may be either *abstract* or *concrete*.

2. The learner must be careful not to confound numbers which are *prime to each other* with *prime* numbers; for numbers that are prime to each other, may themselves be *composite* numbers. Thus 4 and 9 are prime to each other, while they are composite numbers.

6. An *even* number is one which can be divided by 2 without a remainder; as, 4, 6, 8, 10.

7. An *odd* number is one which cannot be divided by 2 without a remainder; as, 1, 3, 5, 7, 9, 15.

OBS. All even numbers except 2, are *composite* numbers. Some odd numbers are *composite*, others are *prime*.

QUEST.—89. Into how many classes are numbers divided? What are abstract numbers? Concrete? What other division of numbers is mentioned? What is a prime number? *Obs.* When is one number said to be prime to another? How many prime numbers are there? What is a composite number? What is an even number? An odd number? *Obs.* Are even numbers prime or composite? *What is true of* odd numbers in this respect?

8. An *integer* is a *whole* number.

9. The *reciprocal* of a number, is the quotient arising from dividing a *unit* by that number. Thus, the reciprocal of 2 is $1 \div 2$, or $\frac{1}{2}$; the reciprocal of 3 is $1 \div 3$, or $\frac{1}{3}$.

10. The *complement* of a number, is the *difference* between that number and a *unit of the next higher order*. Thus, the complement of 8 is 2, because 8 taken from 10, leaves 2; the complement of 93, is 7, &c.

OBS. The complement of a number consisting of *one* integral figure, either with or without decimals, is found by *subtracting* the number from 10. If it has *two* integral figures, it must be subtracted from 100; if *three*, from 1000, &c.

11. One number is a *measure* of another, when the *former* will divide the *latter*, without a remainder. Thus, 2 is a measure of 4; 3 is a measure of 6.

12. A *common measure* is a number, which will divide *two* or *more* numbers, without a remainder. Thus, 2 is a common measure of 4, 6, and 8.

OBS. 1. Any number that *measures* two others, will likewise measure their *sum*, their *difference*, and their *product*. Thus, 3 measures 9 and 15; it also measures $15+9$, $15-9$, and 15×9.

2. A number that *measures* another, will also measure its *multiple*, or its *product* by any *whole* number. Thus, 6 measures 12, and it also measures 12×2, or the product of 12 into any whole number.

13. The *aliquot parts* of a number, are the parts by which it can be divided without a remainder. Thus, 3 and 7 are aliquot parts of 21. Hence,

OBS. The *aliquot parts* of a number, are the *factors*, which being multiplied together, produce the given number.

14. When *two* or *more numbers* are to be subjected to the *same operation*, they are included in a *parenthesis* ( ), or connected by a horizontal line placed over them (——), called a *vinculum*. Thus, the expression $(7+4) \times 3$, or $\overline{7+4} \times 3$, shows that the sum of 7 and 4 (11), is to be multiplied by 3. But $7+4 \times 3$, signifies that 4 only is to be multiplied by 3, and the product added to 7.

QUEST.—What is an integer? What is the reciprocal of a number? What is the complement of a number? *Obs.* How is the complement of a number consisting of one integral figure, found? How, when it has two integral figures? When is one number a measure of another? What is a common measure? What are aliquot parts of a number? When two or more numbers are to be subjected to the same operation, how is it indicated?

ANALYSIS OF COMPOSITE NUMBERS.

89. a. All *composite* numbers are composed of *prime factors*; that is, the *ultimate* or *least* factors into which they may be resolved, are *prime numbers*. Hence,

OBS. All numbers are either *prime numbers*, or are composed of *prime* factors.

Ex. 1. Resolve 60 into its prime factors.

Suggestion.—Divide the 60 by 2, which is the least number that will divide it without a remainder. In like manner divide this quotient by 2; and the next quotient by 3. The divisors, 2, 2, 3, with 5, the last quotient, are the prime factors required. Hence,

Operation.

$$\begin{array}{r} 2)\underline{60} \\ 2)\underline{30} \\ 3)\underline{15} \\ 5 \end{array}$$

89. b. To resolve a *composite* number into its prime factors,

Divide the given number by the smallest number which will divide it without a remainder; then divide this quotient in the same way, and thus continue the operation till a quotient is obtained, which can be divided by no number greater than 1.

The several divisors with the last quotient, will be the prime factors required.

OBS. The *reason* of this rule may be seen from the following considerations:

First, the respective divisors are *prime* factors; for, they are the *least* numbers, which will divide the given number and the successive quotients, without a remainder. (Art. 89. Def. 4. Obs.)

Second, the last quotient is also a *prime* factor; for, it can not be exactly divided by any number except a *unit* and *itself*. (Art. 89. Def. 4.)

2. A *composite* number can be divided by any of its *prime factors* without a remainder, and by the *product* of any two or more of them, but by *no other* number. Thus, the prime factors of 30 are 2, 3, and 5. Now 30 can be divided by 2, 3, and 5; also by 2×3, 2×5, 3×5, and by 2×3×5; but by no other number.

2. Resolve 24, 26, 32, 34, 36, 39, and 44 into prime factors.
3. Resolve 46, 48, 51, 52, 58, 62, 68, 69 into prime factors.
4. Resolve 70, 72, 74, 75, 76, 78, 82, 85 into prime factors.
5. Resolve 120, 124, 136, 156, 208, 145, 225, into prime fact.
6. Resolve 256, 344, 576, 672, 796, 864, 945, into prime fact.
7. Resolve 3420, 18500, 46096, and 96464 into prime factors.

QUEST.—89. *a.* Of what are all composite numbers composed? *Obs.* What then is true of all numbers? 89. *b.* How do you resolve a composite number into its prime factors? *Obs.* How does it appear that the divisors and last quotient are prime factors?

CANCELLATION.*

89. *Cancellation* is the method of contracting arithmetical operations by *rejecting equal factors.*

OBS. 1. The term *cancel*, is from the Latin *cancello*, to *cross out*, or *reject*.

2. Cancellation is applicable to *division*, when the divisor and dividend have *common factors*, and especially to that class of examples which involve both *multiplication* and *division*.

But it is applied with the greatest advantage to reduction of *Compound Fractions* to *Simple* ones, *Multiplication* and *Division* of Fractions, *Simple* and *Compound Proportion*.

90.*a*. We have seen that division is finding a quotient, which multiplied into the divisor will produce the dividend. (Art. 45.) If, therefore, the dividend is resolved into two such factors that one of them is the divisor, the other factor will be the quotient. Suppose 42 is to be divided by 6. Now the factors of 42 are 6 and 7, the first of which being the divisor, the other must be the quotient. Therefore,

Canceling a factor of any number, divides the number by that factor. Hence,

91. When the divisor is a factor of the dividend.

Cancel both the divisor and the factor of the dividend to which it is equal; and the remaining factor of the dividend will be the quotient.

1. Divide the product of 19 into 25 by 19.

Operation.

$\cancel{19})25 \times \cancel{19}$

$\overline{25}$ *Ans.*

Suggestion.—Cancel the 19, which is common both to the divisor and dividend, and 25, the other factor, is the quotient.

OBS. 1. It should be observed, whenever a factor canceled, is equal to the *number itself*, the figure 1 either expressed or understood, is always left; for if any number is divided by itself, the quotient is always 1. (Art 82. Obs.)

2. When the 1 thus left, is in the *multiplier* or *divisor* or *denominator* of a fraction, it may be disregarded; for, multiplying or dividing a number by 1, does not alter the number. (Arts. 45, 82. Obs. 4.) But when the 1 stands in the *dividend*, or *numerator* of a fraction, it performs an important office, and must be preserved.

QUEST.—89. What is Cancellation? *Obs.* What is meant by the term cancel? To what rules is cancellation applicable? 90.a. What is the effect of canceling a factor of any number? 91. When the divisor is a factor of the dividend, how do you proceed?

* Birk's Arithmetical Collections, London, 1764.

2. Divide 85×31 by 85.
3. Divide 76×58 by 58.
4. Divide 75×40 by 40.
5. Divide 63×28 by 7.

Operation.

$$\cancel{7})63 \times 4 \times \cancel{7}$$
$$252 \text{ Ans.}$$

Suggestion.—$28 = 4 \times 7$; consequently the whole dividend is equal to $63 \times 4 \times 7$. We therefore cancel the 7, which is a factor common both to the dividend and the divisor. The product of 63×4, the other factors of the dividend, is the answer required.

6. In 32 times 84, how many times 8? *Ans.* 336.
7. In 35 times 95, how many times 7?
8. In 48 times 133, how many times 8?
9. In 96 times 156, how many times 12?
10. Divide $168 \times 2 \times 7$ by 7×3.

Operation.

$$\cancel{7} \times 3)168 \times 2 \times \cancel{7}$$
$$3)336$$
$$112 \text{ Ans.}$$

Suggestion.—We first cancel the factor 7, which is common to the divisor and dividend, then divide the product of 168 into 2 by 3.

11. Divide the product of 21 into 4 into 9, by the product of 3 into 7 into 2 into 3.

Operation.

$$\cancel{3} \times \cancel{7} \times 2 \times 3)\cancel{21} \times 4 \times 9$$
$$6)36$$
$$6 \text{ Ans.}$$

Suggestion.—The product of 3×7 is 21; we therefore cancel the factors 3 and 7 in the divisor, and the 21 in the dividend. Then dividing the product of 4×9 the remaining factors of the dividend by the product of 2×3, the remaining factors of the divisor, we have 6 for the quotient. Hence,

92. When the divisor and dividend have *common* factors.

Cancel the factors common to both; then divide the product of the remaining factors in the dividend by the product of those remaining in the divisor, and the quotient will be the answer.

OBS. If *two* or *more* factors in the divisor taken together, are equal to *one* factor in the dividend, or vice versa, cancel the single factor in the one, and those which are equal to it in the other.

12. Divide the product of 7, 9, 15, and 8 by the product of 5, 7, and 8.
13. Divide the product of 6, 3, 7, and 4 by 12 into 6.
14. Divide the product of 2, 28, and 15 by 30.
15. Divide the product of 5, 6, and 56 by 7 into 8.

QUEST.—92. When the divisor and dividend have common factors, how proceed?

GREATEST COMMON DIVISOR.

93. A *Common Divisor* is a number which will *divide two* or *more* numbers without a remainder. Thus, 2 is a common divisor of 4, 6, 8, 12, 16.

94. The *Greatest Common Divisor* of two or more numbers, is the *greatest* number which will divide each of them without a remainder. Thus, 6 is the greatest common divisor of 12, 18, and 24.

OBS. 1. A *common divisor* is often called a *common measure.*

2. Numbers which have no *common measure* or *divisor*, greater than 1, are said to be *incommensurable.* Thus, 11 and 17 are incommensurable.

3. It will be seen that a *common divisor* of two or more numbers, is simply a factor which is *common* to those numbers; and the *greatest common divisor* is the *greatest* factor common to them. Hence,

95. To find a *common divisor of two or more numbers.*

Resolve each number into two or more factors, one of which shall be common to all the given numbers. (Art. 86.)

OBS. If the given numbers have not a *common factor*, they cannot have a common divisor greater than 1; consequently, they are either prime numbers, or are prime to each other. (Art. 89, Def. 5, Obs. 2.)

Note.—The following facts may assist the learner in finding common divisors:

1. Any number ending in 0, also any even number, as 2, 4, 6, &c., may be divided by 2.

2. Any number ending in 5 or 0, may be divided by 5.

3. Any number ending in 0, may be divided by 10.

4. When the two right hand figures are divisible by 4, the whole number may be divided by 4.

5. If the three right hand figures of any number are divisible by 8, the whole is divisible by 8.

Ex. 1. Find a common divisor of 8, 10, and 12.

Analysis.—8 may be resolved into the factors 2 and 4; that is, $8=2\times4$; $10=2\times5$; and $12=2\times6$. The factor 2 is common to each number and is therefore a common divisor of them.

2. Find a common divisor of 9, 15, 18, and 24.

3. Find a common divisor of 16, 20, and 36.

4. Find a common divisor of 35, 50, 75, and 80.

5. Find a common divisor of 140 and 184.

6. Find a common divisor of 126 and 4650.

QUEST.—93. What is a common divisor? 94. What is the greatest common divisor of two or more numbers? *Obs.* What is a common divisor sometimes called? 95. How do you find a common divisor of two or more numbers?

7. What is the greatest common divisor of 30 and 42?

Suggestion.—Dividing 42 by 30, the remainder is 12; then dividing 30 (the preceding divisor) by 12 (the last remainder) the remainder is 6; finally, dividing 12 (the preceding divisor) by 6 (the last remainder) nothing remains; consequently 6, the last divisor, is the greatest common divisor. Hence,

Operation.

```
30)42(1
   30
   12)30(2
      24
       6)12(2
         12
```

96. To find the *greatest common divisor* of *two* numbers.

Divide the greater number by the less; then divide the preceding divisor by the last remainder, and so on, till nothing remains. The last divisor will be the greatest common divisor.

Demonstration.—Since 6 is a measure of the last dividend 12, in the solution above, it must therefore be a measure of the preceding dividend 30; because $30=2\times12+6$; now 30 is one of the given numbers. Again, since 6 measures 12 and 30, it must also measure their sum, viz: $30+12$, or 42, which is the other given number. (Art. 89. Def. 12. Obs.)

8. What is the greatest common divisor of 63 and 147?

9. What is the greatest common divisor of 91 and 117?

10. Find the greatest common divisor of 247 and 323.

11. Find the greatest common divisor of 285 and 465.

12. What is the greatest common divisor of 2145 and 3471? Of 464320 and 18945? Of 638296 and 33888? Of 18996 and 29932? Of 260424 and 54423? Of 143168 and 2064888?

97. To find the greatest common divisor of *more* than *two* numbers.

First find the greatest common divisor of any two of the given numbers; then, that of the common divisor thus obtained and of another given number, and so on through all the given numbers. The last common divisor found, will be the one required.

13. What is the greatest com. divisor of 63, 105, and 140?

14. Find the greatest common divisor of 16, 24, and 100.

15. Find the greatest common divisor of 492, 744, and 1044.

QUEST.—*Obs.* If two numbers have not a common factor, what is true of them as to a common divisor? 96. How find the greatest common divisor of two numbers? 97. Of more than two?

LEAST COMMON MULTIPLE.

98. A *multiple* is a number which can be *divided* by another number *without* a *remainder*. Thus, 4 is a multiple of 2; 10 is a multiple of 5.

OBS. The term *multiple*, is from the Latin *multiplex*, which signifies *manifold*, or containing *many times*.

99. A *common multiple* is a number which can be *divided* by *two* or *more numbers* without a remainder. Thus, 12 is a com. multiple of 2, 3, and 4; 15 is a com. multiple of 3 and 5.

OBS. A *common* multiple is a composite number, of which each of the given numbers must be a factor; otherwise it could not be divided by them.

100. The *continued product* of two or more given numbers will always form a common multiple of those numbers.

OBS. The same numbers, therefore, may have an *unlimited number* of common multiples; for, multiplying their continued product by any number, will form a new common multiple. (Art. 99. Obs.)

101. The *least common multiple* of two or more numbers, is the *least number which can be divided by each of them without a remainder*. Thus, 12 is the least com. multiple of 4 and 6.

OBS. The least common multiple of two or more numbers, is the *product* of all the prime factors of the given numbers, each factor being *taken* as many times, as are equal to the *greatest number* of times it is found in either of the given numbers. For, a number which does not contain all the prime factors of either of the given numbers, cannot be divided by that number, and therefore is not a *common* multiple of them. (Art. 89.*b*. Obs. 2.) On the other hand, if any prime factor were employed *more times* than it is repeated as a factor in some one of the given numbers, then the product would not be the *least* common multiple.

15. Find the least common multiple of 6, 10, and 12.

Analysis.—By resolving the given numbers into their prime factors, it will be seen that the different factors are 2, 3, and 5. Now the *greatest number* of times that 2 is found in either, is *two*; therefore the 2 must be taken *twice* in the product, and the other 2s must be rejected. Again, 3 is found only *once* as a factor in either of the given numbers, consequently it must be taken *once* in the product, and the other 3

$$6=\cancel{2}\times\cancel{3}$$
$$10=\cancel{2}\times5$$
$$12=2\times2\times3$$

QUEST.—98. What is a multiple? 99. What is a common multiple? 100 How may a common multiple of two or more numbers be found? *Obs.* How many common multiples may there be of any given numbers? 101. What is the least common multiple of two or more numbers?

be rejected. Finally, 5 is found only *once*, and therefore must be taken *once* in the product. Now, $2 \times 2 \times 3 \times 5 = 60$, which is the least common multiple of the given numbers.

Suggestion.—Write the given numbers in a line, and dividing by 2 the smallest number that will divide any two of them without a remainder, set the quotients 3, 5, and 6 in a line below. Next divide this line by 3 and set the quotients and undivided number 5 in a line below as before. Finally, multiply all the divisors into the quotients and undivided number in the last line, and the product is the answer required. Hence,

Operation.

$$\begin{array}{r|l} 2 & 6'' \; 10'' \; 12 \\ \hline 3 & 3'' \;\; 5'' \;\; 6 \\ \hline & 1'' \;\; 5'' \;\; 2 \end{array}$$

$$2 \times 3 \times 5 \times 2 = 60$$

102. To find the least common multiple of two or more numbers.

I. *Write the given numbers in a horizontal line, and divide by the smallest number which will divide any two or more of them without a remainder, setting the quotients and undivided numbers in a line below.*

II. *Divide this line and set down the results as before; thus continue the operation till there are no two numbers which can be exactly divided by any number greater than* 1.

III. *Finally, multiply all the divisors and numbers in the last line together, and the product will be the least common multiple.*

OBS. 1. The *reason* for dividing by the *smallest* number, is because the *least divisor* of every number is a *prime* number, and we wish to resolve the given numbers into their prime factors. If we divide by a *composite* number, it may have a factor *common* to some of the quotients, or undivided numbers in the last line which must be rejected, or their continued product will not be the *least* common multiple of the given numbers. (See Note 2. p. 99.)

2. The *object* of writing the numbers in a horizontal line, is because this arrangement enables us to resolve all of them into prime factors at once, and at the same time, *reject* those factors which are not to enter into the product.

3. If the given numbers are *prime numbers*, or are *prime* to each other, the *continued product* of them, will be their least common multiple. Thus, the least common multiple of 5 and 7 is 35; of 8 and 9 is 72. (Art. 95. Obs.)

QUEST.—102. How is the least common multiple of two or more numbers found? *Obs.* Why do you divide by the smallest number that will divide two or more of the given numbers without a remainder? How does it appear that dividing by any number that will divide two or more numbers without a remainder, will not always produce the least common multiple? Why write the given numbers in a horizontal line? What is the least common multiple of two *or more prime* numbers, or numbers that are prime to each other?

16. Find the least common multiple of 6, 8, and 12.

First Operation.

2)6″8″12
2)3″4″ 6
3)3″2″ 3
1″2″ 1

$2 \times 2 \times 3 \times 2 = 24$ *Ans.*

Second Operation.

6)6″8″12
2)1″8″ 2
1″4″ 1

Now $6 \times 2 \times 4 = 48$.

Note 1.—In the first operation we divide by the *smallest* numbers which will divide any two of the given numbers without a remainder, and 24 the product of the divisors, and the numbers in the last line, is the true answer.

2. In the second operation, we divide by 6, then by 2. But 6 is a composite number and contains the factor 2, which is common to the 4 in the last line; consequently, the continued product, though a *common* multiple, is not the *least* common multiple of the given numbers. It is *twice* too large. Hence,

3. It will be seen that "dividing by any number, which will divide two or more of the given numbers without a remainder," according to the rule sometimes given, does not always produce the *least* common multiple of the given numbers.

17. Find the least common multiple of 4, 9, and 12.
18. Find the least common multiple of 16, 12, and 24.
19. Find the least common multiple of 15, 9, 6, and 5.
20. Find the least common multiple of 10, 6, 18, 15.
21. Find the least common multiple of 24, 16, 15, 20.
22. Find the least common multiple of 25, 60, 72, 85.
23. Find the least common multiple of 68, 12, 84, 72.
24. Find the least common multiple of 54, 81, 14, 63.
25. Find the least common multiple of 12, 72, 36, 144.
26. Find the least common multiple of the nine digits.
27. Find the least common multiple of 17, 29, and 53.
28. Find the least common multiple of 8, 9, 55, and 49.
29. Find the least common multiple of 720, 336, and 1736.
30. Find the least com. mul. of 8, 12, 16, 24, 36, 48, 72, 144.
31. Three men start from the same place at the same time, to go round a circular field; one of which can travel the distance in 8 hours, another in 10 hours, and the other in 12 hours. In what time will they all meet at the starting place?

32. At the time of meeting, 3 others joined the company, one of whom could perform the distance in 6 hours, another in 16 hours, and the other in 18 hours: when will the six meet at the place from which they started? Ans. 720 h.

SECTION VI.

FRACTIONS.

MENTAL EXERCISES.

ART. 103. When a number or thing is divided into *two equal* parts, one of these parts is called *one half*. If divided into *three equal* parts, one of the parts is called *one third*; if divided into *four equal* parts, one of the parts is called *one fourth* or *one quarter*; if into *ten*, *tenths*; if into a *hundred*, *hundredths*, &c. That is,

When a number or thing is divided into *equal parts*, the parts take their *name* from the *number* of *parts* into which the thing or number is divided.

104. The *value* of *one* of these equal parts manifestly depends upon the *number* of parts into which the given number or thing is divided. Thus, if an orange is successively divided into 2, 3, 4, 5, &c. equal parts, the thirds will be less than the halves; the fourths than the thirds; the fifths than the fourths, &c.

Ex. 1. What is one half of 2 cents? Of 4 cents? 6? 8? 16? 18? 20? 24? 30? 40? 50? 60? 70? 80? 100?

2. What is one third of 6 cents? Of 9? 12? 15?

OBS. A *half* of any number, it will be perceived, is equal to as many units, as 2 is contained times in that number; a *third* of a number is equal to as many units, as 3 is contained times in the given number; a *fourth* is equal to as many, as 4 is contained in it, &c.

3. What is a third of 18? 21? 24? 27? 30? 36? 39? 45? 60? 42? 54? 75?

QUEST.—103. What is meant by one half? What is meant by one third? What is meant by a fourth? 3 fourths? What are fourths sometimes called? What is meant by fifths? By sixths? Eighths? How many sevenths make a whole one? How many tenths? What is meant by twentieths? By hundredths? When a number or thing is divided into equal parts, from what do the parts take their name? 104. Upon what does the value of one of these equal *parts* depend? Which is the greater, a sixth or a fourth? A seventh or a *tenth?*

4. A fourth of 20? 24? 28? 32? 36? 40? 44? 48?
5. A fifth of 25? 30? 35? 40? 45? 50? 55? 60? 100?
6. A sixth of 18? 24? 36? 30? 48? 60? 54? 42? 72?
7. A seventh of 28? 35? 21? 42? 56? 49? 63?
8. An eighth of 24? 40? 32? 64? 48? 56? 72? 88?
9. A ninth of 18? 36? 27? 45? 54? 72? 63? 81? 99?
10. A tenth of 40? 60? 50? 30? 100? 90? 120?
11. What part of 2 is 1? *Ans.* One half.
12. What part of 3 is 1? Of 4? 5? 7? 10? 15? 19?
13. What part of 3 is 2?

Suggestion.—Since 1 is 1 third part of 3, 2 must be two times 1 third part of 3, or two thirds of 3.

14. What part of 5 is 2? 3? 4? 5? 6? 8? 9? 15?
15. What part of 8 is 3? 7? 6? 9? 8? 12? 15? 19?
16. What part of 17 is 5? 8? 9? 13? 15? 16? 20?
17. What part of 100 is 13? 29? 63? 75? 92? 99
18. If 1 half an orange cost 2 cents, what will a whole orange cost?

Analysis.—If 1 half of an orange cost 2 cents, 2 halves or a whole orange, will cost twice as much; and 2 times 2 cents are 4 cents. A whole orange will, therefore, cost 4 cents.

19. If 1 third of a pie cost 4 cents, what will 2 thirds cost? What will a whole pie cost?

20. If 1 fourth of a pound of ginger cost 3 cents, what will 2 fourths of a pound cost? 3 fourths? What will a whole pound cost?

21. If 1 eighth of a yard of cloth cost 2 shillings, what will 8 eighths cost? 5 eighths? 7 eighths? A whole yard?

22. If 1 third of a barrel of flour cost 3 dollars, how much will a whole barrel cost? 5 barrels? 8 barrels?

23. If 1 sixth of a hogshead of molasses cost 5 dollars, what will be the cost of 1 hogshead? Of 10 hogsheads?

24. If 1 pound of sugar cost 12 cents, what will 1 half a pound cost?

Analysis.—If 1 pound cost 12 cents, it is plain that 1 half a pound will cost 1 half of 12 cents; and 1 half of 12 cents is 6 cents. Therefore, 1 half a pound of sugar, at 12 cents a pound, will cost 6 cents.

25. If 1 yard of ribbon cost 15 cents, how much will 1 third of a yard cost?

26. If 1 pound of tea cost 4 shillings, how much will 1 fourth of a pound cost? 2 fourths? 3 fourths?

27. If a ton of hay cost 15 dollars, how much will 1 fifth of a ton cost? 2 fifths? 3 fifths?

28. What will 1 tenth of an acre of land cost, at 30 dollars per acre? 2 tenths? 6 tenths?

29. What will 1 eighth of a ton of iron cost, at 48 dollars per ton? 3 eighths? 5 eighths? 7 eighths?

30. If 1 bushel of corn cost 1 half a dollar, what will 2 bushels cost? 4 bushels? 8 bushels?

Analysis.—If 1 bushel cost 1 half a dollar, 2 bushels will cost twice as much; and 2 times 1 half are 2 halves, which are equal to a whole dollar. Again, 4 bushels will cost 4 times as much as 1 bushel, and 4 times 1 half dollar are 4 halves, equal to 2 whole dollars.

31. If one man eats 1 half a loaf of bread at a meal, how many loaves will 3 men eat?

32. How many whole ones in 4 halves? 5 halves? 6 halves? 8 halves? 9 halves?

33. If I burn 1 third of a ton of coal in a week, how much shall I burn in 3 weeks? 4 weeks? 6 weeks? 10 weeks? 12 weeks?

34. How many whole ones in 4 thirds, and how many over? In 6 thirds? 8 thirds? 11 thirds? 14 thirds?

35. If a horse eat 1 fourth of a bushel of oats a day, how many will he eat in 6 days? In 8? In 10? In 12?

36. If a boy can saw 1 eighth of a cord of wood in a day, how much can he saw in 6 days? In 12 days? In 15 days? In 24 days?

37. If 12 oranges were divided equally among 4 boys, what *part* of them would each boy receive; and how many oranges would each have?

Analysis.—1 is 1 fourth of 4; hence, 1 boy must receive 1 fourth part of the oranges. Now 1 fourth of 12 oranges is 3 oranges. Therefore, each boy would receive 3 oranges.

38. A builder employed 6 men to do a job of work, for

which he gave them 24 dollars: what part of the money did 1 man receive? 2 men? 3 men? 4 men? How many dollars did 1 man receive? 2 men? 3 men? 4 men?

39. If 5 yards of cloth cost 40 dollars, what part of 40 dollars will 1 yard cost? 2 yards? 3 yards? 4 yards? How many dollars will 1 yard cost? 2 yards? 3 yards? 4 yards?

40. 2 is 1 third of what number?

Analysis.—If 2 is 1 third of a number, 3 thirds or the whole number, must be 3 times as many; and 3 times 2 are 6. Therefore 2 is 1 third of 6.

Or thus, 2 is a third of 3 times 2; and 3 times 2 are 6.

41. 4 is 1 fifth of what number? 1 sixth of what number? 1 third? 1 eighth? 1 fourth? 1 seventh?

42. 6 is 1 third of what number? 1 fourth? 1 seventh? 1 tenth? 1 ninth? 1 twelfth?

43. 5 is 1 fourth of what number? 1 sixth? 1 eighth? 1 eleventh? 1 twelfth?

44. 8 is 1 seventh of what number? 1 sixth? 1 tenth? 1 ninth? 1 twelfth?

45. 4 is 2 thirds of what number?

Suggestion.—First find 1 third. Now if 4 is 2 thirds, 1 third is 1 half of 4, which is 2; and 3 thirds is 3 times 2, or 6. Therefore 4 is 2 thirds of 6.

46. 9 is 3 fourths of what number?
47. 8 is 4 fifths of what number?
48. 16 is 4 ninths of what number?
49. 20 is 5 eighths of what number?
50. 32 is 8 twelfths of what number?
51. What is 2 thirds of 24?

Analysis.—1 third of 24 is 8, and 2 thirds is 2 times as many, which is 16. Therefore 16 is 2 thirds of 24.

52. What is 3 fourths of 16? Of 20? Of 28? Of 40?
53. What is 4 fifths of 15? Of 20? Of 25? Of 40? Of 55?
54. What is 5 sixths of 18? Of 30? Of 42? 54? Of 72?
55. What is 3 sevenths of 28? Of 35? Of 42? Of 56? Of 63?
56. What is 3 fifths of 4 times 10? Of 5 times 9?
57. What is 5 sixths of 3 times 12? Of 5 times 12?
58. What is 7 ninths of 6 times 6? Of 8 times 9?

EXERCISES FOR THE SLATE.

105. When a number or thing is divided into *equal parts*, as halves, thirds, fourths, fifths, &c., these parts are called *Fractions*. Hence,

A FRACTION *denotes a part or parts of a number or thing.*

OBS. Fractions are used to express parts of a *collection* of things, as well as of a *single* thing; or parts of any *number* of units, as well as of *one* unit. Thus, we speak of 1 third of *six* oranges; 3 fifths of 75, &c. In this case the *collection*, or *number* to be divided into equal parts, is regarded as a *whole*.

106. Fractions are divided into two classes, *Common* and *Decimal*. (For the illustration of Decimal Fractions, see Section VIII.)

107. *Common* fractions are those which arise from dividing an integer into *any number* of equal parts.

They are expressed by two numbers, one placed over the other, with a line between them. For example, one half is written thus, $\frac{1}{2}$; one third, $\frac{1}{3}$; one fourth, $\frac{1}{4}$; nine tenths, $\frac{9}{10}$.

The number below the line is called the *denominator*, and shows into *how many parts* the number or thing is divided.

The number above the line is called the *numerator*, and shows *how many parts* are expressed by the fraction. Thus, in the fraction $\frac{2}{3}$, the denominator 3, shows that the number is divided into *three* equal parts; the numerator 2, shows that *two* of those parts are expressed by the fraction.

The numerator and denominator together, are called the *terms* of the fraction.

OBS. 1. The term *fraction*, is derived from the Latin *fractio*, which signifies the *act of breaking*, a *broken part* or *piece*. Hence,

Fractions are sometimes called *broken numbers*.

2. *Common* fractions are often called *vulgar* fractions. This term, however, is very properly falling into disuse.

QUEST.—105. What are fractions? 106. Into how many classes are fractions divided? 107. What are Common Fractions? How are they expressed? What is the number below the line called? What does it show? What is the number above the line called? What does it show? What are the numerator and denominator, taken together, called? *Obs.* What is the meaning of the term *fraction*? What are common fractions sometimes called?

3. The number below the line is called the *denominator*, because it gives the name or denomination to the fraction; as, halves, thirds, fifths, &c.

The number above the line is called the *numerator*, because it numbers the parts, or shows how many parts are expressed by the fraction.

108. Common Fractions are divided into *proper*, *improper* *simple*, *compound*, *complex*, and *mixed* numbers.

A *proper* fraction is a fraction whose numerator is *less* than its denominator; as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$.

An *improper* fraction is one whose numerator is *equal to*, or *greater* than its denominator; as, $\frac{3}{3}$, $\frac{7}{2}$.

A *simple* fraction is a fraction which has but *one* numerator and *one* denominator, and may be *proper* or *improper*; as, $\frac{3}{5}$, $\frac{9}{4}$.

A *compound* fraction is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{4}{5}$.

A *complex* fraction is one which has a fraction in its numerator, or denominator, or in both; as, $\dfrac{2\frac{1}{2}}{5}$, $\dfrac{4}{5\frac{1}{3}}$, $\dfrac{2\frac{1}{3}}{8\frac{3}{4}}$, $\dfrac{\frac{2}{3}}{\frac{7}{8}}$.

A *mixed* number is a whole number and a fraction written together; as, $4\frac{2}{3}$, $25\frac{11}{12}$.

OBS. The original notion or definition of a *fraction* appears to have been, that it was a *part* of a *unit*. But it was seen, that those expressions whose numerator is *equal* to, or *greater* than their denominator, as 5 fifths, 9 fourths, &c., did not come under this definition; therefore they were called *improper fractions*.

Although it is not accurate to call 5 fifths, or 9 fourths a *part* of a *unit*, there is no *inaccuracy* in calling them *fractions*; for they denote *parts* of an *integer*. (Art. 105.) The *impropriety*, therefore, belongs not to this class of fractions, but to the definition which limits the meaning of the term *fraction*, to a *part* of a *unit*, and, consequently, is not sufficiently *comprehensive* to cover the whole ground.

Read the following fractions, and name the kind of each:

1. $\frac{6}{7}$; $\frac{9}{10}$; $\frac{17}{35}$; $\frac{115}{23}$; $\frac{235}{164}$; $\frac{4273}{2380}$; $\frac{5670}{8537}$; $\frac{7235}{4276}$; $\frac{6007}{9045}$.
2. $\frac{4}{5}$ of $\frac{25}{73}$; $\frac{13}{88}$ of $\frac{29}{16}$ of 7; $\frac{25}{87}$ of $\frac{99}{100}$ of 1000; $\frac{27}{45}$ of $6\frac{5}{8}$.
3. $7\frac{5}{8}$; $29\frac{6}{7}$; $41\frac{15}{33}$; $273\frac{325}{689}$; $4273\frac{825}{926}$; $706\frac{503}{857}$; $886\frac{45}{79}$.

109. Fractions, it will be seen, both from the definition and the mode of expressing them, arise from *division*, and may be treated as expressions of *unexecuted* division, the numerator answering to the dividend, and the denominator to the divisor. (Arts. 67, 105.)

QUEST.—Why is the lower number called the denominator? Why is the upper one called the numerator? 108. How are common fractions divided? What is a proper fraction? An improper fraction? A simple fraction? A compound fraction? A complex fraction? A mixed number? 109. From what do fractions arise?

110. The *value* of a fraction is the *quotient* of the numerator divided by the denominator. Thus the value of $\frac{6}{3}$ is *two;* of $\frac{4}{4}$ is *one;* of $\frac{1}{3}$ is *one third;* &c. Hence,

111. *If the denominator remains the same, multiplying the numerator by any number, multiplies the value of the fraction by that number.* For, the numerator and denominator answer to the dividend and divisor; therefore, multiplying the numerator is the same as multiplying the dividend. Now multiplying the dividend, we have seen, multiplies the quotient, (Art. 83,) which is the same as the value of the fraction. (Art. 110.) Thus, the value of $\frac{6}{3}=2$. Multiplying the numerator by 3, the fraction becomes $\frac{18}{3}$, whose value is 6, and is the same as 2×3.

112. *Dividing the numerator by any number, divides the value of the fraction by that number.* For, dividing the dividend divides the quotient. (Art. 84.) Thus, $\frac{6}{3}=2$. Now dividing the numerator by 2, the fraction becomes $\frac{3}{3}$, whose value is 1, and is the same as $2\div2$. Hence,

OBS. With a given denominator, the *greater* the *numerator*, the *greater* will be the value of the fraction.

113. *If the numerator remains the same, multiplying the denominator by any number, divides the value of the fraction by that number.* For, multiplying the divisor divides the quotient. (Art. 85.) Thus, $\frac{24}{6}=4$. Now multiplying the denominator by 2, the fraction becomes $\frac{24}{12}$, whose value is 2, and is the same as $4\div2$.

114. *Dividing the denominator by any number, multiplies the value of the fraction by that number.* For, dividing the divisor, multiplies the quotient. (Art. 86.) Thus, $\frac{24}{6}=4$. Now dividing the denominator by 2, the fraction becomes $\frac{24}{3}$, whose value is 8, and is the same as 4×2. Hence,

OBS. With a given numerator, the *greater* the *denominator*, the *less* will be the value of the fraction.

QUEST.—110. What is the value of a fraction? 111. What is the effect of multiplying the numerator while the denominator remains the same? Explain the reason. 112. What is the effect of dividing the numerator? Why? *Obs.* With a given denominator, what is the effect of increasing the numerator? 113. What is the effect of multiplying the denominator? Why? 114. What is the effect of dividing the denominator? Why? *Obs.* With a given numerator, what is the effect of increasing the denominator?

115. It is evident from the preceding articles, that *multiplying the numerator* by any number, has the same effect on the value of the fraction, *as dividing the denominator* by that number. (Arts. 111, 114.) And,

Dividing the numerator has the same effect as *multiplying the denominator*. (Arts. 112, 113.)

116. If the numerator and denominator are both *multiplied* or both *divided* by the same number, *the value of the fraction will not be altered.* (Arts. 88, 109.) Thus, $\frac{12}{4}=3$. Now if the numerator and denominator are both multiplied by 2, the fraction becomes $\frac{24}{8}$; whose value is 3. If both terms are divided by 2, the fraction becomes $\frac{6}{2}$; whose value is 3; that is, $\frac{12}{4}=\frac{24}{8}=\frac{6}{2}=3$.

117. Since the value of a fraction is the quotient of the numerator divided by the denominator, it follows that

If the numerator and denominator are *equal*, the value is a *unit* or *one*. Thus, $\frac{5}{5}=1$, $\frac{7}{7}=1$, &c.

If the numerator is *greater* than the denominator, the value is greater than *one*. Thus, $\frac{4}{2}=2$, $\frac{5}{3}=1\frac{2}{3}$.

If the numerator is *less* than the denominator, the value is less than *one*. Thus, $\frac{1}{3}=1$ *third* of 1, $\frac{4}{5}=4$ *fifths* of 1.

OBS. The best method to estimate the comparative magnitude or value of fractions, is to compare them with a *unit* or *one*, and not with each other. Thus, pupils are often puzzled to tell which is the greater, 7 eighths or 8 sevenths, when they attempt to compare the given fractions with each other; but by comparing each with a *unit*, the difficulty vanishes at once.

118. It will be seen from the preceding exercises that fractions may be *added*, *subtracted*, *multiplied*, and *divided*, as well as whole numbers.

OBS. In order to perform these operations, it is often necessary to make certain changes in the *terms* of the fractions, while the value remains the same.

Thus, the terms of the fraction $\frac{4}{2}$ may be changed into $\frac{2}{1}$, $\frac{8}{4}$, $\frac{16}{8}$, &c., without altering its value; for in each case the value is 2. Hence,

For any given fraction, we may substitute any other fraction of *equal value*.

QUEST.—115. What may be done to the denominator to produce the same effect on the value of the fraction, as multiplying the numerator by any given number? What, to produce the same effect, as dividing the numerator by any given number? 116. What is the effect if the numerator and denominator are both multiplied, or both divided by the same number? 117. When the numerator and denominator are equal, what is the value of the fraction? When the numerator is the larger, what? When smaller, what?

REDUCTION OF FRACTIONS.

119. *Reduction* of *Fractions* is the process of *changing their terms* into others, without altering the *value* of the fractions.

CASE I.—*Reducing fractions to their lowest terms.*

119.a. A fraction is said to be reduced to its *lowest terms*, when its numerator and denominator are expressed in the *smallest* numbers possible.

Ex. 1. Reduce $\frac{6}{12}$ to its lowest terms.

Suggestion.—Dividing both terms of the fraction by 2, it becomes $\frac{3}{6}$. Then, dividing both by 3, we obtain $\frac{1}{2}$, whose terms are the lowest to which the given fraction can be reduced.

First Operation.
$2)\frac{6}{12}=\frac{3}{6}$: then $3)\frac{3}{6}=\frac{1}{2}$ *Ans.*

Or, divide both terms by their greatest common divisor, which is 6, and the given fraction will be reduced to its lowest terms by a single division. (Art. 96.) Hence,

Second Operation.
$6)\frac{6}{12}=\frac{1}{2}$. *Ans.*

120. To reduce a fraction to its *lowest* terms.

Divide the numerator and denominator by any number which will divide them both without a remainder; then divide this result as before, and so on till no number greater than 1 *will exactly divide them; the last two quotients will be the lowest terms to which the given fraction can be reduced.*

Or, divide both the numerator and denominator by their greatest common divisor; and the quotients will be the lowest terms of the given fraction. (Art. 96.)

OBS. 1. The value of a fraction is not altered by reducing it to its lowest terms; for the numerator and denominator are divided by the same number. (Art. 116.)

2. When the terms of the fraction are small, the former method will generally be found to be the shorter and more convenient; but when the terms are large, it is often difficult to determine whether the fraction is in its simplest form, without finding their greatest common divisor.

2. Reduce $\frac{8}{16}$ to its lowest terms. *Ans.* $\frac{1}{2}$.

QUEST.—119. What is reduction of fractions? 119.a. What is meant by lowest terms of a fraction? 120. How is a fraction reduced to its lowest terms? *Obs* Is the value of a fraction altered by reducing it to its lowest terms? Why not?

3. Reduce $\frac{5}{15}$.
4. Reduce $\frac{6}{8}$.
5. Reduce $\frac{16}{18}$.
6. Reduce $\frac{32}{48}$.
7. Reduce $\frac{48}{72}$.
8. Reduce $\frac{75}{87}$.
9. Reduce $\frac{63}{105}$.
10. Reduce $\frac{55}{121}$.
11. Reduce $\frac{240}{312}$.
12. Reduce $\frac{126}{162}$.
13. Reduce $\frac{272}{425}$.
14. Reduce $\frac{435}{957}$.
15. Reduce $\frac{384}{1152}$.
16. Reduce $\frac{1749}{2909}$.
17. Reduce $\frac{594}{2142}$.
18. Reduce $\frac{6465}{7335}$.

CASE II.—*Reducing improper fractions to whole or mixed Nos.*

19. Reduce $\frac{17}{5}$ to a whole or mixed number.

Suggestion.—The object in this example, is to find a whole or mixed number, whose value is equal to the given fraction. But the value of a fraction is the quotient of the numerator divided by the denominator. (Art. 110.) We therefore divide the numerator by the denominator, and the result is $3\frac{2}{5}$. Hence,

Operation.

$$\begin{array}{l} 5)17 \\ \hline \quad 3\frac{2}{5} \textit{ Ans.} \end{array}$$

121. To reduce an *improper* fraction to a *whole*, or *mixed* number.

Divide the numerator by the denominator, and the quotient will be the whole, or mixed number required.

OBS. The remainder placed over the divisor, forms a part of the quotient, and must therefore be annexed to the integral figures. (Arts. 64, 71.)

20. Reduce $\frac{29}{3}$ to a whole or mixed number. *Ans.* $9\frac{2}{3}$.

Reduce the following fractions to whole or mixed numbers:

21. Reduce $\frac{24}{6}$.
22. Reduce $\frac{35}{7}$.
23. Reduce $\frac{21}{8}$.
24. Reduce $\frac{45}{6}$.
25. Reduce $\frac{18}{18}$.
26. Reduce $\frac{500}{12}$.
27. Reduce $\frac{750}{25}$.
28. Reduce $\frac{8437}{298}$.
29. Reduce $\frac{845}{30}$.
30. Reduce $\frac{7243}{320}$.

CASE III.—*Reducing mixed numbers to improper fractions.*

31. Reduce the mixed number $15\frac{3}{4}$ to an improper fraction.

Suggestion.—Since in 1 unit there are 4 fourths, in 15, there are 15 times as many. We therefore reduce the 15 to fourths, by multiplying it by 4, because 4 fourths make a whole one; and adding the 3 fourths we have 63 fourths. Hence,

Operation.

$$\begin{array}{l} 15\frac{3}{4} \\ \underline{\ \ 4} \\ \frac{63}{4} \textit{ Ans.} \end{array}$$

QUEST.—121. How reduce an improper fraction to a whole or mixed number?

122. To reduce a *mixed* number to an *improper* fraction.

Multiply the whole number by the denominator of the fraction, and to the product add the given numerator. The sum placed over the given denominator, will form the improper fraction required.

OBS. 1. A whole number may be expressed in the form of a fraction without altering its value, by *making 1 the denominator.* This fraction, in all cases, will be an *improper* fraction.

2. A whole number may be reduced to a fraction of a given denominator, by *multiplying* the given number by the proposed denominator; the product will be the numerator of the fraction required. Thus, 25 may be expressed by $\frac{25}{1}$ $\frac{100}{4}$, or $\frac{400}{16}$, &c., for $25=\frac{25}{1}=\frac{100}{4}=\frac{400}{16}$, &c.

32. Reduce $8\frac{1}{3}$ to an improper fraction. *Ans.* $\frac{25}{3}$.

Reduce the following mixed numbers to improper fractions:

33. Reduce $9\frac{2}{3}$.
34. Reduce $16\frac{5}{8}$.
35. Reduce $23\frac{7}{9}$.
36. Reduce $45\frac{5}{12}$.
37. Reduce $64\frac{8}{15}$.
38. Reduce $56\frac{23}{45}$.
39. Reduce $804\frac{1}{3}$.
40. Reduce $725\frac{1}{5}$.
41. Reduce 45 to fifths.
42. Reduce 72 to eighths.
43. Reduce 830 to sixths.
44. Reduce 743 to 15ths.

CASE IV.—*Reducing compound fractions to simple ones.*

45. Reduce $\frac{2}{3}$ of $\frac{6}{7}$ to a simple fraction.

Analysis.—2 thirds of $\frac{6}{7}$ is 2 times as much as 1 third of $\frac{6}{7}$. Now 1 third of $\frac{6}{7}$ is $\frac{6}{21}$; for, multiplying the denominator divides the value of the fraction. (Art. 113.) And 2 thirds is 2 times $\frac{6}{21}$, which is equal to $\frac{12}{21}$, or $\frac{4}{7}$. (Art. 120.)

Or, we may reason thus: multiplying the numerator of the second fraction by 2, the numerator of the first, the result $\frac{12}{7}$, is *three* times too large; for we wish to find only *two thirds* of $\frac{6}{7}$, instead of *two times* $\frac{6}{7}$, and *two* thirds of a number, is manifestly the same as *one* third *two times* that number. To correct this, we take *one third* of $\frac{12}{7}$, by multiplying the denominator 7, by 3 the other denominator. (Art. 113.) Hence,

Operation.

$\frac{2}{3}\times\frac{6}{7}=\frac{12}{21}$

and $\frac{12}{21}=\frac{4}{7}$ *Ans.*

QUEST.—122. How reduce a mixed number to an improper fraction? *Obs.* How express a whole number in the form of a fraction? How reduce a whole number to a fraction of a given denominator?

123. To reduce *compound* fractions to *simple* ones.

Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

OBS. If the example contains *whole* or *mixed* numbers, they must be reduced to *improper fractions*, before the numerators or denominators are multiplied together.

46. Reduce $\frac{4}{7}$ of $\frac{1}{8}$ of $1\frac{2}{9}$ of 3 to a simple fraction.

Solution.—$\frac{4}{7}$ of $\frac{1}{8}$ of $1\frac{2}{9}$ of $3 = \frac{4}{7} \times \frac{1}{8} \times \frac{11}{9} \times \frac{3}{1} = \frac{132}{504}$, or $\frac{11}{42}$ *Ans.*

47. Reduce $\frac{1}{3}$ of $\frac{10}{12}$ of $\frac{2}{15}$ to a simple fraction.

48. Reduce $\frac{1}{5}$ of $\frac{2}{7}$ of $\frac{4}{9}$ to a simple fraction.

49. Reduce $\frac{3}{8}$ of $\frac{7}{15}$ of $\frac{11}{13}$ to a simple fraction.

Contraction by Cancellation.

50. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{7}$ to a simple fraction.

Operation.

$$\frac{1}{\cancel{2}} \text{ of } \frac{\cancel{2}}{\cancel{3}} \text{ of } \frac{\cancel{3}}{4} \text{ of } \frac{5}{7} = \frac{5}{28}$$

Suggestion.—Since the product of the numerators is to be divided by the product of the denominators, we cancel the factors 2 and 3, which are common to both; for, this divides the terms of the new fraction by the same number, and therefore does not alter its value. Then, multiplying the remaining factors together, we have $\frac{5}{28}$, which is the answer required. (Arts. 90, 116.) Hence,

124. To reduce *compound* fractions to *simple* ones by CANCELLATION.

Cancel all the factors common to the numerators and denominators; then multiply the remaining factors together as before. (Art. 123.)

OBS. 1. If *two* or *more* factors in the numerators taken together, are equal to *one* factor in the denominators, or vice versa, cancel the single factor and those which are equal to it.

2. The *reason* of this contraction is evident from the fact, that the numerator and denominator of the simple fraction are both divided by the same number; consequently its value is not altered. (Arts. 90, 116.)

3. The advantage of this method is two-fold: it *shortens* the operation of multiplying, and at the same time *reduces* the answer to its *lowest terms*.

QUEST.—123. How reduce a compound fraction to a simple one? Explain the solution of the forty-fifth example? 124. How may the operation be contracted? *Obs.* How does it appear that this method does not alter the value of the fraction? What is the advantage of this method?

51. Reduce $\frac{4}{5}$ of $\frac{10}{12}$ of $\frac{3}{7}$ to a simple fraction.

Suggestion.—The factors 4 and 8 in the numerators taken together, are equal to the 12 in the denominator.

$$\frac{\cancel{4}}{\cancel{5}} \text{ of } \frac{\overset{2}{\cancel{10}}}{\cancel{12}} \text{ of } \frac{\cancel{3}}{7}=\frac{2}{7}\ \textit{Ans.}$$

52. Reduce $\frac{2}{7}$ of $\frac{6}{9}$ of $\frac{10}{12}$ to a simple fraction.
53. Reduce $\frac{4}{7}$ of $\frac{1}{4}$ of $\frac{7}{14}$ of $\frac{5}{8}$ to a simple fraction.
54. Reduce $\frac{5}{8}$ of $\frac{2}{9}$ of $\frac{4}{5}$ of $\frac{7}{10}$ to a simple fraction.
55. Reduce $\frac{8}{10}$ of $\frac{12}{15}$ of $\frac{20}{32}$ to a simple fraction.
56. Reduce $\frac{6}{10}$ of $\frac{15}{18}$ of $\frac{7}{9}$ of $\frac{9}{20}$ to a simple fraction.
57. Reduce $\frac{2}{9}$ of $\frac{17}{12}$ of $\frac{13}{17}$ of $\frac{9}{26}$ to a simple fraction.
58. Reduce $\frac{3}{12}$ of $\frac{6}{17}$ of $\frac{9}{7}$ of $\frac{2}{3}$ to a simple fraction.
59. Reduce $\frac{5}{7}$ of $\frac{14}{25}$ of $6\frac{1}{5}$ of 17 to a simple fraction.
60. Reduce $\frac{17}{35}$ of $\frac{27}{68}$ of $29\frac{7}{8}$ of 75 to a simple fraction.
61. Reduce $\frac{13}{16}$ of $\frac{12}{26}$ of $\frac{21}{45}$ of $\frac{16}{24}$ of $84\frac{3}{4}$ to a simple fraction.
62. Reduce $\frac{7}{8}$ of $\frac{13}{14}$ of $\frac{9}{26}$ of $\frac{4}{7}$ of $85\frac{6}{11}$ to a simple fraction.
63. Reduce $\frac{42}{54}$ of $\frac{18}{14}$ of $\frac{3}{5}$ of $178\frac{2}{3}$ to a simple fraction.
64. Reduce $\frac{17}{19}$ of $\frac{38}{34}$ of $\frac{75}{87}$ of $\frac{2}{3}$ of $\frac{1}{2}$ to a simple fraction.
65. Reduce $\frac{25}{103}$ of $\frac{3}{7}$ of $\frac{21}{37}$ of $\frac{3}{5}$ of 37 to a simple fraction.

Note.—For method of reducing *Complex Fractions* to *Simple* ones, see Art. 143.

CASE V.—*Reducing fractions to a common denominator.*

124. *a.* Two or more fractions have a *common denominator*, when they have the *same* denominator.

Ex. 1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$ to a common denominator.

Operation.

$$\frac{1}{2}=\frac{1\times3\times5}{2\times3\times5}=\frac{15}{30}.$$
$$\frac{2}{3}=\frac{2\times2\times5}{2\times3\times5}=\frac{20}{30}.$$
$$\frac{3}{5}=\frac{3\times2\times3}{2\times3\times5}=\frac{18}{30}.$$

Suggestion.—After each numerator, we write all the denominators of the given fractions, except its own, with the sign × between them, and under each line place all the denominators in like manner. Then performing the multiplications indicated, the products form fractions equal to the given fractions, and are the answer required. (Art. 116.) Hence,

QUEST.—124.*a.* What is meant by a common denominator? 125. How reduce fractions to a common denominator?

125. To reduce fractions to a *common* denominator.

Multiply each numerator into all the denominators except its own for a new numerator, and all the denominators together for a common denominator.

OBS. 1. The *reason* that reducing fractions to a common denominator does not change their value, is manifest from the fact, that the numerator and denominator of each fraction are multiplied by the *same numbers*. (Art. 116.)

2. *Compound* fractions must be reduced to *simple* ones, and *mixed* numbers to *improper* fractions, before attempting to reduce them to a common denominator.

3. A *whole* number and a *fraction* may be reduced to a common denominator by first reducing the whole number to the form of a fraction; then proceeding according to the rule above. (Art. 122. Obs. 1.)

2. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ $2\frac{1}{2}$, and 3 to a common denominator.

Suggestion.—$\frac{2}{3}$ of $\frac{3}{4}=\frac{6}{12}$, $2\frac{1}{2}=\frac{5}{2}$ and $3=\frac{3}{1}$. Now reducing $\frac{6}{12}$, $\frac{5}{2}$, $\frac{3}{1}$ to a common denominator, they become $\frac{12}{24}$, $\frac{60}{24}$, $\frac{72}{24}$.

3. Reduce $\frac{3}{5}$, $\frac{1}{4}$, and $\frac{6}{7}$ to a common denominator.

Ans. $\frac{84}{140}$, $\frac{35}{140}$, $\frac{120}{140}$.

4. Reduce $\frac{1}{9}$, $\frac{2}{3}$, and $\frac{3}{4}$ to a common denominator.

5. Reduce $\frac{2}{3}$, $\frac{1}{2}$, $\frac{4}{5}$, and $\frac{2}{3}$.

6. Reduce $\frac{3}{8}$, $\frac{4}{7}$, $\frac{6}{9}$, and $\frac{2}{5}$.

7. Reduce $\frac{7}{8}$, $\frac{5}{7}$, $\frac{6}{10}$, and $\frac{7}{12}$.

8. Reduce $\frac{9}{10}$, $\frac{6}{7}$, $\frac{12}{15}$, and $\frac{2}{5}$.

9. Reduce $\frac{12}{25}$, $\frac{63}{70}$, and $\frac{27}{40}$.

10. Reduce $\frac{8}{20}$, $\frac{70}{100}$, and $\frac{52}{25}$.

11. Reduce $\frac{7}{21}$, $8\frac{5}{7}$, and 4.

12. Reduce $\frac{3}{70}$, $\frac{5}{8}$ of $2\frac{3}{7}$.

CASE VI.—*Reducing fractions to their least common denominator.*

13. Reduce $\frac{3}{4}$, $\frac{2}{6}$, and $\frac{5}{8}$ to the least common denominator.

Suggestion.—We first find the least common multiple of all the given denominators, which is 24; and this is the least common denominator required. (Art. 102.) The next step is to reduce the given fractions to *twenty-fourths* without altering their value.

Operation.

$$\begin{array}{r} 2)\underline{4 \;''\; 6 \;''\; 8} \\ 2)\underline{2 \;''\; 3 \;''\; 4} \\ 1 \;''\; 3 \;''\; 2 \end{array}$$

$2 \times 2 \times 3 \times 2 = 24$

This is done, by multiplying both terms of each fraction by such a number as will make its denominator 24. (Art. 116.) Now 4, the denominator of the first fraction, is contained in 24, 6 times; and multiplying both terms of the fraction $\frac{3}{4}$ by 6, it becomes $\frac{18}{24}$. The denominator 6 is contained in 24, 4 times; and multiplying the second fraction $\frac{2}{6}$ by 4, it becomes $\frac{8}{24}$. The de-

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QUEST.—*Obs.* Does this process alter the value of the fractions? Why not?

nominator 8 is contained in 24, 3 times; and multiplying the third fraction $\frac{5}{8}$ by 3, it becomes $\frac{15}{24}$. Now $\frac{18}{24}$, $\frac{8}{24}$, and $\frac{15}{24}$ are respectively equal to the given fractions $\frac{3}{4}$, $\frac{2}{6}$, and $\frac{5}{8}$, and are the answer required. Hence,

126. To reduce fractions to their *least* common denominator.

I. *Find the least common multiple of all the denominators of the given fractions, and it will be the least common denominator.*

II. *Divide the least common denominator by the denominator of each of the given fractions, and multiply the numerator by the quotient; the products will be the numerators required.*

OBS. 1. If the example contains *compound* fractions, *whole* or *mixed* numbers, they must first be reduced to simple fractions, then all must be reduced to their lowest terms; otherwise the least common multiple of their denominators, may not be the least common denominator.

2. It is evident this process does not alter the value of the given fractions: for the numerator and denominator of each fraction are multiplied by the same number, consequently their value remains the same. (Art. 116.)

14. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to the least common denominator.

$2 \times 3 \times 2 = 12$, the least common denominator.

Now $12 \div 3 = 4$, and $4 \times 2 = 8$, the 1st numerator.

$12 \div 4 = 3$, and $3 \times 3 = 9$, " 2d "

$12 \div 6 = 2$, and $2 \times 5 = 10$, " 3d "

Ans. $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$.

Operation.

2)3 " 4 " 6

3)3 " 2 " 3

1 " 2 " 1

15. Reduce $\frac{7}{8}$ and $\frac{9}{10}$ to the least common denominator.

Ans. $\frac{35}{40}$ and $\frac{36}{40}$.

Reduce the following to the least common denominator:

16. $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{9}$.

17. $\frac{3}{7}$, $\frac{1}{6}$, and $\frac{11}{14}$.

18. $\frac{8}{9}$, $\frac{7}{8}$, $\frac{5}{12}$, and $\frac{2}{24}$.

19. $\frac{1}{5}$, $\frac{3}{8}$, $\frac{7}{10}$, and $\frac{4}{20}$.

20. $\frac{2}{12}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{2}{6}$.

21. $\frac{5}{16}$, $\frac{7}{24}$, and $\frac{1}{36}$.

22. $\frac{2}{15}$, $\frac{9}{20}$, and $\frac{6}{120}$.

23. $\frac{11}{14}$, $\frac{6}{8}$, and $\frac{13}{18}$.

24. $\frac{5}{9}$, $\frac{3}{24}$, and $\frac{13}{45}$.

25. $\frac{6}{18}$, $\frac{9}{15}$, and $\frac{38}{60}$.

26. $14\frac{1}{2}$, $\frac{3}{7}$, and 19.

27. $28\frac{5}{8}$, $\frac{2}{3}$ of $\frac{3}{4}$ and 45.

28. $\frac{28}{27}$ of 4, and $19\frac{1}{2}$.

29. $\frac{63}{84}$, $\frac{9}{13}$ of $4\frac{1}{3}$, and 63.

30. $\frac{35}{42}$ of $\frac{14}{7}$, and $29\frac{3}{4}$.

31. $\frac{92}{96}$, $\frac{11}{34}$ of 17, and $73\frac{1}{7}$.

QUEST.—126. How reduce fractions to the least common denominator? *Obs.* Does this process alter the value of the given fractions? Why not?

ADDITION OF FRACTIONS.

126.*a*. If two or more fractions have a *common* denominator, the parts of a unit expressed by their numerators, are of the *same value* or *denomination*. (Art. 107. Obs. 3.) Hence,

When fractions have a common denominator, their numerators are added like whole numbers, and the result placed over the common denominator, will be the sum of the given fractions.

Note.—This and the following articles refer to fractions which arise from Simple Numbers. For the method of adding and subtracting Fractional Compound Numbers, see Arts. 168, 169.

Ex. 1. A man gave $\frac{1}{8}$ of a dollar to one of his children, $\frac{2}{8}$ to another, $\frac{3}{8}$ to another, and $\frac{5}{8}$ to another: how much did he give to all?

Suggestion.—Write the fractions one after another with the sign + between them, and add their numerators. Thus, 1 eighth and 2 eighths are 3 eighths, and 3 are 6 eighths, and 5 are 11 eighths. He therefore gave $\frac{11}{8}$, or $1\frac{3}{8}$ dollars to all.

Operation.

$\frac{1}{8}+\frac{2}{8}+\frac{3}{8}+\frac{5}{8}=\frac{11}{8}$, or $1\frac{3}{8}$ *Ans.*

2. A man bought $\frac{1}{2}$ a barrel of flour at one time, $\frac{2}{3}$ of a barrel at another, and $\frac{3}{4}$ of a barrel at another: how much did he buy in all?

Suggestion.—Since these fractions have not a common denominator, it is evident their numerators cannot be added as in the last example; for 1 half, 2 thirds, and 3 fourths will make neither 6 halves, nor 6 thirds, nor 6 fourths. (Art. 22.) We therefore reduce them to a common denominator, then add their numerators as above.

When reduced, the fractions become $\frac{12}{24}$, $\frac{16}{24}$, and $\frac{18}{24}$. Now placing the sum of the numerators 46, over the common denominator, the result is $\frac{46}{24}$, which reduced to a mixed number becomes $1\frac{22}{24}$, or $1\frac{11}{12}$.

Operation.

$1\times3\times4=12$, 1st numerator.
$2\times2\times4=16$, 2d "
$3\times2\times3=18$, 3d "
$2\times3\times4=24$, com. denom.
$\frac{46}{24}$ or $1\frac{11}{12}$ bar. *Ans.*

QUEST.—126.*a*. How do you add fractions which have a common denominator? Explain the reason?

127. From these illustrations and principles, we deduce the following general

RULE FOR ADDITION OF FRACTIONS.

Reduce the fractions to a common denominator; add their numerators, and place the sum over the common denominator.

OBS. 1. *Compound* fractions must be reduced to simple ones, *whole* and *mixed* numbers to improper fractions, and all of them to a common denominator, then add them as above. (Art. 125. Obs. 2, 3.)

2. In adding *mixed* numbers, it is generally more convenient to add the whole numbers and fractional parts separately, and then unite their sums.

3. The operation may frequently be shortened by reducing the given fractions to their *least common denominator*, and adding their numerators. (Art. 126.)

3. What is the sum of $\frac{1}{2}$ of $\frac{2}{3}$, $2\frac{1}{4}$, and 7?

Suggestion.—$\frac{1}{2}$ of $\frac{2}{3}=\frac{2}{6}$, $2\frac{1}{4}=\frac{9}{4}$, and $7=\frac{7}{1}$. Now reducing $\frac{2}{6}$, $\frac{9}{4}$, and $\frac{7}{1}$ to a common denominator as in the margin, and adding their numerators, the result is $\frac{230}{24}$, which is equal to $9\frac{14}{24}$, or $9\frac{7}{12}$.

Operation.

$\frac{2}{6}=\frac{8}{24}$

$\frac{9}{4}=\frac{54}{24}$

$\frac{7}{1}=\frac{168}{24}$.

Ans. $9\frac{14}{24}$, or $9\frac{7}{12}$.

4. Add $\frac{3}{4}$, and $\frac{5}{3}$.
5. Add $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{2}{3}$.
6. Add $\frac{4}{9}$, $\frac{11}{15}$, and $\frac{1}{5}$.
7. Add $\frac{3}{12}$, $\frac{6}{7}$, and $\frac{9}{13}$.
8. Add $\frac{3}{5}$, $\frac{2}{11}$, and $\frac{6}{18}$.
9. Add $\frac{1}{10}$, $\frac{2}{7}$, and $\frac{5}{6}$.
10. Add $\frac{4}{13}$, $\frac{7}{8}$, and $\frac{12}{7}$.
11. Add $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{8}$, and $\frac{9}{2}$.
12. Add $\frac{1}{8}$, $\frac{3}{5}$, $\frac{6}{3}$, and $\frac{8}{7}$.
13. Add $\frac{6}{8}$, $\frac{2}{3}$ of $\frac{1}{2}$ and $\frac{7}{12}$.
14. Add $\frac{3}{5}$, $\frac{2}{3}$, $\frac{7}{8}$ of $\frac{1}{4}$ and $\frac{1}{6}$.
15. Add $\frac{1}{7}$ of 3, $\frac{3}{5}$ of $\frac{1}{3}$, and $\frac{5}{8}$.
16. Add $2\frac{1}{2}$, $6\frac{1}{3}$, and $\frac{2}{3}$.
17. Add $\frac{2}{3}$ of 2, $3\frac{1}{2}$, and $5\frac{2}{7}$.
18. Add $\frac{24}{30}$, $\frac{81}{45}$, and $\frac{19}{10}$.
19. Add $35\frac{1}{8}$, $\frac{53}{12}$, and $\frac{2}{3}$ of $\frac{7}{8}$.
20. Add $\frac{25}{4}$, $6\frac{1}{2}$, $1\frac{2}{3}$, and $\frac{5}{8}$.
21. Add $25\frac{3}{8}$, $18\frac{11}{19}$, and $\frac{23}{34}$.
22. Add $\frac{11}{45}$ of 85, $\frac{21}{35}$ of $3\frac{5}{8}$.
23. Add $\frac{141}{263}$, $\frac{375}{487}$, and $\frac{831}{977}$.
24. Add $24\frac{7}{8}$, $82\frac{3}{7}$, and $\frac{19}{20}$.
25. Add $263\frac{3}{5}$, $\frac{4}{5}$ of $\frac{19}{20}$ of $385\frac{2}{3}$.

26. A grocer sold $47\frac{3}{7}$ pounds of sugar to one customer, $83\frac{3}{5}$ pounds to another, and $68\frac{5}{9}$ pounds to another: how much did he sell to all?

27. If you travel $85\frac{5}{12}$ miles in one day, $78\frac{9}{16}$ in another, and $125\frac{17}{35}$ in another, how far will you travel in all?

28. If a man buys 3 pieces of cloth, containing $127\frac{7}{8}$ yards, $168\frac{9}{16}$ yards, and $256\frac{3}{5}$ yards, how much will he then have?

QUEST.—127. What is the rule for addition of fractions? *Obs.* What must be done with compound fractions, whole and mixed numbers? How else can *mixed numbers* be added? How may the operation be shortened?

SUBTRACTION OF FRACTIONS.

128. When two fractions have *a common denominator, the less numerator may be subtracted from the greater, as in whole numbers, and the result placed over the common denominator, will be the difference between the fractions.* (Art. 126.*a*.)

Ex. 1. If I buy $\frac{35}{57}$ of an acre of land, and afterwards sell $\frac{19}{57}$ of an acre, how much shall I have left?

Suggestion.—We write the less fraction after the greater with the sign − between them, then taking 19 fifty-sevenths from 35 fifty-sevenths, the answer is $\frac{16}{57}$ of an acre.

Operation.

$\frac{35}{57}-\frac{19}{57}=\frac{16}{57}$ *Ans.*

2. From $\frac{5}{6}$ of a yard of cloth, take $\frac{3}{4}$ of a yard.

Suggestion.—Since these fractions have not a common denominator, it is plain that one numerator cannot be taken from the other; for 3 fourths taken from 5 sixths, will leave neither 2 sixths, nor 2 fourths. We must, therefore, reduce them to a common denominator, then subtract as above. When reduced, the fractions become $\frac{20}{24}$, and $\frac{18}{24}$; and 18 twenty-fourths from 20 twenty-fourths leave $\frac{2}{24}$, or $\frac{1}{12}$ yard, which is the answer.

Operation.

$$\left.\begin{array}{l}5\times4=20\\3\times6=18\end{array}\right\}\text{ numerators.}$$

$6\times4=24$ com. denom.

$\frac{20}{24}-\frac{18}{24}=\frac{2}{24}$, or $\frac{1}{12}$ yard.

129. From these illustrations and principles, we deduce the following general

RULE FOR SUBTRACTION OF FRACTIONS.

Reduce the fractions to a common denominator; subtract the less numerator from the greater, and place the remainder over the common denominator.

OBS. 1. *Compound* fractions must be reduced to simple ones, *whole* and *mixed* numbers to improper fractions, and all of them to a common denominator, as in addition.

2. In subtracting *mixed* numbers, it is sometimes more convenient to take the fractional part of the less from the fractional part of the greater, then the integral part of the less from that of the greater.

QUEST.—129. What is the rule for subtraction of fractions? *Obs.* What must be done with compound fractions, whole and mixed numbers? How else are mixed numbers subtracted?

3. In subtracting a *proper* fraction from a whole number, we may borrow a unit and take the fraction from this, then diminish the whole number by 1. (Art. 36.)

3. From $9\frac{2}{3}$ subtract $5\frac{1}{2}$.

Suggestion.—Reducing the mixed numbers to improper fractions, then to a common denominator, they become $\frac{58}{6}$, and $\frac{33}{6}$. Subtracting the less from the greater, we have $\frac{25}{6}$, or $4\frac{1}{6}$.

First operation.

$9\frac{2}{3}=\frac{29}{3}$, and $\frac{29}{3}=\frac{58}{6}$.

$5\frac{1}{2}=\frac{11}{2}$, and $\frac{11}{2}=\frac{33}{6}$.

$\frac{58}{6}-\frac{33}{6}=\frac{25}{6}$, or $4\frac{1}{6}$ *Ans.*

Or, reducing the fractional parts to a common denominator, then subtracting the numerator of the less from that of the greater, the result is $4\frac{1}{6}$, the same as before. (Art. 129. Obs. 2.)

Second operation.

$9\frac{2}{3}=9\frac{4}{6}$.

$5\frac{1}{2}=5\frac{3}{6}$.

Ans. $\overline{4\frac{1}{6}}$.

4. From $\frac{8}{9}$ take $\frac{5}{9}$.
5. From $\frac{10}{12}$ take $\frac{7}{12}$.
6. From $\frac{16}{20}$ take $\frac{2}{5}$.
7. From $\frac{18}{24}$ take $\frac{5}{8}$.
8. From $\frac{37}{42}$ take $\frac{13}{125}$.
9. From $\frac{16}{21}$ take $\frac{8}{15}$.
10. From $\frac{9}{55}$ take $\frac{1}{15}$.
11. From $\frac{2}{7}$ take $\frac{3}{11}$.
12. From $\frac{27}{35}$ take $\frac{23}{45}$.
13. From $\frac{14}{75}$ take $\frac{3}{25}$.
14. From $8\frac{1}{3}$ take $5\frac{2}{3}$. *Ans.* $2\frac{2}{3}$.
15. From $12\frac{5}{8}$ take $7\frac{1}{4}$.
16. From $15\frac{3}{7}$ take $9\frac{1}{2}$.
17. From $25\frac{3}{5}$ take $17\frac{4}{5}$.
18. From $37\frac{4}{7}$ take $19\frac{3}{4}$.
19. From 2 take $\frac{3}{5}$. *Ans.* $\frac{7}{5}$, or $1\frac{2}{5}$.
20. From 6 take $\frac{2}{3}$.
21. From 65 take $25\frac{3}{15}$.
22. From $\frac{2}{3}$ of $\frac{3}{4}$ take $\frac{1}{2}$ of $\frac{2}{6}$.
23. From $\frac{5}{8}$ of $\frac{3}{4}$ take $\frac{1}{6}$ of $\frac{2}{12}$.
24. From $\frac{3}{5}$ of 10 take $\frac{2}{3}$ of 6.
25. From $\frac{6}{8}$ of 24 take $\frac{6}{9}$ of 27.

26. A man bought a wagon for $85\frac{5}{8}$ dollars, and a sleigh for $69\frac{3}{4}$ dollars: how much more did he pay for one than the other?

27. A man having $246\frac{7}{16}$ acres of land, sold $\frac{2}{3}$ of 195 acres: how many acres did he have left?

28. If from a piece of cloth containing $125\frac{19}{20}$ yards, you cut $87\frac{7}{16}$ yards, how many yards will be left?

29. From $563\frac{7}{8}$ pounds, take $\frac{16}{27}$ of $260\frac{1}{2}$ pounds.

30. From $1678\frac{5}{7}$ bushels, take $\frac{5}{9}$ of $\frac{11}{23}$ of 356 bushels.

31. From $\frac{2}{3}$ of $\frac{7}{8}$ of 256 miles, take $\frac{3}{4}$ of 33 miles.

32. From $\frac{5}{9}$ of $\frac{4}{5}$ of $385\frac{1}{4}$ rods, take $\frac{4}{7}$ of $67\frac{1}{3}$ rods.

33. From $\frac{11}{16}$ of $\frac{19}{20}$ of $573\frac{4}{5}$ tons, take $\frac{13}{44}$ of $216\frac{5}{8}$ tons.

QUEST.—In what other manner is a fraction subtracted from a whole number?

MULTIPLICATION OF FRACTIONS.

130. We have seen that multiplying by a *whole number*, is taking the multiplicand as many times, as there are *units* in the multiplier. (Art. 45.) On the other hand,

If the multiplier is only a *part* of a unit, it is plain we must take only a *part* of the multiplicand. That is,

131. *Multiplying by a fraction is taking a certain* PORTION *of the multiplicand as many times, as there are like portions of a unit in the multiplier.* Thus,

Multiplying by $\frac{1}{2}$, is taking 1 *half* of the multiplicand *once*. Thus, $6\times\frac{1}{2}=3$. (Art. 104. Obs.)

Multiplying by $\frac{1}{3}$, is taking 1 *third* of the multiplicand *once*. Thus, $6\times\frac{1}{3}=2$.

Multiplying by $\frac{2}{3}$, is taking 1 *third* of the multiplicand *twice*. Thus, $6\times\frac{2}{3}=4$. Hence,

OBS. 1. To find a *half* of a number, *divide it by* 2.
To find a *third* of a number, *divide it by* 3.
To find a *fourth* of a number, *divide it by* 4, &c.

2. If the multiplier is a *unit* or 1, the product is *equal* to the multiplicand; if the multiplier is *greater* than a unit, the product is *greater* than the multiplicand; and if the multiplier is *less* than a unit, the product is *less* than the multiplicand. (Art. 45.)

CASE I.—*Multiplying a fraction and whole number together.*

Ex. 1. If a bushel of corn is worth $\frac{1}{2}$ of a dollar, how much is 5 bushels worth?

Suggestion.—Multiplying the numerator of the fraction $\frac{1}{2}$, which denotes the price of 1 bushel, by the number of bushels, it gives $\frac{5}{2}$, which is equal to $2\frac{1}{2}$ dollars.

Operation.

$$\frac{1\times5}{2}=\frac{5}{2}, \text{ or } 2\tfrac{1}{2}.$$

Ans. $2\frac{1}{2}$ dolls.

2. If 1 yard of silk costs $\frac{7}{8}$ of a dollar, how much will 4 yards cost?

QUEST.—130. What is meant by multiplying by a whole number? 131. By a fraction? By $\frac{1}{2}$? By $\frac{1}{3}$? By $\frac{2}{3}$? By $\frac{3}{5}$? By $\frac{7}{8}$? *Obs.* How do you find a half of a number? A third? A fourth? An eighth? A hundredth? If the multiplier is a unit or 1, what is the product equal to? When the multiplier is greater than 1, how is the product compared with the multiplicand? When less, how?

Suggestion.—We have seen, that dividing the denominator of a fraction by any number, multiplies the value of the fraction by that number. (Art. 114.) Instead of multiplying the numerator, we may, therefore, divide the denominator by the whole number, and the result is $\frac{7}{2}$, or $3\frac{1}{2}$ dollars. Hence,

Operation.

$\frac{7}{8\div4}=\frac{7}{2}$, and

$\frac{7}{2}=3\frac{1}{2}$ dolls.

132. To multiply a *fraction* by a *whole* number.

Multiply the numerator of the fraction by the whole number, and write the product over the denominator.

Or, divide the denominator by the whole number, when this can be done without a remainder. (Art. 114.)

OBS. 1. A fraction is multiplied into a number *equal* to its *denominator* by *canceling* the denominator. Thus $\frac{4}{7}\times7=4$. (Arts. 89, 91.)

2. On the same principle, a fraction is multiplied into *any factor* in its *denominator*, by *canceling* that factor. Thus, $\frac{3}{15}\times3=\frac{3}{5}$. (Arts. 91, 114.)

3. Multiply $\frac{6}{12}$ by 8.
4. Multiply $\frac{7}{8}$ by 12.
5. Multiply $\frac{5}{15}$ by 18.
6. Multiply $\frac{12}{26}$ by 10.
7. Multiply $\frac{8}{12}$ by 6.
8. Multiply $\frac{1}{4}$ by 12.
9. Multiply $\frac{3}{4}$ by 10.
10. Multiply $\frac{7}{8}$ by 15.
11. Multiply $\frac{5}{8}$ by 2.
12. Multiply $\frac{15}{25}$ by 5.
13. Multiply $\frac{24}{36}$ by 9.
14. Multiply $\frac{18}{25}$ by 25.
15. Multiply $\frac{12}{15}$ by 36.
16. Multiply $\frac{18}{25}$ by 120.
17. Multiply $\frac{256}{375}$ by 25.
18. Multiply $\frac{420}{650}$ by 50.
19. Multiply $9\frac{1}{2}$ by 5.

Suggestion.—In this example we have a *mixed* number to be multiplied by a *whole* number. Now, 5 times $\frac{1}{2}$ are $\frac{5}{2}$, which are equal to 2 and $\frac{1}{2}$. Set down the $\frac{1}{2}$. 5 times 9 are 45, and 2 make 47. Hence,

Operation.

$$\begin{array}{r} 9\frac{1}{2} \\ 5 \\ \hline \end{array}$$

Ans. $47\frac{1}{2}$

133. To multiply a *mixed* number by a *whole* one.

Multiply the fractional part and the whole number separately, and unite the products.

QUEST.—132. How multiply a fraction by a whole number? *Obs.* How is a fraction multiplied by a number equal to its denominator? How by any factor in its denominator? 133. How multiply a mixed number by a whole one?

20. Multiply $15\frac{3}{4}$ by 7. *Ans.* $110\frac{1}{4}$.

21. Multiply $25\frac{4}{5}$ by 10.

22. Multiply $48\frac{10}{16}$ by 8.

23. What will $\frac{2}{3}$ of an acre of land cost, at 26 dollars per acre?

Suggestion.—Multiplying by $\frac{2}{3}$, we have seen, is taking 1 *third* of the multiplicand *twice*. (Art. 131.) Now 1 third of 26 is $8\frac{2}{3}$; and 2 thirds is 2 times as much. 2 times $\frac{2}{3}$ are $\frac{4}{3}$, equal to 1 and $\frac{1}{3}$; and 2 times 8 are 16, and 1 makes 17. *Ans.* $17\frac{1}{3}$ dolls.

First operation.

$$\begin{array}{r} 3\overline{)26} \text{ dollars.} \\ \overline{8\frac{2}{3}} \\ 2 \\ \hline \textit{Ans. } 17\frac{1}{3} \text{ dolls.} \end{array}$$

Or, we may first multiply the whole number by the numerator, and then divide this product by the denominator; for *one third* of *two* times 26, is obviously the same as *two times one third* of it. Hence,

Second operation.

$$\begin{array}{r} 26 \text{ dollars.} \\ 2 \\ \hline 3\overline{)52} \\ \overline{\textit{Ans. } 17\frac{1}{3}} \text{ dolls.} \end{array}$$

134. To multiply a *whole* number by a *fraction*.

Divide the whole number by the denominator, and multiply the quotient by the numerator.

Or, multiply the whole number by the numerator, and divide the product by the denominator.

OBS. 1. When the *whole* number cannot be divided by the denominator without a remainder, the latter method is generally preferred.

2. Since the product of any two numbers is the same, whichever is taken for the multiplier, the fraction may be taken for the multiplicand, and the whole number for the multiplier, whenever it is more convenient. (Art. 47.)

24. Multiply 23 by $\frac{4}{7}$. *Ans.* $13\frac{1}{7}$.

25. Multiply 24 by $\frac{3}{8}$.

26. Multiply 27 by $\frac{3}{5}$.

27. Multiply 35 by $\frac{5}{7}$.

28. Multiply 41 by $\frac{4}{5}$.

29. Multiply 45 by $\frac{7}{9}$.

30. Multiply 65 by $\frac{8}{11}$.

31. Multiply 72 by $\frac{9}{13}$.

32. Multiply 93 by $\frac{9}{16}$.

33. Multiply 25 by $3\frac{1}{2}$.

Suggestion.—In this example it is required to multiply a *whole* by a *mixed* number. We first multiply 25 by 3, then by $\frac{1}{2}$, and the sum of the products is $87\frac{1}{2}$. To find a half of a number we divide the number by 2. (Art. 131. Obs. 1.) Hence,

Operation.

$$\begin{array}{r} 2\overline{)25} \\ 3\frac{1}{2} \\ \hline 75 \\ 12\frac{1}{2} \\ \hline \textit{Ans. } 87\frac{1}{2}. \end{array}$$

QUEST.—134. How multiply a whole number by a fraction?

134.a. To multiply a *whole* by a *mixed* number.

Multiply first by the whole number, then by the fraction, and add the products together.

34. Multiply 27 by $3\frac{1}{3}$. *Ans.* 90.
35. Multiply 63 by $10\frac{3}{7}$.
36. **Multiply 75 by $12\frac{3}{5}$.**

CASE II.—*Multiplying a fraction by a fraction.*

37. A man owning $\frac{2}{3}$ of a ship, sold $\frac{3}{5}$ of what he owned. What part of the ship did he sell?

Analysis.—$\frac{1}{5}$ of $\frac{2}{3}$ is $\frac{2}{15}$; for, multiplying the denominator by any number, divides the value of the fraction. (Art. 113.) Now 3 fifths of $\frac{2}{3}$ is 3 times as much; and $\frac{2}{15}\times 3=\frac{6}{15}$, which, reduced to its lowest terms, is $\frac{2}{5}$. *Ans.*

Or, we may reason thus: Since he owned $\frac{2}{3}$, and sold $\frac{3}{5}$ of what he owned, he must have sold $\frac{3}{5}$ of $\frac{2}{3}$ of the ship. Now $\frac{3}{5}$ of $\frac{2}{3}$ is a compound fraction, whose value is found by multiplying the numerators together for a new numerator, and the denominators for a new denominator. (Art. 123.)

Multiplying the two numerators together, and the two denominators, the result is $\frac{6}{15}$, which reduced to its lowest terms, becomes $\frac{2}{5}$. Hence,

Operation.
$\frac{2}{3}\times\frac{3}{5}=\frac{6}{15}$, or $\frac{2}{5}$ *Ans.*

135. To multiply a *fraction* by a *fraction.*

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

OBS. 1. The *reason* of this rule may be thus seen: Multiplying the numerator of the multiplicand by the numerator of the multiplier, the product is as many times too large as there are units in the denominator of the multiplier. To correct this, we multiply the two denominators together. (Art. 113.) Thus, in the example above, multiplying $\frac{2}{3}$ by 3, the result $\frac{6}{3}$, is 5 times too large; for we wished to find a *fifth* part of 3. We therefore take a fifth of $\frac{6}{3}$ by multiplying the denominator by 5.

2. The process of multiplying one fraction by another, is the same as that of reducing compound fractions to simple ones. (Art. 123.)

38. Multiply $\frac{1}{3}$ by $\frac{3}{5}$. *Ans.* $\frac{3}{15}=\frac{1}{5}$.
39. Multiply $\frac{2}{7}$ by $\frac{2}{3}$.
40. Multiply $\frac{7}{9}$ by $\frac{5}{8}$.
41. Multiply $\frac{4}{12}$ by $\frac{5}{8}$.
42. Multiply $\frac{13}{15}$ by $\frac{6}{8}$.

QUEST.—134.*a.* How multiply a whole by a mixed number? 135. How is a fraction multiplied by a fraction? *Obs.* Explain the reason of the rule. To what is the process of multiplying one fraction by another similar?

Contraction by Cancellation.

43. Multiply $\frac{1}{7}$ and $\frac{2}{3}$ and $\frac{3}{4}$ and $\frac{4}{5}$ together.

Suggestion.—Since the factors 3 and 4 are common both to the numerators and denominators, we may *cancel* them, and multiply the remaining factors together as in reducing compound fractions to simple ones. (Art. 124.) Hence,

Operation.

$$\frac{1}{7}\times\frac{2}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{5}=\frac{2}{35}$$

Ans. $\frac{2}{35}$.

136. To multiply fractions by CANCELLATION.

Cancel all the factors common to the numerators and denominators; then multiply the remaining factors together as in the rule above. (Art. 135.)

OBS. 1. The *reason* of this contraction is manifest from the fact, that it divides the product of the numerators and that of the denominators by the same numbers, and therefore does not alter the value of the answer. (Art. 116.)

2. Care must be taken that the factors canceled in the numerators are *exactly equal* to those canceled in the denominators.

44. Multiply $\frac{3}{4}$ by $\frac{4}{5}$. *Ans.* $\frac{3}{5}$.
45. Multiply $\frac{2}{3}$ by $\frac{1}{6}$ and $\frac{3}{4}$. *Ans.* $\frac{1}{12}$.
46. Multiply $\frac{5}{12}$ by $\frac{4}{5}$ and $\frac{3}{7}$.
47. Multiply $\frac{8}{3}$ by $\frac{3}{13}$ and $\frac{1}{4}$ and $\frac{13}{12}$.
48. Multiply $\frac{75}{80}$ by $\frac{16}{25}$ and $\frac{5}{3}$ and $\frac{4}{9}$ and $\frac{3}{7}$.
49. Multiply $\frac{37}{63}$ by $\frac{9}{14}$ and $\frac{7}{37}$ and $\frac{6}{21}$ and $\frac{3}{2}$.
50. Multiply $7\frac{1}{2}$ by $3\frac{1}{3}$.

Suggestion.—Reducing the mixed numbers to improper fractions, $7\frac{1}{2}=\frac{15}{2}$, and $3\frac{1}{3}=\frac{10}{3}$; then multiplying them together, the result is $\frac{150}{6}$, or 25. Hence,

Operation.

$$\frac{15}{2}\times\frac{10}{3}=\frac{150}{6}$$

Ans. 25.

137. To multiply a *mixed* number by a *mixed* number.

Reduce the mixed numbers to improper fractions, and proceed as in multiplying a fraction by a fraction. (Arts. 135, 136.)

51. Multiply $9\frac{3}{4}$ by $7\frac{1}{2}$.
52. Multiply $28\frac{2}{5}$ by $17\frac{2}{3}$.
53. Multiply $85\frac{7}{8}$ by $23\frac{2}{7}$.
54. Multiply $93\frac{7}{8}$ by $46\frac{4}{5}$.

QUEST.—136. How multiply fractions together by cancellation? *Obs.* How does it appear that this process will give the true answer? What is necessary to be observed with regard to canceling factors? 137. How multiply a mixed number by a mixed number?

137.a. The preceding cases may be summed up in the following general

RULE FOR MULTIPLICATION OF FRACTIONS.

Reduce whole and mixed numbers to improper fractions, then canceling the common factors, multiply the remaining numerators together and the remaining denominators, and the result will be the answer required.

EXAMPLES FOR PRACTICE.

1. What will 12 bushels apples cost, at $\frac{1}{3}$ of a dollar a bushel?
2. If a bushel of wheat weighs $\frac{3}{5}$ of a hundred weight, how much will $10\frac{1}{2}$ bushels weigh?
3. If a man earns $\frac{3}{4}$ of a dollar per day, how much can he earn in $12\frac{1}{4}$ days?
4. If a family consume $\frac{4}{7}$ of a barrel of flour in a week, how much will they consume in $15\frac{1}{3}$ weeks?
5. If I burn $\frac{7}{8}$ of a cord of wood in a month, how much shall I burn in $12\frac{3}{4}$ months?
6. If a man can reap $\frac{11}{12}$ of an acre of grain in a day how many acres can he reap in $29\frac{2}{3}$ days?
7. If a pound of powder is worth 6 shillings, how much is $7\frac{2}{5}$ pounds worth?
8. If a gallon of oil is worth 7 shillings, how much is $8\frac{3}{4}$ gallons worth?
9. When beef is 10 dollars a barrel, how much will $9\frac{5}{8}$ barrels cost?
10. What will $\frac{3}{8}$ of a firkin of butter cost, at $15\frac{4}{5}$ dollars a firkin?
11. At $\frac{5}{8}$ of a dollar a cord, how much will the sawing of $20\frac{1}{4}$ cords of wood amount to?
12. What cost 16 pounds of cheese, at $8\frac{1}{2}$ cents a pound?
13. What cost 9 dozen of eggs, at $12\frac{1}{2}$ cents per dozen?
14. What cost $15\frac{2}{3}$ yards of cambric, at 15 pence per yard?
15. What cost $11\frac{1}{9}$ cords of wood, at $1\frac{1}{2}$ dollar per cord?
16. At $12\frac{1}{2}$ cents a pound, what cost $2\frac{2}{3}$ pounds of pepper?
17. At 5 shillings a pound, what cost $12\frac{3}{8}$ pounds of tea?
18. What cost 16 pounds of starch, at $22\frac{1}{2}$ cents per pound?

QUEST.—137.a. What is the general rule for multiplication of fractions?

19. What cost 18 ounces of nutmegs, at $16\frac{1}{4}$ cents an ounce?
20. At $12\frac{3}{4}$ cents a yard, what will 17 yards of cotton cost?
21. At $3\frac{11}{15}$ dollars a yard, what cost 15 yards of broadcloth?
22. What cost $15\frac{3}{4}$ yards of ribbon, at 10 cents per yard?
23. What cost 22 penknives, at $\frac{13}{16}$ of a dollar apiece?
24. At $\frac{5}{16}$ of a dollar a yard, what cost $8\frac{3}{4}$ yards of lace?
25. At $\frac{3}{8}$ dollar a yard, what will $9\frac{7}{9}$ yards of muslin cost?
26. At $\frac{3}{4}$ dollar a bushel, what cost $\frac{9}{10}$ bushel of wheat?
27. What will $\frac{6}{7}$ pound of tea cost, at $\frac{5}{8}$ of a dollar a pound?
28. What cost 66 bushels of apples, at $18\frac{3}{4}$ cents a bushel?
29. At $62\frac{1}{2}$ cents a yard, what cost $12\frac{1}{2}$ yards of balzorine?
30. What cost $18\frac{1}{2}$ yards of lace, at $16\frac{1}{4}$ cents per yard?
31. What cost 13 bushels of oats, at $18\frac{3}{4}$ cents a bushel?
32. What cost $31\frac{1}{2}$ yards of sheeting, at $\frac{1}{9}$ dollar per yard?
33. At $\frac{7}{12}$ dollar a quart, what cost $18\frac{1}{2}$ quarts of cherries?
34. At $3\frac{3}{4}$ shillings a yard, what cost $7\frac{1}{2}$ yards of gingham?
35. What cost $14\frac{3}{5}$ bushels of potatoes, at $18\frac{3}{4}$ cents a bushel?
36. At $7\frac{3}{8}$ shillings a yard, what cost $8\frac{3}{8}$ yards of silk?
37. At $\frac{7}{8}$ dollar a bushel, what cost $47\frac{4}{5}$ bushels of pears?
38. What cost $63\frac{3}{4}$ pounds of sugar, at $9\frac{3}{4}$ cents per pound?
39. What cost $22\frac{3}{4}$ yards of velvet, at $3\frac{2}{5}$ dollars a yard?
40. What cost $39\frac{3}{4}$ yards of calico, at $1\frac{2}{3}$ shillings a yard?
41. What cost $25\frac{1}{4}$ pounds of figs, at $15\frac{1}{2}$ cents a pound?
42. What cost $35\frac{3}{5}$ cords of wood, at $18\frac{1}{3}$ shillings per cord?
43. What cost $175\frac{1}{2}$ bushels of corn, at $\frac{3}{8}$ of a dollar a bushel?
44. What cost $38\frac{3}{4}$ tons of hay, at $15\frac{7}{8}$ dollars a ton?
45. At $42\frac{1}{2}$ miles a day, how far can you travel in $17\frac{1}{2}$ days?
46. Multiply $85\frac{26}{35}$ by $\frac{3}{4}$ of 19.
47. Multiply 126 by $\frac{5}{9}$ of 33.
48. Multiply $\frac{46}{75}$ by $14\frac{1}{2}$.
49. Multiply $69\frac{3}{4}$ by $\frac{15}{17}$.
50. Multiply $46\frac{3}{4}$ by $31\frac{7}{8}$.
51. Multiply $\frac{14}{15}$ by $\frac{2}{3}$ of 34.
52. Mult. $\frac{5}{9}$ of $\frac{3}{4}$ by $\frac{2}{3}$ of $\frac{4}{5}$.
53. Mult. $\frac{6}{7}$ of 9 by $\frac{3}{5}$ of 7.
54. Multiply $\frac{7}{8}$ of $3\frac{1}{2}$ by $17\frac{3}{4}$.
55. Mult. $\frac{5}{9}$ of $18\frac{1}{7}$ by $\frac{4}{5}$ of $24\frac{1}{2}$.
56. Multiply $16\frac{2}{3}$ by $\frac{4}{5}$ of 6.
57. Multiply $\frac{7}{8}$ of $8\frac{3}{4}$ by $\frac{53}{64}$.
58. Multiply $\frac{7}{8}$ of $9\frac{1}{2}$ by $8\frac{3}{4}$.
59. Multiply $146\frac{3}{4}$ by $\frac{11}{13}$ of $\frac{21}{35}$.
60. Mult. $256\frac{1}{9}$ by $\frac{5}{7}$ of $25\frac{1}{6}$.
61. Mult. $217\frac{1}{8}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of 8.
62. Mult. $316\frac{13}{55}$ by $\frac{6}{19}$ of 38.
63. Mult. $\frac{856}{945}$ by $\frac{5}{7}$ of $\frac{119}{428}$.
64. Mult. $\frac{786}{967}$ by $\frac{3}{10}$ of $\frac{245}{393}$.
65. Mult. $468\frac{5}{11}$ by $\frac{3}{4}$ of $\frac{21}{47}$.
66. Multiply $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 by $\frac{7}{8}$ of $\frac{5}{6}$ of 45.
67. Multiply $\frac{5}{7}$ of $\frac{6}{13}$ of $\frac{26}{35}$ of $\frac{1}{2}$ of 29 by $\frac{11}{13}$ of $\frac{6}{10}$ of $\frac{3}{11}$.
68. Multiply $\frac{7}{8}$ of $\frac{14}{42}$ of $\frac{8}{56}$ of $16\frac{1}{2}$ by $\frac{21}{33}$ of $\frac{28}{35}$ of $\frac{3}{8}$ of 49.

DIVISION OF FRACTIONS.

CASE I.—*Dividing a fraction by a whole number.*

Ex. 1. If 3 bushels of oats cost $\frac{6}{7}$ of a dollar, what will 1 bushel cost?

Suggestion.—1 is 1 third of 3; therefore, 1 bushel will cost 1 third part as much as 3 bushels. Now, dividing the numerator of the fraction $\frac{6}{7}$ by 3, and setting the quotient over the denominator, the result $\frac{2}{7}$, is the answer required.

Operation.
$\frac{6}{7}\div 3=\frac{2}{7}$ *Ans.*

2. If 4 yards of calico cost $\frac{5}{6}$ of a dollar, what will 1 yard cost?

Suggestion.—In this case we cannot divide the numerator by 4, without a remainder. We therefore multiply the denominator by the 4, which divides the fraction. (Art. 118.) Hence,

Operation.
$$\frac{5}{6}\div 4=\frac{5}{6\times 4}, \text{ or } \frac{5}{24} \text{ Ans.}$$

138. To divide a *fraction* by a *whole* number.

Divide the numerator by the whole number, when it can be done without a remainder. Or, if this cannot be done,
Multiply the denominator by the whole number.

3. Divide $\frac{6}{8}$ by 3.

First Method.
$\frac{6}{8}\div 3=\frac{2}{8}$, or $\frac{1}{4}$. *Ans.*

Second Method.
$\frac{6}{8}\div 3=\frac{6}{24}$, or $\frac{1}{4}$. *Ans.*

4. Divide $\frac{12}{15}$ by 6.
5. Divide $\frac{16}{19}$ by 8.
6. Divide $\frac{15}{9}$ by 7.
7. Divide $\frac{17}{11}$ by 12.
8. Divide $\frac{45}{40}$ by 9.
9. Divide $\frac{72}{85}$ by 8.
10. Divide $\frac{125}{231}$ by 25.
11. Divide $\frac{150}{238}$ by 30.

CASE II.—*Dividing a fraction by a fraction.*

12. At $\frac{1}{4}$ of a dollar a pound, how many pounds of honey can be bought for $\frac{3}{4}$ of a dollar?

QUEST.—138. How is a fraction divided by a whole number?

Suggestion.—Since $\frac{1}{4}$ of a dollar will buy 1 pound, $\frac{3}{4}$ of a dollar will buy as many pounds as $\frac{1}{4}$ is contained times in $\frac{3}{4}$. Now 1 fourth is contained in 3 fourths 3 times; that is, dividing one numerator by the other, the quotient is 3. Therefore, $\frac{3}{4}$ of a dollar will buy 3 pounds.

Operation.

$\frac{3}{4} \div \frac{1}{4} = 3$

Ans. 3 pounds.

13. At $\frac{2}{5}$ of a dollar a bushel, how much barley can be bought for $\frac{3}{4}$ of a dollar?

Suggestion.—Since these fractions have different denominators, the parts denoted by their numerators are of *different value;* consequently one numerator cannot be divided by the other. We therefore reduce them to a common denominator; then divide the numerator of the dividend by the numerator of the divisor as above.

First operation.

$\frac{3}{4} = \frac{15}{20}$

$\frac{2}{5} = \frac{8}{20}$

$\frac{15}{20} \div \frac{8}{20} = 1\frac{7}{8}$.

Second operation.

$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$.

Ans. $\frac{15}{8}$, or $1\frac{7}{8}$ bu.

OBS. After the fractions are reduced to a common denominator, it will be perceived that no use is made of the common denominator itself. In practice, therefore, we simply multiply the *numerator* of the dividend by the *denominator* of the divisor, and the *denominator* of the dividend by the *numerator* of the divisor, as in reducing fractions to a common denominator. Or, what is the same in effect, we *invert* the divisor as in the sec. operation, and proceed as in multiplication of fractions. (Art. 135.) Hence,

139. To divide a *fraction* by a *fraction.*

I. *If the given fractions have a common denominator, divide the numerator of the dividend by the numerator of the divisor.*

II. *When the fractions have not a common denominator, invert the divisor, and proceed as in multiplication of fractions.*

OBS. 1. *Compound fractions* must be reduced to simple ones, and *mixed numbers* to improper fractions.

2. When two fractions have a *common denominator*, it is plain *one numerator* can be divided by the *other*, as well as *one whole* number by *another;* for the *parts* denoted by their numerators, are of the *same kind* or *value.*

3. When the fractions do not have a *common* denominator, the *reason* that inverting the divisor, and proceeding as in multiplication, will produce the *true*

QUEST.—139. How is one fraction divided by another when they have a common denominator? How, when they have not a common denominator? *Obs.* What must be done with compound fractions and mixed numbers? When fractions have a common denominator, how does it appear that dividing one numerator by the other will give the true answer?

answer, is because this process, in effect, reduces the two fractions to a *common denominator*, then the numerator of the dividend is divided by the numerator of the divisor.

We do not multiply the two denominators together for a common denominator; for no use is made of a common denominator when found.

The object of *inverting the divisor* is simply for *convenience* in multiplying.

Note.—To *invert* a fraction is to put the numerator in the place of the denominator, and the denominator in the place of the numerator.

139. *a.* The *reason* of the preceding rule may also be explained in the following manner:

Dividing the dividend $\frac{3}{4}$ by 2, the quotient is $\frac{3}{8}$. (Art. 118.) But it is required to divide it by only $\frac{1}{5}$ of 2; consequently $\frac{3}{8}$ is 5 times too small for the true quotient. To correct this, we multiply $\frac{3}{8}$ by 5, and the result will be the answer required. Now $\frac{3}{8}\times 5=\frac{15}{8}$, or $1\frac{7}{8}$, *Ans.*

Operation.

$\frac{3}{4}\div 2=\frac{3}{8}$

$\frac{3}{8}\times 5=\frac{15}{8}$

And $\frac{15}{8}=1\frac{7}{8}$ *Ans.*

14. Divide $\frac{2}{3}$ of $\frac{4}{7}$ by $1\frac{1}{2}$.

Solution.—$\frac{2}{3}$ of $\frac{4}{7}=\frac{8}{21}$, and $1\frac{1}{2}=\frac{3}{2}$. Now $\frac{8}{21}\times\frac{2}{3}=\frac{16}{63}$. *Ans.*

15. Divide $7\frac{1}{2}$ by $2\frac{1}{4}$. *Ans.* $3\frac{1}{3}$.

16. Divide $13\frac{1}{2}$ by $\frac{3}{5}$.

17. Divide $\frac{5}{7}$ by $1\frac{2}{3}$.

18. Divide $\frac{25}{37}$ by $\frac{17}{40}$.

19. Divide $\frac{42}{57}$ by $\frac{19}{20}$.

Contraction by Cancellation.

20. Divide $\frac{1}{3}$ of $\frac{5}{8}$ by $\frac{1}{2}$ of $\frac{2}{3}$.

Suggestion.—We may arrange the numerators, (which answer to dividends,) on the right of a perpendicular line, and the denominators, (which answer to the divisors,) on the left; then canceling the factors 3 and 2, which are common to both sides, (Art. 91.) we multiply the remaining factors in the numerators together, and those remaining in the denominators.

Operation.

$$\begin{array}{r|l} \cancel{3} & 1 \\ 8 & 5 \\ 1 & \cancel{2} \\ \cancel{2} & \cancel{3} \\ \hline 8 & 5=\frac{5}{8} \text{ Ans.} \end{array}$$

Or, thus: $\frac{1}{3}$ of $\frac{5}{8}\div\frac{1}{2}$ of $\frac{2}{3}=\frac{1}{\cancel{3}}$ of $\frac{5}{8}\times\frac{\cancel{2}}{1}$ of $\frac{\cancel{3}}{\cancel{2}}=\frac{5}{8}$. *Ans.*

QUEST.—When the fractions have not a common denominator, how does it appear that inverting the divisor and proceeding as in multiplication will give the true answer? What is the object of inverting the divisor? *Note.* What is *meant by inverting* a fraction?

140. Hence, to divide fractions by *Cancellation.*

Invert the divisor, then cancel the factors common to the numerators and denominators, and proceed as in multiplication of fractions. (Art. 136.)

OBS. 1. Before arranging the terms of the divisor for cancellation, it is always necessary to invert them, or suppose them to be inverted.

2. The *reason* of this contraction is manifest from the fact, that it simply divides the numerator and denominator of the quotient by the same numbers, and therefore does not alter its value. (Art. 116.)

21. Divide $4\frac{1}{2}$ by $2\frac{1}{4}$. *Ans.* 2.
22. Divide $\frac{4}{7}$ of 6 by $\frac{1}{6}$ of 4.
23. Divide $4\frac{1}{3}$ by $\frac{1}{3}$ of $\frac{33}{4}$.
24. Divide $\frac{1}{7}$ of $\frac{25}{28}$ by $\frac{1}{3}$ of $\frac{4}{7}$.
25. Divide $\frac{5}{12}$ of $\frac{3}{4}$ by $\frac{3}{7}$.
26. Divide $\frac{7}{8}$ of $15\frac{3}{4}$ by $4\frac{2}{3}$.
27. Divide $\frac{7}{9}$ by $\frac{23}{37}$ of $\frac{7}{46}$.
28. Divide $\frac{3}{5}$ by $\frac{3}{34}$ of $2\frac{5}{7}$.
29. Divide $25\frac{1}{4}$ by $\frac{1}{4}$ of 26.

CASE III.—*Dividing a whole number by a fraction.*

30. A merchant sent 12 barrels of flour to supply some destitute people, allowing $\frac{2}{3}$ of a barrel to each family. How many families shared in his bounty?

Analysis.—If $\frac{2}{3}$ of a barrel supplied 1 family, 12 barrels will supply as many families as $\frac{2}{3}$ is contained times in 12. Reducing the dividend 12 to the form of a fraction, it becomes $\frac{12}{1}$; then inverting the divisor, we proceed as in dividing a fraction by a fraction.

First operation.

$\frac{12}{1}\times\frac{3}{2}=\frac{36}{2}$

$\frac{36}{2}=18$ *Ans.*

Or, we may reason thus: $\frac{1}{3}$ is contained in 12, as many times as there are thirds in 12, and $12\times3=36$. Now, 2 thirds are contained in 12, only half as many times as 1 third; and $36\div2=18$. (Art. 85.) That is, we multiply the whole number by the denominator, and divide the product by the numerator. Hence,

Second operation.

$$\begin{array}{r} 12 \\ \underline{3} \\ 2)\underline{36} \\ \textit{Ans. } 18 \text{ fam.} \end{array}$$

QUEST.—140. How divide fractions by cancellation? *Obs.* What must be done to the divisor before arranging its terms? How does it appear that this contraction gives the true answer?

141. To divide a *whole* number by a *fraction.*

Reduce the whole number to the form of a fraction, and proceed according to the rule for dividing a fraction by a fraction.

Or, multiply the whole number by the denominator, and divide the product by the numerator.

OBS. When the divisor is a mixed number, it must be reduced to an improper fraction, then proceed as above.

31. Divide 120 by $3\frac{3}{5}$. *Ans.* $33\frac{1}{3}$.
32. Divide 35 by $\frac{2}{5}$.
33. Divide 47 by $\frac{5}{8}$.
34. Divide 165 by $\frac{7}{8}$.
35. Divide 237 by $\frac{11}{15}$.
36. Divide 475 by $128\frac{5}{7}$.
37. Divide 643 by $241\frac{7}{8}$.

141.*a.* The preceding cases may be summed up in the following general

RULE FOR DIVISION OF FRACTIONS.

Reduce compound fractions to simple ones, whole and mixed numbers to improper fractions; then invert the divisor, and proceed as in multiplication of fractions. (Art. 137.*a.*)

EXAMPLES FOR PRACTICE.

1. At $\frac{1}{2}$ dollar per bushel, how many bushels of pears can be bought for 65 dollars?

2. At $\frac{3}{4}$ of a penny apiece, how many apples can be bought for 78 pence?

3. At [illegible] of a dollar a pound, how many pounds of tea will 87 dollars buy?

4. How many bushels of wheat, at $1\frac{1}{4}$ dollar a bushel, can be purchased for 215 dollars?

5. How many gallons of molasses, at $2\frac{1}{2}$ dimes per gallon, will 310 dimes buy?

6. How many yards of satinet, at $1\frac{3}{8}$ of a dollar per yard, can be purchased for 120 dollars?

7. At $4\frac{5}{8}$ dollars per yard, how many yards of cloth can be obtained for $25\frac{1}{2}$ dollars?

8. At $6\frac{1}{4}$ cents a mile, how far can you ride for $62\frac{1}{2}$ cents?

QUEST.—141. How is a whole number divided by a fraction? *Obs.* How by a *mixed number?* 141.*a.* What is the general rule for division of fractions?

9. At $12\frac{1}{2}$ cents a pound, how many pounds of flax will $67\frac{3}{4}$ cents buy?

10. At $16\frac{1}{4}$ cents per pound, how many pounds of figs can you buy for $87\frac{1}{2}$ cents?

11. How many cords of wood, at $6\frac{1}{4}$ dollars per cord, will it take to pay a debt of $67\frac{1}{2}$ dollars?

12. How many barrels of beer, at $11\frac{3}{5}$ dollars per barrel, can be obtained for $95\frac{1}{2}$ dollars?

13. A man bought $15\frac{1}{2}$ barrels of beef for $124\frac{5}{8}$ dollars, how much did he give per barrel?

14. A man bought $13\frac{1}{2}$ pounds of sugar for $94\frac{1}{2}$ cents: how much did his sugar cost him a pound?

15. A lady bought $15\frac{3}{4}$ yards of silk for $145\frac{5}{12}$ shillings: how much did she pay per yard?

16. Bought $15\frac{1}{8}$ baskets of peaches for $24\frac{1}{4}$ dollars: how much was the cost per basket?

17. Bought $30\frac{1}{4}$ yards of breadcloth for $181\frac{1}{2}$ dollars: what was the price per yard?

18. Paid 375 dollars for $125\frac{1}{5}$ pounds of indigo: what was the cost per pound?

19. How many tons of hay, at $16\frac{1}{2}$ dollars per ton, can be bought for $196\frac{1}{8}$ dollars?

20. How many sacks of wool, at $17\frac{1}{8}$ dollars per sack, can be purchased for 1500 dollars?

21. How many bales of cotton, at $15\frac{7}{8}$ dollars per bale, can be bought for 2500 dollars?

22. Divide $145\frac{7}{12}$ by 16.
23. Divide $16\frac{75}{95}$ by 25.
24. Divide 8526 by $45\frac{9}{10}$.
25. Divide 12563 by $68\frac{11}{15}$.
26. Divide $85\frac{63}{73}$ by $18\frac{4}{9}$.
27. Divide $105\frac{2}{65}$ by $82\frac{6}{25}$.
28. Divide $\frac{3}{5}$ of $\frac{8}{15}$ by $6\frac{1}{2}$.
29. Divide $\frac{4}{7}$ of 16 by $\frac{2}{3}$ of $\frac{4}{5}$.
30. Divide $\frac{9}{10}$ of 30 by 19.
31. Divide $\frac{3}{4}$ of $\frac{2}{9}$ by 21.
32. Divide $\frac{8}{11}$ of $\frac{19}{26}$ by $\frac{2}{7}$ of 31.
33. Divide $\frac{4}{15}$ of $\frac{2}{21}$ by $\frac{1}{2}$ of $\frac{5}{8}$.
34. Divide $4\frac{4}{5}$ by $\frac{2}{3}$ of 12.
35. Divide $223\frac{7}{8}$ by $\frac{3}{4}$ of 51.
36. Divide $\frac{12}{23}$ by $18\frac{1}{2}$.
37. Divide $\frac{2}{3}$ of $\frac{3}{5}$ by 48.
38. Divide $25\frac{1}{3}$ by $\frac{4}{5}$ of 7.
39. Divide $42\frac{1}{4}$ by $\frac{4}{5}$ of $53\frac{1}{3}$.
40. Divide $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{8}{14}$ of $\frac{2}{5}$ of $\frac{19}{20}$ by $\frac{3}{5}$ of $\frac{15}{26}$.
41. Divide $\frac{5}{12}$ of $\frac{8}{9}$ of $\frac{12}{15}$ of $\frac{9}{10}$ by $\frac{14}{25}$ of $\frac{15}{11}$ of 18.
42. Divide $\frac{13}{25}$ of $\frac{16}{43}$ of 67 by $\frac{31}{43}$ of $\frac{67}{95}$ of 25.
43. Divide $\frac{23}{24}$ of $\frac{31}{60}$ of $41\frac{1}{2}$ by $\frac{82}{60}$ of $\frac{31}{24}$ of 31.
44. Divide $\frac{47}{85}$ of $\frac{23}{47}$ of $\frac{34}{91}$ of $82\frac{4}{5}$ by $\frac{69}{91}$ of $\frac{34}{23}$ of $42\frac{3}{5}$.

COMPLEX FRACTIONS.

142. From the definition of *complex* fractions, and the manner of expressing them, it will be seen that they arise from *division* of fractions. (Art. 108.)

1. Reduce $\frac{\frac{2}{3}}{\frac{3}{4}}$ to a simple fraction.

Suggestion.—Since the numerator of a fraction answers to the dividend, and the denominator to the divisor, the operation is the same as dividing $\frac{2}{3}$ by $\frac{3}{4}$. We therefore invert the denominator $\frac{3}{4}$, and then multiply the numerators together, and the denominators, as in dividing a fraction by a fraction. The result $\frac{8}{9}$, is the answer required. Hence,

Operation.

$$\frac{\frac{2}{3}}{\frac{3}{4}}=\tfrac{2}{3}\div\tfrac{3}{4}$$

$$\tfrac{2}{3}\div\tfrac{3}{4}=\tfrac{2}{3}\times\tfrac{4}{3}, \text{ or } \tfrac{8}{9}.$$

143. To reduce a complex fraction to a simple one.

Consider the denominator as a divisor, and proceed as in division of fractions. (Art. 139.)

2. Reduce $\frac{2\frac{1}{3}}{5\frac{3}{4}}$ to a simple fraction. *Ans.* $\frac{28}{69}$.

3. Reduce $\frac{6}{8\frac{1}{2}}$.

4. Reduce $\frac{5\frac{1}{2}}{6}$.

5. Reduce $\frac{\frac{3}{5}}{\frac{4}{5}}$.

6. Reduce $\frac{4\frac{5}{9}}{6}$.

7. Reduce $\frac{8}{5\frac{1}{2}}$.

8. Reduce $\frac{9\frac{1}{2}}{7\frac{1}{8}}$.

9. Reduce $\frac{12\frac{1}{4}}{6\frac{1}{5}}$.

10. Reduce $\frac{18\frac{1}{3}}{12\frac{1}{2}}$.

11. Reduce $\frac{20\frac{3}{8}}{25\frac{3}{4}}$.

12. Reduce $\frac{\frac{23}{27}}{\frac{31}{35}}$.

13. Reduce $\frac{251}{\frac{4}{5}}$.

14. Reduce $\frac{\frac{25}{35}}{476}$.

QUEST.—*142.* From what do complex fractions arise? 143. How reduce them to simple fractions?

144. To add, subtract, multiply, or divide *complex* fractions.

Reduce them to simple ones, then proceed according to the rules of Simple fractions.

Note.—If the following examples are found too difficult for beginners, they can be omitted till review.

1. Add $\frac{4\frac{1}{2}}{8}$ and $\frac{12}{5\frac{1}{3}}$.
2. Add $\frac{\frac{22}{34}}{\frac{18}{25}}$ and $\frac{25\frac{1}{2}}{43}$.
3. Add $\frac{12\frac{3}{4}}{16\frac{5}{7}}$ and $\frac{22\frac{1}{2}}{38\frac{2}{3}}$.
4. Add $\frac{51\frac{11}{12}}{63\frac{14}{15}}$ and $\frac{17\frac{4}{5}}{13\frac{1}{2}}$.
5. From $\frac{13\frac{4}{5}}{17\frac{3}{7}}$ take $\frac{2\frac{1}{2}}{8\frac{3}{4}}$.
6. From $\frac{18\frac{5}{9}}{21}$ take $\frac{11}{23\frac{1}{2}}$.
7. From $\frac{25\frac{3}{4}}{\frac{11}{12}}$ take $\frac{13\frac{3}{4}}{24\frac{2}{3}}$.
8. From $\frac{48\frac{13}{27}}{51\frac{16}{35}}$ take $\frac{19\frac{1}{2}}{37\frac{3}{4}}$.
9. Multiply $\frac{2\frac{1}{3}}{2\frac{1}{4}}$ by $\frac{4\frac{1}{2}}{1\frac{3}{4}}$.
10. Multiply $\frac{4\frac{3}{4}}{5\frac{7}{8}}$ by $\frac{7\frac{1}{5}}{10\frac{1}{4}}$.
11. Multiply $\frac{5\frac{1}{3}}{2\frac{2}{5}}$ by $\frac{2\frac{1}{2}}{\frac{5}{12}}$.
12. Multiply $\frac{3\frac{5}{8}}{6\frac{1}{2}}$ by $\frac{5\frac{5}{6}}{2\frac{2}{3}}$.
13. Multiply $\frac{2\frac{2}{9}}{\frac{3}{5}} \times \frac{5\frac{1}{3}}{2\frac{1}{2}}$ by $\frac{\frac{3}{4}}{6\frac{2}{3}}$.
14. Multiply $\frac{3\frac{3}{4}}{2\frac{5}{6}} \times \frac{2\frac{2}{3}}{3\frac{3}{8}}$ by $\frac{\frac{3}{8}}{\frac{2}{7}}$.
15. Divide $\frac{15\frac{3}{7}}{63}$ by $\frac{18\frac{1}{2}}{21\frac{2}{3}}$.
16. Divide $\frac{18\frac{11}{12}}{17\frac{15}{16}}$ by $\frac{\frac{42}{35}}{65}$.
17. Divide $\frac{27\frac{3}{4}}{48\frac{5}{7}}$ by $\frac{21}{7\frac{5}{8}}$.
18. Divide $\frac{45\frac{21}{43}}{24\frac{9}{20}}$ by $\frac{39\frac{4}{5}}{43\frac{7}{8}}$.
19. Multiply the sum of $\frac{118\frac{2}{3}}{71\frac{1}{5}}$ and $\frac{50\frac{5}{8}}{45}$ by $\frac{68\frac{4}{9}}{85\frac{3}{4}}$.
20. What is the quotient of $\frac{83\frac{11}{14}}{95\frac{13}{15}}$ divided by $\frac{7\frac{1}{2}}{12}$ of $\frac{5\frac{1}{2}}{7\frac{1}{4}}$.
21. Divide the product of $\frac{65\frac{3}{8}}{87\frac{1}{2}}$ into $\frac{46\frac{1}{3}}{140}$ by $\frac{16\frac{3}{4}}{13\frac{2}{5}}$.
22. Find the sum of $\frac{3}{4}$ of $\frac{44\frac{2}{5}}{63\frac{3}{7}}$, $\frac{4}{5}$ of $\frac{17}{25\frac{1}{2}}$, and $\frac{1}{3}$ of $\frac{92\frac{4}{7}}{81}$.
23. Divide $\frac{35\frac{1}{2}}{49\frac{2}{3}}$ of $119\frac{1}{3}$ by $\frac{86\frac{5}{7}}{607}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{48\frac{1}{2}}{57}$.

QUEST.—144. How do you add, subtract, multiply, or divide complex fractions?

EXERCISES IN FRACTIONS.

1. A man bought 3 pieces of cloth, one of which contained $45\frac{3}{4}$ yards, another $63\frac{5}{8}$ yards, and the other $56\frac{4}{7}$ yards: how many yards did he buy?

2. If you travel $85\frac{7}{8}$ miles in one day, $95\frac{11}{12}$ miles the second, and $115\frac{5}{16}$ miles the third day, how far will you travel in all?

3. What is the sum of $\frac{7}{12}$ of $\frac{9}{21}$ of 6, $75\frac{3}{5}$, $163\frac{26}{35}$, and $185\frac{7}{10}$?

4. What is the sum of $69\frac{3}{4}$, $\frac{8}{9}$ of 360, $\frac{2}{3}$ of $\frac{3}{4}$ of $46\frac{1}{2}$, and $76\frac{4}{5}$?

5. A man having $178\frac{3}{4}$ acres of land, sold $87\frac{3}{5}$ acres: how many acres had he left?

6. A young man having $685\frac{7}{8}$ dollars, lost $263\frac{4}{5}$ dollars by gambling: how much did he have left?

7. If I buy $\frac{11}{16}$ of a ship, and sell $\frac{2}{3}$ of what I buy, how much shall I then own?

8. A miller having 675 barrels of flour, sold 17 barrels more than $\frac{3}{4}$ of it: how much did he have left?

9. What will $175\frac{3}{4}$ gallons of oil cost, at $1\frac{3}{8}$ dollar per gallon?

10. What cost $78\frac{4}{5}$ boxes of sugar, at $24\frac{3}{4}$ dollars per box?

11. What is 29 times $\frac{3}{8}$ of 14641?

12. What is the product of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{6}{7}$ of 23 into $\frac{2}{3}$ of $45\frac{3}{4}$?

13. If you pay $125\frac{3}{8}$ dollars for $89\frac{2}{5}$ baskets of peaches, how much is that per basket?

14. If $213\frac{3}{4}$ barrels of beef cost $1835\frac{7}{8}$ dollars, how much is that per barrel?

15. How many times is $\frac{3}{4}$ of $\frac{7}{8}$ of 32 contained in $563\frac{1}{8}$?

16. How often is $27\frac{1}{5}$ contained in $\frac{4}{5}$ of $\frac{7}{8}$ of $84\frac{3}{7}$?

17. What is the sum of $156\frac{7}{8}$ and $83\frac{2}{3}$? What is their difference?

18. What is the product of $\frac{2}{3}$ of $\frac{4}{5}$ of 156, and $11\frac{3}{7}$? What is the quotient?

19. What is the sum, difference, product, and quotient of $268\frac{5}{9}$, and $75\frac{4}{5}$?

20. What cost $35\frac{3}{8}$ cords of wood, at $3\frac{3}{4}$ dollars a cord?

21. What cost $68\frac{9}{10}$ tons of coal, at $6\frac{5}{8}$ dollars a ton?

22. At $17\frac{3}{5}$ dollars per week, how much will it cost a family to board $43\frac{1}{2}$ weeks?

23. A drover bought 45 cows, at $19\frac{3}{5}$ dollars apiece, and *68 oxen*, at $39\frac{5}{8}$ dollars: how much did he pay for both?

24. If from $\frac{4}{5}$ of $\frac{19}{20}$ of $78\frac{1}{2}$ miles, you subtract $\frac{1}{7}$ of $\frac{3}{8}$ of $16\frac{3}{4}$ miles, how many miles will be left?

25. If you divide $87\frac{1}{2}$ by $16\frac{3}{4}$, and multiply the quotient by $27\frac{3}{4}$, what will the product be?

26. If the sum of $87\frac{11}{12}$ and $117\frac{16}{27}$ is divided by their difference, what will be the quotient?

27. If the difference between $91\frac{4}{7}$ and $65\frac{3}{8}$ is multiplied by $\frac{3}{5}$ of the former, what will be the product?

28. If you pay $188\frac{3}{7}$ pounds sterling for $168\frac{1}{3}$ tons of Liverpool coal, how much is that per ton?

29. A merchant paid $1468\frac{4}{5}$ pounds sterling for $178\frac{1}{2}$ packages of goods: what was that a package?

30. If you pay $\frac{1}{4}$ of $\frac{7}{11}$ of $4\frac{2}{3}$ cents apiece for pears, how many can you buy for $\frac{2}{3}$ of $\frac{5}{8}$ of $18\frac{3}{4}$ cents?

31. What number must be taken from $185\frac{13}{16}$, that the difference may be 63?

32. What number must be taken from $256\frac{19}{25}$, that the remainder may be $116\frac{2}{3}$?

33. By what number must $12\frac{2}{3}$ be multiplied, that the product may be 56?

34. What number must be multiplied by $15\frac{2}{3}$, that the product may be $56\frac{1}{2}$?

35. What number must be divided by $28\frac{3}{5}$, that the quotient may be 35?

36. What number must be divided by $37\frac{1}{2}$, that the quotient may be $\frac{2}{5}$ of $\frac{5}{7}$ of 8?

37. What number must be added to $26\frac{3}{5}$, and the sum multiplied by $7\frac{3}{5}$, that the product may be 496?

38. Bought a piece of silk containing $96\frac{3}{4}$ yards, and having used $\frac{2}{3}$ of it, sold $\frac{3}{4}$ of the remainder, at $1\frac{1}{4}$ dollar a yard; the remnant was put at $\frac{7}{8}$ dollar a yard: how much did the parts sold come to?

39. A man having $56\frac{1}{2}$ tons of hay, sold $\frac{3}{5}$ of it at $10\frac{5}{8}$ dollars per ton, and the remainder at $9\frac{3}{4}$ dollars: how much did he receive for his hay?

40. A grocer bought $84\frac{5}{8}$ barrels of flour, at $5\frac{3}{4}$ dollars per barrel, and sold it at $6\frac{1}{4}$ dollars per barrel: how much did he gain?

41. What number must be multiplied by $45\frac{3}{5}$, that the product may be $17\frac{1}{2}$ times $128\frac{4}{7}$?

SECTION VII.

COMPOUND OR DENOMINATE NUMBERS.

ART. **145.** SIMPLE *Numbers* are those which express *units* of the *same kind* or *denomination;* as one, two, three; 3 pears, 5 feet, 7 roses, &c.

COMPOUND *Numbers* are those which express *units* of *different kinds* or *denominations;* as the divisions of money, weight, and measure. Thus, 6 shillings and 7 pence; 7 feet and 3 inches, &c., are compound numbers.

Note.—Compound Numbers are frequently called *Denominate Numbers.*

FEDERAL MONEY.

146. *Federal Money* is the currency of the United States. Its denominations are *Eagles, dollars, dimes, cents,* and *mills.*

10 mills (*m.*)	make 1 cent,	marked	*ct.*
10 cents	" 1 dime,	"	*d.*
10 dimes	" 1 dollar,	"	*doll.* or $.
10 dollars	" 1 eagle,	"	*E.*

OBS. Federal Money was established by Congress, Aug. 8th, 1786. Previous to this, English or sterling money was the principal currency of the country.

146.*a*. The national coins of the United States are of three kinds, viz: gold, silver, and copper.

1. The gold coins are the *eagle, the double eagle*, half eagle, quarter eagle, and gold dollar.**

The eagle contains 258 grains of *standard* gold; the double eagle, half eagle, and quarter eagle, like proportions.

2. The silver coins are the *dollar, half dollar, quarter dollar,* the *dime, half dime,* and *three-cent-piece.*†

The dollar contains $412\frac{1}{2}$ grs.; the half dollar 192 grs.; the quarter dollar 96 grs.; the dime 38.4 grs.; the half dime 19.2 grs.‡

QUEST.—145. What are simple numbers? What are compound numbers? 146. What is Federal Money? What are its denominations? Recite the Table. *Obs.* When and by whom was it established? 146.*a.* Of how many kinds are the coins of the United States? What are they? What are the gold coins? The silver coins?

* *By Act of* Congress, Feb. 20th, 1849. † March 3d, 1851. ‡ Feb. 1853.

3. The copper coins are the *cent*, and *half cent*.

The cent contains 168 grains of *pure* copper; the half cent, a like proportion. Mills are not coined.

OBS. 1. The fineness of gold used for coin, jewelry, and other purposes, also the gold of commerce, is estimated by the number of parts of gold which it contains. Pure gold is commonly supposed to be divided into 24 equal parts, called *carats*. Hence, if it contains 10 parts of *alloy*, or some *baser* metal, it is said to be 14 carats fine; if 5 parts of alloy, 19 carats fine; and when absolutely pure, it is 24 carats fine.

2. The present *standard* for both *gold* and *silver coins* of the United States, by Act of Congress, 1837, is 900 parts of pure metal by weight to 100 parts of alloy. The alloy of gold coin is composed of silver and copper, the silver not to exceed the copper in weight. The alloy of silver coin is pure copper.

STERLING MONEY.

147. *English* or *Sterling Money* is the national currency of *Great Britain*. Its denominations are *pounds*, *shillings*, *pence*, and *farthings*.

4 farthings (*qr.* or *far.*)	make	1 penny,	marked	*d.*
12 pence	"	1 shilling,	"	*s.*
20 shillings	"	1 pound, or sovereign,		£.
21 shillings	"	1 guinea.		*G.*

OBS. 1. It is customary to express farthings in fractions of a penny. Thus 1 qr. is written ¼d.; 2 qrs., ½d.; 3 qrs., ¾d.

2. The Pound Sterling is represented by a gold coin, called a *Sovereign*. According to *Act of Congress*, 1842, the *value* of a pound sterling is 4 *Dollars* and 84 *cents*. Its *intrinsic* value, according to assays at the United States Mint, is 4 dollars, 86 cents, and 1 mill. The *legal* value of an English shilling is $24\frac{1}{3}$ cents.

Note.—Most children have very *erroneous* and *indistinct ideas* of the *weights* and *measures* in common use. It is, therefore, strongly recommended for teachers to illustrate them *practically*, by referring to some visible object of equal magnitude, or by exhibiting the ounce; the pound; the *linear* inch, foot, yard, and rod; also a *square* and *cubic* inch, foot, and yard; the pint, quart, &c.

QUEST.—The copper? *Obs.* How is the fineness of gold estimated? Into how many carats is pure gold supposed to be divided? When it contains 10 parts of alloy, how fine is it said to be? 5 parts of alloy? 2 parts? 4 parts? What is the present standard for the gold and silver coins of the United States? What is the alloy of gold coins? What of silver coins? 147. What is Sterling Money? What are its denominations? Repeat the Table. *Obs.* How are farthings usually expressed? How is a pound sterling represented? What is its value in dollars and cents? What is the legal value of an English shilling? How many farthings in 5 pence? How many farthings in 8d.? In 10d.? In 12d.? How many pence in 3 shillings? In 5s.? In 7s.? In 11s.?

Note.—It will be found profitable to drill the class in mental exercises like the preceding, upon each of the subsequent Tables.

TROY WEIGHT.

148. *Troy Weight* is used in weighing gold, silver, jewels, liquors, &c., and is generally adopted in philosophical experiments. Its denominations are *pounds*, *ounces*, *pennyweights*, and *grains*.

24 grains (*gr.*)	make	1 pennyweight,	marked	*pwt.*
20 pennyweights	"	1 ounce,	"	*oz.*
12 ounces	"	1 pound,	"	*lb.*

The *carat*, by which the weight and value of diamonds are estimated, is equal to 4 *grains*.

OBS. 1. The *standard* of Weights and Measures is different in different countries and in different States of the Union. In 1834, the Government of the United States adopted a uniform standard, for the use of the several custom houses and other purposes. Since that, many of the States have adopted the same.

2. The *standard unit* of *Weight* adopted by the *United States*, is the *Troy Pound* of the United States Mint, which is equal to $22\frac{794}{1000}$ cubic inches of distilled water, at its maximum density, the barometer standing at 30 inches, and is identical with the Imperial Troy pound of England, established by Act of Parliament, A. D. 1826.* The maximum density of water is at the temperature of about 40° Fahrenheit.—*Olmsted's Philosophy.*

3. Troy Weight was formerly used in weighing articles of every kind. It was introduced into Europe from Cairo in Egypt, about the time of the Crusades, in the 12th century. Some suppose its name was derived from *Troyes*, a city in France, which first adopted it; others think it was derived from *Troy-novant*, the former name of London.†

AVOIRDUPOIS WEIGHT.

149. *Avoirdupois Weight* is used in weighing *groceries* and *coarse articles;* as sugar, tea, coffee, butter, cheese, flour, hay, &c., and *all metals*, except gold and silver. Its denominations are *tons*, *hundreds*, *quarters*, *pounds*, *ounces*, and *drams*.

16 drams (*dr.*)	make	1 ounce,	marked	*oz.*
16 ounces	"	1 pound,	"	*lb.*
25 pounds	"	1 quarter,	"	*qr.*
4 quarters, or 100 lbs.	"	1 hundred weight,		*cwt.*
20 hundred weight	"	1 ton, marked		*T.*

QUEST.—148. To what is Troy Weight applied? What are its denominations? Repeat the Table. *Obs.* When was Troy Weight introduced into Europe? From what was its name derived? Do all the States have the same standard of weights and measures? What is the standard unit of weight adopted by the Government of the United States? 149. In what is Avoirdupois Weight used? What are its denominations? Repeat the Table.

* Hassler on Weights and Measures, p. 10. Also, Reports of the Secretary of the Treasury, March 3d, 1831; and June 20th, 1832.

† Hind's Arithmetic, Art. 224. Also, North American Review, Vol. XLV.

OBS. 1. The *Avoirdupois Pound* of the *United States*, is equal to $27\frac{7015}{10000}$ cubic inches of distilled water, at the maximum density, and at 30 inches barometer. It is determined from the Troy pound, by the legal proportions of 5760 grains, which constitute the Troy pound, to 7000 grains Troy, which constitute the avoirdupois pound.* That is,

5760 grains	Troy make		1 pound	Troy.
7000 grains	"	"	1 pound	Avoirdupois.
$437\frac{1}{2}$ grains	"	"	1 ounce	"
$27\frac{11}{32}$ grains	"	"	1 dram	"

2. The British Imperial Pound Avoirdupois is defined to be the weight of $27\frac{7274}{10000}$ cubic inches of distilled water, at the temperature of 62° Fahrenheit, when the barometer stands at 30°.†

3. Formerly it was customary to allow 112 pounds for a hundred weight, and 28 pounds for a quarter; but this practice has become nearly or quite obsolete. The laws of most of the States, as well as general usage, call 100 lbs. a hundred weight, and 25 lbs. a quarter.

In estimating duties, and weighing a few coarse articles, as iron, dye-woods, and coal at the mines, 112 lbs. are still allowed for a hundred weight. Coal, however, is sold in cities, at 100 lbs. for a hundred weight.

4. *Gross* weight is the weight of goods with the boxes, casks, or bags which contain them, and allows 112 lbs. for a hundred weight.

Net weight is the weight of goods only, and allows 100 lbs. for a hundred weight.

Note.—The term *Avoirdupois*, is thought by some to be derived from the French *avoir du poids*, a phrase signifying to have weight. Others think it is from *avoirs*, the ancient name of *goods* or *chattels*, and *poids* signifying *weight* in the Norman dialect.‡

APOTHECARIES' WEIGHT.

150. *Apothecaries' Weight* is used by apothecaries and physicians in *mixing medicines*. Its denominations are *pounds*, *ounces*, *drams*, *scruples*, and *grains*.

20 grains (*gr.*)	make	1 scruple,	marked	*sc.*, or ℈.
8 scruples	"	1 dram,	"	*dr.*, or ʒ.
8 drams	"	1 ounce,	"	*oz.*, or ℥.
12 ounces	"	1 pound,	"	℔.

QUEST.—Point to an object that weighs an ounce. A pound. *Obs.* How is the Avoirdupois pound of the United States determined? How many pounds were formerly allowed for a hundred weight? For a quarter? What is gross weight? Net weight? 150. In what is Apothecaries' Weight used? What are its denominations? Recite the Table.

* Reports of Secretary of Treasury, March 3d, 1832; June 20, 1832. Also, Congressional Documents of 1833.

† Hind's Arithmetic, Art. 223.

‡ Adams' Report on Weight and Measures; also, Hind's Arithmetic.

OBS. 1. The pound and ounce in this weight are the same as the *Troy* pound and ounce. The only difference between Apothecaries' and Troy weight, is in the *divisions* and *subdivisions* of the *ounce.*

2. Drugs and medicines are sold at wholesale by *avoirdupois* weight.

LONG MEASURE.

151. *Long Measure* is used in measuring *length* or *distances only*, without regard to *breadth* or *depth.* Its denominations are *leagues*, *miles*, *furlongs*, *rods*, *yards*, *feet*, and *inches.*

12 inches (*in.*)	make	1 foot,	marked	*ft.*
3 feet	"	1 yard,	"	*yd.*
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	"	1 rod, perch, or pole,	"	*r.* or *p.*
40 rods	"	1 furlong,	"	*fur.*
8 furlongs, or 320 rods	"	1 mile,	"	*m.*
3 miles	"	1 league,	"	*l.*
60 geographical miles, or $69\frac{1}{2}$ statute miles	"	1 degree,	"	*deg.* or °

360 degrees make a great circle, or the circumference of the earth.

Note.—4 inches make 1 hand; 9 inches, 1 span; 18 inches, 1 cubit; 6 feet, 1 fathom; 4 rods or 100 links, 1 chain; 25 links, 1 rod, $7\frac{92}{100}$ inches, 1 link.

The chain is commonly used in measuring roads and land, and is called *Gunter's Chain*, from the name of its inventor.

A *knot*, in sea phrase, answers to a *nautical* or *geographical* mile.

OBS. 1. The *standard unit* of *Length* adopted by the *United States*, is the *Yard* of 3 feet, or 36 inches, and is identical with the British Imperial Yard. It is made of brass, and is determined from the scale of Troughton,* at the temperature of 62° Fahrenheit. For the method of determining the *standard unit* of length adopted by Great Britain, France, and the State of New York, see Higher Arithmetic, Arts. 255, 273.

2. Long Measure is frequently called *linear*, or *lineal* measure. Formerly the inch was divided into 3 *barleycorns*; but the barleycorn, as a measure, has become obsolete.

The inch is commonly divided either into *eighths* or *tenths*; sometimes, however, it is divided into *twelfths*, which are called *lines.*

QUEST.—*Obs.* To what are the apothecaries' ounce and pound equal? How are drugs and medicines bought and sold? 151. In what is Long Measure used? What are its denominations? Recite the Table. Draw a line an inch long upon the blackboard. Draw one a foot, and another a yard long. How long is your desk? Your teacher's table? How wide? How long is the school room? How wide? *Obs.* What is the standard unit of Length adopted by the United States? What is Long Measure frequently called?

* A celebrated English artist.

CLOTH MEASURE.

152. *Cloth Measure* is used in measuring *cloth*, *lace*, and all kinds of goods which are bought or sold by the *yard*. Its denominations are *ells*, *yards*, *quarters*, *nails*, and *inches*.

$2\frac{1}{4}$ inches (*in.*)	make	1 nail,	marked	*na.*
4 nails, or 9 in.	"	1 quarter of a yard,	"	*qr.*
4 quarters	"	1 yard,	"	*yd.*
3 quarters, or $\frac{3}{4}$ of a yard	"	1 Flemish ell,	"	*Fl. e.*
5 quarters, or $1\frac{1}{4}$ yard	"	1 English ell,	"	*E. e.*
6 quarters, or $1\frac{1}{2}$ yard	"	1 French ell,	"	*F. e.*

OBS. *Cloth* measure is a species of *long* measure. The yard is the same in both. Cloths, laces, &c., are bought and sold by the *linear* yard without regard to their width. Ells are seldom used.

SQUARE MEASURE.

153. *Square Measure* is used in measuring *surfaces*, or things whose *length* and *breadth* are considered without regard to *heighth* or *depth;* as land, flooring, plastering, &c. Its denominations are *acres*, *roods*, *square rods*, *square yards*, *square feet*, and *square inches*.

144 square inches (*sq. in.*)	make	1 square foot,	marked	*sq. ft.*
9 square feet	"	1 square yard.	"	*sq. yd.*
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet	"	1 square rod, perch or pole,	"	*sq. r.*
40 square rods	"	1 rood,	"	*R.*
4 roods, or 160 square rods	"	1 acre	"	*A.*
640 acres	"	1 square mile,	"	*M.*

Note.—16 square rods make 1 square chain; 10 square chains, or 100,000 square links, make an acre. Flooring, roofing, plastering, &c., are frequently estimated by the "square" which contains 100 square feet.

OBS. 1. A *square* is a figure which has *four equal* sides, and all its angles *right angles*, as seen in the following diagram. Hence,

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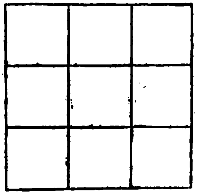
QUEST.—152. In what is Cloth Measure used? What are its denominations? Repeat the Table. *Obs.* Of what is cloth measure a species? What is the kind of yard by which cloths, laces, &c., are bought and sold? 153. To what use is Square Measure applied? What are its denominations? Recite the Table. *Obs.* What is a square? Draw a square upon the blackboard.

2. A *Square Inch* is a square, whose sides are each a *linear* inch in length.

A *Square Foot* is a square, whose sides are each a *linear* foot in length, and is equal to 144 square inches, or the product of 12 in. × 12.

A *Square Yard* is a square, whose sides are each a *linear* yard, or three *linear* feet in length, and is equal to 9 *square* feet, or the product of 3 ft. × 3 as represented in the adjacent figure. Hence,

3 *ft.* × 3 = 1 *sq. yd.*



3. *The area of squares, and all right-angled parallelograms, is found by multiplying the length by the breadth.*

CUBIC MEASURE.

154. *Cubic Measure* is used in measuring *solid* bodies, or things which have *length*, *breadth*, and *thickness*, such as timber, boxes of goods, the capacity of rooms, &c. Its denominations are *tons*, *cords*, *cubic yards*, *cubic feet*, and *cubic inches*.

1728 cubic inches (*cu. in.*)	make	1 cubic foot,	marked	*cu. ft.*
27 cubic feet	"	1 cubic yard,	"	*cu. yd.*
40 feet of round, or 50 ft. of hewn timber	"	1 ton,	"	*T.*
42 cubic feet	"	1 ton of shipping,	"	*T.*
16 cubic feet	"	1 foot of wood, or a cord foot,	"	*c. ft.*
8 cord feet, or 128 cubic feet	"	1 cord,	"	*C.*

OBS. 1. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 cord. For, 8 × 4 × 4 = 128.

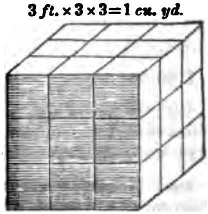
2. A *Cube* is a solid body bounded by *six equal squares*. It is often called a *hexaedron*. Hence,

A *Cubic Inch* is a cube, each of whose sides is a *square* inch, as represented by the adjoining figure.

A *Cubic Foot* is a cube, each of whose sides is a *square* foot.

A *Cubic Yard* is a cube, each of whose sides is a *square* yard, and is equal to 27 *cubic* feet, or the product of 3 ft. × 3 × 3. Hence,

3 *ft.* × 3 × 3 = 1 *cu. yd.*



QUEST.—What is a square inch? A square foot? What is a square yard? Draw a square inch; a square foot; a square yard. 154. To what use is Cubic Measure applied? What are its denominations? Recite the Table. *Obs.* What is a cube? What is a cubic inch? A cubic foot? Draw a cubic inch upon the *blackboard*. What is meant by a ton of round timber?

3. *The solidity of a cube, and all bodies having six sides perpendicular to each other, is found by multiplying the length, breadth and thickness together.*

4. The *Cubic Ton* is chiefly used for estimating the cartage and transportation of timber. By a *ton of round timber* is meant, such a quantity of timber in its rough or natural state, as when hewn, will make 40 cubic feet, and is supposed to be equal in weight to 50 feet of hewn timber.

The *cubic ton*, (formerly called *load*,) is by no means an *accurate* or *uniform* standard of estimating weight; for, different kinds of timber, are of very different degrees of density.

WINE MEASURE.

155. *Wine Measure* is used in measuring *wine*, *alcohol*, *molasses*, *oil*, and all other liquids except beer, ale, and milk. Its denominations are *tuns*, *pipes*, *hogsheads*, *gallons*, *quarts*, *pints*, and *gills*.

4 gills (*gi.*)	make 1 pint,	marked	*pt.*
2 pints	" 1 quart,	"	*qt.*
4 quarts	" 1 gallon,	"	*gal.*
$31\frac{1}{2}$ gallons	" 1 barrel,	"	*bar.* or *bbl.*
42 gallons	" 1 tierce,	"	*tier.*
63 gallons, or 2 barrels	" 1 hogshead,	"	*hhd.*
2 hogsheads	" 1 pipe or butt,	"	*pi.*
2 pipes	" 1 tun,	"	*tun.*

OBS. The *standard unit* of *Liquid Measure* of the *United States*, is the *Wine Gallon*, which contains 231 cubic inches, and is equal to $8\frac{339}{1000}$ pounds avoirdupois of distilled water, at the maximum density.

BEER MEASURE.

156. *Beer Measure* is used in measuring *beer*, *ale*, and *milk*. Its denominations are *hogsheads*, *barrels*, *gallons*, *quarts*, *pints*.

2 pints (*pt.*)	make 1 quart,	marked	*qt.*
4 quarts	" 1 gallon,	"	*gal.*
36 gallons	" 1 barrel,	"	*bar.* or *bbl.*
$1\frac{1}{2}$ barrels, or 54 gals.	" 1 hogshead,	"	*hhd.*

OBS. The beer gallon contains 282 cubic inches. In many places milk is measured by wine measure.

QUEST.—155. In what is Wine Measure used? What are its denominations? Recite the Table. *Obs.* What is the standard unit of Liquid Measure of the United States? How many cubic inches in a wine gallon? 156. In what is Beer Measure used? What are its denominations? Recite the Table. *Obs.* How many cubic inches in a beer gallon?

DRY MEASURE.

157. *Dry Measure* is used in measuring *grain*, *fruit*, *salt*, &c. Its denominations are *chaldrons*, *quarters*, *bushels*, *pecks*, *quarts*, and *pints*.

2 pints (*pt.*)	make	1 quart,	marked	*qt.*
8 quarts	"	1 peck,	"	*pk.*
4 pecks, or 32 qts.	"	1 bushel,	"	*bu.*
8 bushels	"	1 quarter,	"	*qr.*
32 bushels	"	1 chaldron,	"	*ch.*

OBS. 1. In England, 36 bushels of coal make a chaldron.

2. The *standard unit* of *Dry Measure* of the *United States* is the *Winchester Bushel*, which contains $2150\frac{4}{10}$ cubic inches, and is equal to $77\frac{6274}{10000}$ pounds avoirdupois of distilled water, at the maximum density.

3. The Winchester bushel is so called, because the standard measure was formerly kept at *Winchester*, England. By statute, it is an upright cylinder, $18\frac{1}{2}$ inches in diameter, and 8 inches deep.

TIME.

158. *Time* is a measured portion of *duration*. It is naturally divided into *days* and *years*; the former being caused by the revolution of the Earth on its axis, the latter by its revolution round the Sun. Its denominations are *centuries*, *years*, *months*, *weeks*, *days*, *hours*, *minutes*, and *seconds*.

60 seconds (*sec.*)	make	1 minute,	marked	*min.*
60 minutes	"	1 hour,	"	*hr.*
24 hours	"	1 day,	"	*d.*
7 days	"	1 week,	"	*wk.*
4 weeks	"	1 lunar month,	"	*mo.*
12 calendar months, or 365 days, 6 hrs., (nearly)	"	1 civil year,	"	*yr.*
13 lunar mo., or 52 weeks	"	1 year,	"	*yr.*
100 years	"	1 century,	"	*cen.*

OBS. 1. Time is measured by clocks, watches, chronometers, dials, hour-glasses, &c.

QUEST.—157. To what use is Dry Measure applied? What are its denominations? Repeat the Table. *Obs.* What is the standard unit of Dry Measure adopted by the government? 158. What is Time? How is Time naturally divided? How are the former caused? How the latter? What are its denominations? Repeat the Table. *Obs.* How is Time measured?

2. A *civil* year is a *legal* or *common* year; a period of time established by government for civil or common purposes. The *civil* year is often called *Julian* year, from the Emperor Julius Cæsar, who adapted the *calendar* of the *civil* year to the *supposed* length of the *solar* year, by adding 1 day to every *fourth* year.

3. A *solar* year is the time in which the earth revolves round the sun, and contains 365 days, 5 hours, 48 minutes, and 48 seconds.

4. A *leap* year, sometimes called *bissextile*, contains 366 days, and occurs once in *four* years.

It is caused by the excess of 6 hours, which the civil year contains above 365 days, and is so called because it *leaps* or *runs* over one day more than a common year. The odd day is added to February, because it is the shortest month. Every leap year, therefore, February has 29 days.

The following are the names of the 12 calendar months, with the number of days in each:

January,	(Jan.)	*first*	month, has	31	days.
February,	(Feb.)	*second*	"	28	"
March,	(Mar.)	*third*	"	31	"
April,	(Apr.)	*fourth*	"	30	"
May,	(May)	*fifth*	"	31	"
June,	(June)	*sixth*	"	30	"
July,	(July)	*seventh*	"	31	"
August,	(Aug.)	*eighth*	"	31	"
September,	(Sept.)	*ninth*	"	30	"
October,	(Oct.)	*tenth*	"	31	"
November,	(Nov.)	*eleventh*	"	30	"
December,	(Dec.)	*twelfth*	"	31	"

The number of days in each month may be easily remembered from the following lines:

"Thirty days hath September,
April, June, and November;
February twenty-eight alone,
All the rest have thirty-one;
Except in Leap year, then is the time,
When February has twenty-nine."

Note.—The terms *September*, *October*, *November*, and *December*, which in our year denote the *ninth*, *tenth*, *eleventh*, and *twelfth* months, are derived from the Latin numerals *septem*, *octo*, *novem*, and *decem*, signifying *seven*, *eight*, *nine*, *ten*.

This discrepancy arose in this way: The Romans, from whom the names of the months were borrowed, originally divided the year into but *ten* months, commencing with *March* and ending with *December*; consequently, the numerals *septem*, *octo*, *novem*, *decem*, were correctly employed in denoting the last four months of their year.

At length they added January and February to the number of months, and commenced the year with January, while they retained the former names of the other months. Thus, the commencement of the year being set back *two months*, *March* became the *third* month, instead of the *first* as previously; *September* the *ninth*, instead of the *seventh*; *October* the *tenth*, instead of the *eighth*, &c.

QUEST.—What is a civil year? A solar year? A leap year? How is leap year caused? To which month is the odd day added? Name the calendar months, and the number of days in each?

TABLE *showing the number of days* from any day of one month to any day of any other month within a year.

From	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
To Jan.	365	334	306	275	245	214	184	153	122	92	61	31
" Feb.	31	365	337	306	276	245	215	184	153	123	92	62
" March	59	28	365	334	304	273	243	212	181	151	120	90
" April	90	59	31	365	335	304	274	243	212	182	151	121
" May	120	89	61	30	365	334	304	273	242	212	181	151
" June	151	120	92	61	31	365	335	304	273	243	212	182
" July	181	150	122	91	61	30	365	334	303	273	242	212
" Aug.	212	181	153	122	92	61	31	365	334	304	273	243
" Sept.	243	212	184	153	123	92	62	31	365	335	304	274
" Oct.	273	242	214	183	153	122	92	61	30	365	334	304
" Nov.	304	273	245	214	184	153	123	92	61	31	365	335
" Dec.	334	303	275	244	214	183	153	122	91	61	30	365

To find the number of days from any day of one month to the *same day* of any other month, look in the *upper line* for the *month* of the *first date*, and at the *left hand* for the month of the *second date*; the number of days between the two dates is found in the angle formed by the intersection of these columns. Thus, if we wish to know the number of days from May 21st to July 21st, look at the top for May, then at the left side for July, and tracing these lines to the point of intersection, we find 61, which is the number of days between the two dates.

When the day of the month is *different* in the two dates, this *difference* must be added to or subtracted from the tabular number, according as the second date is *greater* or *less* than the first. Thus, from the 7th of November to the 20th of June, the number of days is 212 + 13, or 225. The number of days from the 15th of October to the 10th of March, is 151—5, or 146.

In *leap* years, if the end of February is included in the time, 1 day must be added to the tabular number.

When the time *exceeds* 1 year, 365 days must be added for each year.

CIRCULAR MEASURE.

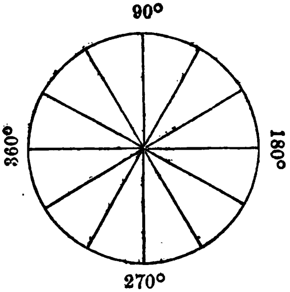
159. *Circular Measure* is applied to the divisions of the circle, and is used in reckoning *latitude* and *longitude*, and the *motion* of the heavenly bodies. Its denominations are *circles*, *signs*, *degrees*, *minutes*, and *seconds*.

60 seconds (″)	make 1 minute,	marked	′
60 minutes	" 1 degree,	"	°
30 degrees	" 1 sign,	"	*s.*
12 signs, or 360°	" 1 circle,	"	*c.*

QUEST.—159. To what is Circular Measure applied? What are its denominations? Repeat the Table.

OBS. 1. Circular Measure is often called *Angular Measure*, and is chiefly used by astronomers, navigators, and surveyors.

2. The circumference of every circle is divided, or supposed to be divided, into 360 equal parts, called *degrees*, as in the subjoined figure.



3. Since a degree is $\frac{1}{360}$ part of the circumference of a circle, it is obvious that its *length* must depend on the *size* or *magnitude* of the circle. Thus, a degree of longitude is the 360th part of the distance round the earth, which is very different from the 360th part of the circumference of the adjoining circle.

Note.—The term *minute*, is from the Latin *minutum*, which signifies a small part. The term *second*, is an abbreviated expression for *second* minutes, or minutes of the *second order*.

MISCELLANEOUS TABLE.

159.a. The following denominations not included in the preceding Tables, are frequently used.

12 units	make 1 dozen, (*doz.*)
12 dozen, or 144	" 1 gross.
12 gross, or 1728	" 1 great gross.
20 units	" 1 score.
56 pounds	" 1 firkin of butter.
100 pounds	" 1 quintal of fish.
30 gallons	" 1 bar. of fish in Mass.
200 lbs. of shad, or salmon	" 1 bar. in N. Y. and Ct.
196 pounds	" 1 barrel of flour.
200 pounds	" 1 barrel of pork.
14 pounds of iron, or lead	" 1 stone.
21½ stone	" 1 pig.
8 pigs	" 1 fother.

OBS. Formerly 112 pounds were allowed for a quintal.

QUEST.—*Obs.* What is Circular Measure sometimes called? By whom is it chiefly used? Into what is the circumference of every circle divided? On what does the length of a degree depend? 159.*a.* How many units make a dozen? How many dozen a gross? A great gross? How many units make a score? Pounds a firkin?

PAPER AND BOOKS.

159.*b*. The terms *folio*, *quarto*, *octavo*, &c., applied to books, denote the *number* of leaves into which a sheet of paper is folded.

24 sheets of paper	make	1 quire.
20 quires	"	1 ream.
2 reams	"	1 bundle.
5 bundles	"	1 bale.

A sheet folded in two leaves, is called a *folio*.
A sheet folded in four leaves, is called a *quarto*, or 4*to*.
A sheet folded in eight leaves, is called an *octavo*, or 8*vo*.
A sheet folded in twelve leaves, is called a *duodecimo*.
A sheet folded in sixteen leaves, is called a 16*mo*.
A sheet folded in eighteen leaves, is called an 18*mo*.
A sheet folded in thirty-two leaves, is called a 32*mo*.
A sheet folded in thirty-six leaves, is called a 36*mo*.
A sheet folded in forty-eight leaves, is called a 48*mo*.

159.*c*. Previous to the adoption of Federal money in 1786, accounts in the United States were kept in pounds, shillings, pence, and farthings.

In New England currency, Virginia, Kentucky, Tennessee, Indiana, Illinois, Missouri, and Mississippi,	6 shil. make $1.
In New York currency, North Carolina, Ohio, and Michigan,	8 shil. make $1.
In Pennsylvania currency, New Jersey, Delaware, and Maryland,	7s. 6d. make $1.
In Georgia currency, and South Carolina,	4s. 8d. make $1.
In Canada currency, and Nova Scotia,	5 shil. make $1.

OBS. At the time Federal money was adopted, the *colonial currency* or *bills of credit* issued by the colonies, had more or less *depreciated* in value. This depreciation being greater in some colonies than in others, gave rise to the *different values* of the *State currencies*.

QUEST.—159.*b*. When a sheet of paper is folded in two leaves, what is it called? When in four leaves, what? When in eight? In twelve? In sixteen? In eighteen? In thirty-six? 159.*c*. Previous to the adoption of Federal Money, in what were accounts kept in the U. S.? How many shillings make a dollar in N. E. currency? In N. Y. currency? In Penn. currency? In Georgia currency? In Canada currency?

ALIQUOT PARTS OF $1 IN FEDERAL MONEY.

50 cents = $\$\frac{1}{2}$.	$12\frac{1}{2}$ cents = $\$\frac{1}{8}$.
$33\frac{1}{3}$ cents = $\$\frac{1}{3}$.	10 cents = $\$\frac{1}{10}$.
25 cents = $\$\frac{1}{4}$.	$8\frac{1}{3}$ cents = $\$\frac{1}{12}$.
20 cents = $\$\frac{1}{5}$.	$6\frac{1}{4}$ cents = $\$\frac{1}{16}$.
$16\frac{2}{3}$ cents = $\$\frac{1}{6}$.	5 cents = $\$\frac{1}{20}$.

PARTS OF $1 IN NEW YORK CURRENCY.

4 shillings = $\$\frac{1}{2}$.	1 shilling 4 pence = $\$\frac{1}{6}$.
2 shillings 8 pence = $\$\frac{1}{3}$.	1 shilling = $\$\frac{1}{8}$.
2 shillings = $\$\frac{1}{4}$.	6 pence = $\$\frac{1}{16}$.

PARTS OF $1 IN NEW ENGLAND CURRENCY.

3 shillings = $\$\frac{1}{2}$.	1 shilling = $\$\frac{1}{6}$.
2 shillings = $\$\frac{1}{3}$.	9 pence = $\$\frac{1}{8}$.
1 shil. and 6 pence = $\$\frac{1}{4}$.	6 pence = $\$\frac{1}{12}$.

ALIQUOT PARTS OF STERLING MONEY.

10 shillings = £$\frac{1}{2}$.	1s. 8d. = £$\frac{1}{12}$.
6s. 8d. = £$\frac{1}{3}$.	1 shilling = £$\frac{1}{20}$.
5 shillings = £$\frac{1}{4}$.	6 pence = $\frac{1}{2}$ shilling.
4 shillings = £$\frac{1}{5}$.	4 pence = $\frac{1}{3}$ shilling.
3s. 4d. = £$\frac{1}{6}$.	3 pence = $\frac{1}{4}$ shilling.
2s. 6d. = £$\frac{1}{8}$.	2 pence = $\frac{1}{6}$ shilling.
2 shillings = £$\frac{1}{10}$.	1 penny = $\frac{1}{12}$ shilling.

ALIQUOT PARTS OF A TON.

10 hundred lbs. = $\frac{1}{2}$ ton.	2 hundred 2 qrs. = $\frac{1}{8}$ ton.
5 hundred lbs. = $\frac{1}{4}$ ton.	2 hundred lbs. = $\frac{1}{10}$ ton.
4 hundred lbs. = $\frac{1}{5}$ ton.	1 hundred lbs. = $\frac{1}{20}$ ton.

ALIQUOT PARTS OF A POUND AVOIRDUPOIS.

8 ounces = $\frac{1}{2}$ pound.	2 ounces = $\frac{1}{8}$ pound.
4 ounces = $\frac{1}{4}$ pound.	1 ounce = $\frac{1}{16}$ pound.

ALIQUOT PARTS OF TIME.

6 months = $\frac{1}{2}$ year.	15 days = $\frac{1}{2}$ month.
4 months = $\frac{1}{3}$ year.	10 days = $\frac{1}{3}$ month.
3 months = $\frac{1}{4}$ year.	6 days = $\frac{1}{5}$ month.
2 months = $\frac{1}{6}$ year.	5 days = $\frac{1}{6}$ month.
$1\frac{1}{2}$ month = $\frac{1}{8}$ year.	3 days = $\frac{1}{10}$ month.
$1\frac{1}{3}$ month = $\frac{1}{9}$ year.	2 days = $\frac{1}{15}$ month.
1 month = $\frac{1}{12}$ year.	1 day = $\frac{1}{30}$ month.

REDUCTION.

160. REDUCTION is the process of changing *Compound Numbers* from one denomination to another, without altering their *value.* It is of two kinds, *Descending* and *Ascending.*

Reduction Descending is the process of reducing *higher* denominations to *lower*, as pounds to shillings, &c.

Reduction Ascending is the process of reducing *lower* denominations to *higher*, as farthings to pence, pence to shillings, &c.

Ex. 1. Reduce £2, 13s. 6d. 2 far. to farthings.

Operation.

£	s.	d.	far.
2	13	6	2

20s. in £1.
53 shillings.
12d. in 1s.
642 pence.
4 far. in 1d.
2570 far. *Ans.*

Suggestion.—To reduce the pounds to shillings, we multiply them by 20, because 20s. make £1; for if there are 20s. in £1, in £2 there must be twice as many, or 40s., and adding the given shillings (13) makes 53s. We next reduce the 53s. to pence, by multiplying them by 12, because 12d. make 1s., and adding in the given pence (6), we have 642d. Finally, we reduce the 642d. to farthings, by multiplying them by 4, because 4 far. make 1d. and adding in the given farthings (2), we have 2570 far. That is, £2, 13s. 6d. 2 far.=2570 farthings.

161. Hence, we derive the following general

RULE FOR REDUCTION DESCENDING.

Multiply the highest denomination given by the number required of the next lower denomination to make ONE *of this higher, and to the product add the given number of this lower denomination. Proceed in this manner with each successive denomination, till you come to the one required.*

OBS. Reduction *Descending*, it will be observed, consists in *successive multiplications*, and may with propriety be called *Reduction by Multiplication.*

2. Reduce £3, 2s. 5d. to pence. *Ans.* 749.

QUEST.—160. What is Reduction? Of how many kinds is it? What is Reduction Descending? Reduction Ascending? Explain the solution of the first example from your slate. How are pounds reduced to shillings? Why multiply by 20? How reduce shillings to pence? Why? How pence to farthings? Why? 161. What is the rule for Reduction Descending? *Obs.* Which of the fundamental rules is employed in Reduction Descending?

3. How many pounds in 2570 farthings?

Suggestion.—This example requires *lower* denominations to be reduced to *higher*, and therefore belongs to reduction *ascending*. To reduce the farthings to pence, we divide them by 4, because 4 far. make 1d.; for if 4 far. make 1d., in 2570 far. there must be as many pence as 4 is contained times in 2570, which is 642 and 2 far. over. We next reduce the 642d. to shillings, by dividing them by 12, because 12d. make 1s. Finally, we reduce the 53s. to pounds, by dividing them by 20, because 20s. make £1. Therefore 2570 far.=£2, 13s. 6d. 2 far.

Operation.

4)2570 far.
12)642d. 2 far. over.
2|0)5|3s. 6d. over.
£2, 13 s. over.
Ans. £2, 13s. 6d. 2 far.

162. Hence, we derive the following general

RULE FOR REDUCTION ASCENDING.

Divide the given denomination by that number which it takes of this denomination to make ONE *of the next higher.*

Proceed in this manner with each successive denomination, till you come to the one required. The last quotient, with the several remainders, will be the answer.

PROOF.—*Reduction Ascending and Descending mutually prove each other.*

OBS. 1. Each remainder is of the same denomination as the dividend from which it arose. (Art. 66. Obs. 2.)

2. Reduction *Ascending*, it will be observed, consists in *successive divisions*, and may with propriety be called *Reduction by Division.*

4. Reduce 4237 feet to rods.

Suggestion.—Having reduced the feet to yards, we have 1412 yds, and 1 ft. over. We next reduce the yards to rods, by dividing them by $5\frac{1}{2}$. In dividing by $5\frac{1}{2}$, we reduce both the divisor and dividend to *halves;* then performing the division, the result is

Operation.

3)4237 feet.
$5\frac{1}{2}$)1412 yds. 1 ft. over.
2
11)2824 half yd.
256 r. 4 yds. over.
Ans. 256 r. 4 yds. 1 ft.

QUEST.—Explain the solution of the third example from your slate. How are farthings reduced to pence? Why? How reduce pence to shillings? Why? How shillings to pounds? Why? 162. What is the rule for Reduction Ascending? How is Reduction proved? *Obs.* Of what denomination is each remainder? Which of the fundamental rules is employed in Reduction Ascending?

256 rods, and 8 rem. (Art. 141. Obs.) Now this 8 rem. is *half* yards, and is equal to 4 whole yards. (Art. 162. Obs. 1.)

Suggestion.—Reverse the operation; that is, reduce the rods to yds., by multiplying them by $5\frac{1}{2}$; (Art. 134.*a.*) then reduce the yards to feet, and the result is 4237 feet, which is equal to the given number of feet. The work is therefore correct.

Proof.

$$\begin{array}{r} 2)256 \text{ r. } 4 \text{ yds. } 1 \text{ ft.} \\ \underline{5\frac{1}{2}} \\ 1284 \\ \underline{128} \\ 1412 \text{ yards.} \\ \underline{3} \\ \textit{Result } 4237 \text{ feet.} \end{array}$$

5. In £35, 4s. 6d. how many pence? *Ans.* 8454d.
6. In 57600 far., how many pounds? *Ans.* £60.
7. In £43, 12s, how many shillings?
8. In 217 shillings, how many farthings?
9. In 1176 pence, how many pounds?
10. In 12356 farthings, how many shillings?
11. In £675, how many farthings?
12. In £84, 16s. $7\frac{1}{2}$d., how many farthings?
13. In 25256 pence, how many pounds?
14. In 56237 farthings, how many pounds?
15. In £425, 9s. $7\frac{1}{2}$d., how many farthings?
16. In 411 lbs., 2 oz. 3 pwts., how many pennyweights?
17. In 715 ounces, how many grains?
18. In 10 lbs. 5 oz. 6 pwts., how many grains?
19. In 512 pwts., how many pounds?
20. In 2156 grains, how many ounces?
21. In 35210 grains, how many pounds?
22. Reduce 425 pounds 3 oz. 5 drs. Avoirdupois, to drams.
23. Reduce 36 cwt. 2 qrs. to pounds.
24. Reduce 35 tons, 7 cwt. 15 lbs. to ounces.
25. Reduce 3 quarters, 15 lbs. 10 oz. to drams.
26. Reduce 875 ounces to pounds.
27. Reduce 1565 pounds to hundred weight.
28. Reduce 1728 drams to pounds.
29. Reduce 5672 ounces to hundred weight.
30. Reduce 15285 pounds to tons.
31. Reduce 8526720 drams to hundred weight.
32. How many drams in 170 lbs. Apothecaries weight?
33. How many scruples in 156 pounds?

34. How many ounces in 726 scruples?
35. How many pounds in 1260 drams?
36. In 896 rods, 3 yds. 1 ft., how many feet? *Ans.* 14794 ft.
37. In 45 furlongs, how many inches?
38. In 1584 feet, how many rods? *Ans.* 96 rods.
39. In 9728 inches, how many rods?
40. In 26400 feet, how many miles?
41. In 25 leagues, how many inches?
42. In 40 leagues, 6 furlongs, 2 in., how many inches?
43. In 750324 inches, how many miles?
44. How many inches in the circumference of the earth?
45. How many inches in 845 yds. cloth measure?
46. How many nails in 53 Flemish ells?
47. How many nails in 81 English ells?
48. Reduce 563 quarters to yards.
49. Reduce 1324 nails to French ells.
50. Reduce 5208 nails to English ells.
51. In 1766 square rods and 19 yards, how many feet?
52. In 56 acres and 3 roods, how many square feet?
53. In 1275 square miles, how many acres?
54. How many square rods in 25640 feet?
55. How many acres in 1865 roods?
56. How many acres in 2118165½ yards?

57. How many square feet in a table 21 feet long and 6 feet wide? *Ans.* 126 sq. ft. (Art. 153. Obs. 3.)

58. What is the area of a garden, which is 18 rods long and 15 rods wide? *Ans.* 270 square rods.

59. How many square feet in a floor, 18 feet long and 17 feet wide?

60. How many square yards in a ceiling, 20 feet long and 18 feet wide?

61. What is the area of a field, which is 36 rods long and 25 rods wide?

62. How many acres are there in a piece of land, 80 rods long and 48 rods wide?

63. In 75 cubic feet, how many inches?
64. In 37 tons of round timber, how many inches?
65. In 28124 cubic feet, how many tons of hewn timber?
66. In 16568 cubic feet of wood, how many cords?
67. In 65 cords of wood, how many cubic feet?

68. How many cubic inches are there in a box, whose length is 30 inches, its breadth 18, and its depth 15 inches?

69. How many cubic inches in a block of marble, 43 inches long, 18 inches broad, and 12 inches thick?

70. How many cubic feet in a room, 16 feet long, 15 feet wide, and 9 feet high?

71. How many cubic feet in a load of wood, 8 feet long, 4 feet wide, and $3\frac{1}{2}$ feet high?

72. How many cubic feet in a pile of wood, 16 feet long, 6 feet wide, and 5 feet high? How many cords?

73. How many cords of wood in a pile, 140 feet long, $4\frac{1}{2}$ feet wide, and $6\frac{1}{2}$ feet high?

74. In 4624 gills, how many gallons wine measure?

75. In 24260 quarts, how many hogsheads?

76. How many pints in 15 hogsheads, and 20 gallons?

77. How many gills in 40 bar. 3 gals. 2 qts. of wine?

78. How many barrels of beer in 5000 pints?

79. How many hogsheads in 7800 quarts of beer?

80. How many quarts in 25 hhds. and 7 gals. of beer?

81. How many pints in 110 gals. 3 qts. and 1 pt. beer?

82. Reduce 536 bushels, and 3 pecks to quarts.

83. Reduce 821 quarters to pints.

84. Reduce 6912 pints to bushels.

85. Reduce 85600 quarts to bushels.

86. In 15 days, 6 hours, and 9 min., how many seconds?

87. In 365 days and 6 hours, how many minutes?

88. How many seconds in a solar year?

89. Allowing 365 d. 6 h. to a year, how many minutes has a person lived who is 21 years old?

90. How many hours in 568240 seconds?

91. How many weeks in 8568456 minutes?

92. How many lunar months in 6925600 hours?

93. How many years in 56857200 hours?

94. How many years in 1000000000 seconds?

95. In 75 degrees, how many seconds?

96. In 8 signs, and 15 degrees, how many minutes?

97. In 12 signs, how many seconds?

98. In 86860 seconds, how many degrees?

99. In 567800 minutes, how many signs?

100. In 25000000 seconds, how many signs?

101. A stationer bought 25 doz. slates at 9 pence apiece: how many pounds sterling did they come to?

102. A company of 15 persons spent 29 guineas and 6s. for a supper: how many shillings apiece did the supper cost them?

103. A farmer sold 12 cows for $5\frac{1}{4}$ guineas apiece, and took his pay in sheep at 22 shillings a head: how many sheep did he receive?

104. Bought a quarter of beef, weighing 2 cwt. and $17\frac{1}{2}$ lbs., at 7 pence a pound: what did it amount to?

105. What will a load of hay containing 1 ton and 17 cwt. amount to, at $1\frac{1}{2}$ dollar a hundred?

106. A silversmith having 7 lbs. $8\frac{1}{2}$ oz. of silver, made it into tea spoons each weighing $2\frac{1}{2}$ oz., which he sold for 7 eighths of a dollar apiece. How many spoons did he make; and how much did they come to?

107. How many dollars can be made out of 50 lbs. 9 oz. of silver, allowing $412\frac{1}{2}$ grains to a dollar?

108. How many eagles of standard weight and purity, can be made out of 100 lbs. 10 oz. of gold? How many half eagles? How many double eagles?

109. Required to reduce 5 m. 6 fur. 28 rods, 5 yds. and 8 in. to inches, and prove the operation.

110. Required to reduce 8 m. 5 fur. 25 rods, 3 yds. 0 ft. 8 in. to inches, and prove the operation.

111. What will 4 acres, 2 roods and 15 rods of land cost, at $2\frac{1}{2}$ dollars a rod?

112. If you count 65 per minute, how many can you count in 6 weeks, 4 days, and 5 hours; allowing 6 days to a week and 10 hours to a day?

113. If a pendulum vibrates 65 times per minute, how many times will it vibrate in 253 days, 16 hours?

114. A grocer bought 13 tons, 9 cwt. 2 qrs. and 18 lbs. of butter, which he packed in firkins: how many firkins did it require?

115. A farmer having 5 hhds. 1 bbl. 16 gals. of cider, put it up in bottles holding 3 pints each: how many bottles did it take?

116. How many yards of carpeting a yard wide, will it take to cover a floor 22 feet long and 18 feet wide?

117. How many acres are there in a field 865 rods long, and 66 feet wide?

118. How many square yards in the four sides of a room 18 feet long, $17\frac{1}{2}$ feet wide, and $14\frac{1}{2}$ feet high?

119. How many square yards of plastering will it take to cover the four sides and the ceiling of a room 18 feet square, and 15 feet high?

120. How many yards of muslin 3 qrs. wide, are equal to 36 yds. brocatelle, which is $1\frac{1}{2}$ yard wide?

121. How many yards of silk 3 qrs. wide, will 51 yds. of cambric line, which is $1\frac{1}{4}$ yd. wide?

122. What will it cost to pave a street 3 m. 115 rods long, and 2 rods wide, at $15\frac{1}{2}$ dollars a square rod?

123. A man having 15 acres and 60 rods of land, laid it out in lots each containing 12 sq. rods, and sold the lots at \$350 apiece: how much did he realize for his land?

124. What is the worth of a pile of wood 18 ft. long, $10\frac{1}{2}$ ft. high, and $9\frac{1}{2}$ wide, at $3\frac{1}{2}$ dollars a cord?

125. How many times will a wheel of a railroad car, 9 ft. in circumference, turn round in going 1500 miles?

126. How long would it take a cannon ball, flying at the rate of 8 miles per minute, to reach the moon, a distance of 240000 miles?

127. The velocity of light is 11875000 miles per minute, and it takes 8 minutes for it to pass from the sun to the earth: how far from the sun is the earth; and how many weeks would it take a man to travel this distance, traveling 30 miles an hour?

128. How many bricks will it take to pave a side walk 75 feet long and 8 feet wide, each brick being 8 inches long and 4 inches wide?

129. How many suits of clothes can be made from 648 yards, allowing 4 yds. 2 qrs. to a suit?

130. Allowing 1 shingle to cover 24 sq. inches, how many shingles will be required to cover the roof of a house 50 feet long, the rafters on each side being 20 feet long?

131. In a township 6 miles square, how many farms are there of 160 acres each?

132. How many bricks will it take to build a prison 60 feet long, 25 feet wide, and 48 feet high, whose walls are 1 foot *thick,* the bricks 8 in. long, 4 in. wide, and 2 in. thick?

FRACTIONAL COMPOUND NUMBERS.

163. In order that one *concrete* number may properly be said to be a *part* of another, the two numbers must necessarily express objects of the *same kind*, or objects which can be *reduced* to the *same kind* or *denomination*. Thus, 1 *penny* is $\frac{1}{240}$ of a *pound*, but 1 penny cannot properly be said to be part of a *foot*, or of a *year;* for, feet and years cannot be reduced to pence. So 1 orange is $\frac{1}{5}$ of 5 oranges; but 1 orange cannot be said to be $\frac{1}{5}$ of 5 apples, or 5 pumpkins; for apples and pumpkins cannot be reduced to oranges.

CASE I.—*Reducing Compound numbers to fractions of higher denominations.*

Ex. 1. Reduce 7s. 6d. to the fraction of £1.

Operation.

7s. 6d.
12
90 pence.
£1 × 20 × 12 = 240d.
Ans. £$\frac{90}{240}$, or £$\frac{3}{8}$.

Suggestion.—The object of this example is to find what part of 1 pound, 7s. 6d. is equal to. But in order to find *what part* one number is of another, the numbers must be reduced to the *same denomination*. (Art. 163.) Now 7s. 6d.=90d., and £1=240d. The question therefore resolves itself into this: What part of 240d. is 90d. The answer is $\frac{90}{240}$, which reduced to its lowest terms, is £$\frac{3}{8}$. In the operation, we reduce the 7s. 6d. to pence, (the lowest denomination mentioned,) for the numerator, and £1 to pence for the denominator. Hence,

164. To reduce a compound number to the *fraction* of a *higher* denomination.

First reduce the given compound number to the lowest denomination mentioned for the numerator; then reduce a UNIT *of the denomination of the required fraction to the same denomination as the numerator, and the result will be the denominator.*

OBS. When the given number contains but one denomination, it of course requires no reduction.

If the given number contains a fraction, the denominator of the fraction is the lowest denomination mentioned. Thus, in $6\frac{3}{4}$s., the lowest denomination is *fourths* of a shilling; in $2\frac{3}{5}$ far., the lowest denomination is *fifths* of a farthing.

QUEST.—163. When may one concrete number be said to be a part of another? 164. How is a compound number reduced to a fraction of a higher denomination?

2. Reduce 3s. 7d. 2 far. to the fraction of £1.

Ans. £$\frac{174}{960}$, or $\frac{29}{160}$.

3. Reduce 9d. 3 far. to the fraction of 1s.
4. What part of a bushel is 3 pecks and 5 qts.?
5. What part of a peck is 5 qts. and 1 pt.?
6. What part of a gallon is 3 qts. 1 pt. and 3 gills?
7. What part of 1 gallon is 1 pt. and 1 gill?
8. What part of 1 hogshead is 15 gals. and 3 qts.?
9. What part of 1 ton is 5 cwt. and 2 qrs.?
10. What part of 1 hundred weight is 2 qrs. and 7 lbs.?
11. What part of 1 quarter is 1 lb. and 5 oz.?
12. What part of 1 mile is 45 rods 2 yds. and 2 ft.?
13. What part of 1 mile is 10 fur. and 35 rods?
14. What part of 1 league is 2 m. 1 fur. and 1 r.?
15. What part of 1 yard is 2 qrs. and 3 nails?
16. What part of £1 is 7d. 3 far.? *Ans.* £$\frac{31}{960}$.
17. What part of £1 is 3$\frac{2}{3}$d? *Ans.* £$\frac{11}{720}$.
18. What part of £1 is 5$\frac{3}{4}$ shillings? *Ans.* £$\frac{23}{80}$.
19. What part of 1 day is 2$\frac{1}{2}$ hours?
20. What part of 1 day is 4 h. and 8$\frac{1}{2}$ min.?
21. What part of 1 hour is 3 min. and 40 sec.?
22. What part of a week is 1 hour and 15$\frac{3}{4}$ sec.?
23. What part of a hundred weight is 1 pound and 5$\frac{3}{4}$ oz.?
24. What part of 1 ton is $\frac{5}{8}$ of a pound?
25. What part of 1 hogshead is 15$\frac{3}{8}$ of a gallon?
26. What part of 1 gallon is 3 qts. 1 pt. and 3$\frac{2}{5}$ gills?
27. What part of 30 days is 11 days 3 hours and 14 min.?
28. What part of 5 tons is 1 ton 11 cwt. and 16 lbs.?
29. What part of 4 yards square is 4 square yards?
30. What part of $\frac{1}{6}$ of a square yard is $\frac{1}{6}$ of a square foot?
31. What part of 4 cubic yards is 4 cubic feet?
32. What part of $\frac{1}{6}$ of a cubic yard is $\frac{1}{6}$ of a cubic foot?

33. A young man having £17, 15s. 8d., spent £5, 6s. 7d. in dissipation: what part of his money did he spend?

34. A man having 3 tons, 5 cwt. 16 lbs. of flax, sold 1 ton, 7 cwt. 19 lbs.: what part of his flax did he sell?

35. From a cask of vinegar containing 43 gals. 3 qts., a grocer sold 19 gals. 2 qts. and 1$\frac{1}{2}$ pints: what part of the whole cask did he sell?

CASE II.—*Reducing fractions of higher denominations to whole numbers of lower denominations.*

Ex. 1. Reduce $\frac{3}{8}$ of £1 to shillings and pence.

Suggestion.—3 *eighths* of £1 is the same as 1 *eighth* of £3. Now, reducing the numerator £3, to shillings, and dividing it by the denominator 8, the result is 7s. and 4 remainder, or $\frac{4}{8}$s. Next, reducing the numerator 4s. to pence, and dividing again by the denominator 8, the quotient is 6d. The quotients 7s. and 6d. form the answer required. That is, $\frac{3}{8}$ of £1=7s. 6d. Hence,

Operation.

£3
20
8)60s.
7s. and 4 rem.
12
8)48d.
6d.
Ans. 7s. 6d.

165. To reduce a *fraction* of a *higher* denomination to *whole* numbers of *lower* denominations.

Reduce the numerator of the given fraction to the next lower denomination, and divide the result by the denominator; then reduce the remainder to the next lower denomination still, and divide by the denominator as before.

Proceed in this manner with each remainder, and the several quotients will be the whole numbers required.

2. Reduce $\frac{3}{5}$ of £1 to shillings. *Ans.* 12s.
3. How many shillings and pence in £$\frac{1}{8}$?
4. How many shillings, &c., in £$\frac{5}{7}$?
5. In $\frac{4}{5}$ of 1 week, how many days, hours, &c.?
6. In $\frac{7}{18}$ of 1 day, how many hours, minutes, &c.?
7. Reduce $\frac{2}{5}$ of 1 league to miles, &c.
8. Reduce $\frac{4}{9}$ of 1 mile to furlongs. &c.
9. Reduce $\frac{7}{16}$ of 1 hundred weight to quarters, &c.
10. In $\frac{3}{7}$ of 1 ton, how many hundred weight, &c.?
11. In $\frac{2}{3}$ of 1 bushel, how many pecks, quarts, &c.?
12. Reduce $\frac{3}{7}$ of a square mile to acres, rods, yards, and feet.
13. How many cubic feet in $\frac{5}{8}$ of a cord?

QUEST.—165. How is a fraction of a higher denomination reduced to whole numbers of lower denominations?

CASE III.—*Reducing fractions of higher denominations to fractions of lower denominations.*

14. Reduce $\frac{1}{480}$ of £1 to the fraction of a penny.

Analysis.—Since £1 is equal to 20s., it is plain that $\frac{1}{480}$ of £1 is equal to $\frac{1}{480}$ of 20s., which is $\frac{20}{480}$s. Again, since 1s. is equal to 12d., $\frac{20}{480}$s. must be equal to $\frac{20}{480}$ of 12d. which is $\frac{240}{480}$d. or $\frac{1}{2}$d. In the operation, we reduce the numerator of the given fraction to the denomination required, which is pence, and the result placed over the given denominator forms the fraction required. Hence,

Operation.

£1 numerator.
$$\begin{array}{r} 20 \\ \hline 20\text{s.} \\ 12 \\ \hline 240\text{d.} \end{array}$$
Ans. $\frac{240}{480}$d. or $\frac{1}{2}$d.

166. To reduce a fraction of a *higher* denomination to the fraction of a *lower* denomination.

Reduce the given numerator to the denomination of the required fraction, and place the result over the given denominator.

OBS. 1. This process is the same in principle as reducing a *whole* compound number to a lower denomination. (Art. 161.)

2. When factors common to the numerator and denominator occur, the operation may be shortened by canceling those factors. (Art. 136.)

15. Reduce $\frac{3}{5760}$ of £1 to the fraction of a farthing.

Solution.—$\frac{3\times20\times12\times4}{5760}=\frac{\cancel{3}\times\cancel{20}\times\cancel{12}\times\cancel{4}}{\cancel{5760},\ 2}=\frac{1}{2}$ far. *Ans.*

16. Reduce $\frac{7}{180}$ of 1 week to the fraction of a day.
17. Change $\frac{4}{1184}$ of 1 mile to the fraction of a rod.
18. Change $\frac{2}{165}$ of 1 rod to the fraction of a foot.
19. Change $\frac{10}{1787}$ of 1 yard to the fraction of a nail.
20. Change $\frac{1}{100000}$ of 1 ton to the fraction of a pound.
21. What part of a second is $\frac{2}{172800}$ of a day?
22. What part of a pint is $\frac{4}{2565}$ of a bushel?
23. What part of a square foot is $\frac{3}{174240}$ of an acre?
24. What part of a cubic foot is $\frac{4}{1152}$ of a cord?

QUEST.—166. How is a fraction of a higher denomination reduced to the fraction of a lower denomination? *Obs.* How may the operation be shortened?

CASE IV.—*Reducing fractions of lower denominations to fractions of higher denominations.*

Ex. 1. Reduce $\frac{2}{3}$ of a penny to the fraction of a pound.

Analysis.—Since 1 penny is $\frac{1}{12}$ of a shilling, it is plain that $\frac{2}{3}$ of 1d. is equal to $\frac{2}{3}$ of $\frac{1}{12}$s. which is $\frac{2}{36}$s. Again, since 1 shilling is $\frac{1}{20}$ of a pound, $\frac{2}{36}$s. must be equal to $\frac{2}{36}$ of £$\frac{1}{20}$, which is £$\frac{2}{720}$, or £$\frac{1}{360}$. In the operation we reduce £1 to the same denomination as the given fraction, (*thirds* of a penny), and the result 720, placed under the given numerator, forms the fraction required. Hence,

Operation.

$$\begin{array}{r} £1 \\ \underline{20} \\ 20\text{s.} \\ \underline{12} \\ 240\text{d.} \\ \underline{3} \\ 720 \end{array}$$

Ans. $\frac{2}{720}$, or £$\frac{1}{360}$.

167. To reduce a fraction of a *lower* denomination to the fraction of a *higher* denomination.

Reduce a unit of the denomination of the required fraction to the same denomination as the given fraction, and the result will be the denominator.

Or, divide the given fraction by the same numbers as in reducing whole compound numbers to higher denominations.

OBS. 1. This Case is similar in principle to Case first.

2. When factors common to the numerator and denominator occur, the operation may be *shortened* by *canceling* those factors. (Art. 136.)

2. Reduce $\frac{4}{5}$ of a pint to the fraction of a bushel.

Solution.—$\frac{4}{5\times2\times8\times4}=\frac{\cancel{4}}{5\times2\times8\times\cancel{4}}=\frac{1}{80}$ bu. *Ans.*

3. Reduce $\frac{4}{7}$ of a farthing to the fraction of a pound.
4. What part of a pound Troy is $\frac{3}{4}$ of a grain?
5. What part of a ton is $\frac{5}{8}$ of an ounce?
6. What part of a mile is $\frac{8}{11}$ of a foot?
7. What part of an acre is $\frac{4}{5}$ of a square foot?
8. What part of a cord is $\frac{1}{2}$ of a cubic foot?
9. What part of a hogshead is $\frac{4}{5}$ of a pint?
10. What part of 2 square yards is $\frac{1}{2}$ of a square yard.?
11. What part of $\frac{1}{2}$ of a square yard is $\frac{1}{2}$ yard square?

QUEST.—167. How is a fraction of a lower denomination reduced to the fraction of a higher? *Obs.* How may the operation be shortened?

COMPOUND ADDITION.

167.a. *Compound Addition* is the process of uniting two or more *compound numbers in one sum.*

1. What is the sum of £4, 9s. 6d. 2 far.; £3, 12s. 8d. 3 far.; and £8, 6s. $9\frac{1}{4}$ pence?

Operation.

	£	s.	d.	far.
	4 "	9 "	6 "	2
	3 "	12 "	8 "	3
	8 "	6 "	9 "	1
Ans.	16 "	9 "	0 "	2

Suggestion.—Write the numbers under each other, pounds under pounds, shillings under shillings, &c. Then beginning with the lowest denomination, we find the sum is 6 far., which is equal to 1d. and 2 far. over. Write the 2 far. under the column of farthings, and carry the 1d. to the column of pence. The sum of the pence is 24, which is equal to 2s. and nothing over. Place a cipher under the column of pence, and carry the 2s. to the column of shillings. The sum of the shillings is 29, which is equal to £1 and 9s. over. Write the 9s. under the column of shillings, and carry the £1 to the column of pounds. The sum of the pounds is 16, which we set down in full, as in simple addition. (Art. 29.) The answer is £16, 9s. 0d. 2 far.

168. Hence, we derive the following general

RULE FOR COMPOUND ADDITION.

I. *Write the numbers so that the same denominations shall stand under each other.*

II. *Beginning at the right hand, add each column separately, and divide its sum by the number required to make* ONE *of the next higher denomination. Setting the remainder under the column added, carry the quotient to the next column, and thus proceed as in Simple Addition.* (Art. 23.)

PROOF.—*The proof is the same as in Simple Addition.*

OBS. 1. *Compound* Addition is the same in principle as *Simple* Addition, and the *reasons* of the rule are the same. The apparent difference between them arises from the fact, that in simple numbers, the *ratio* of increase being 10, we

QUEST.—167.*a.* What is Compound Addition? 168. How do you write compound numbers for addition? Where do you begin to add, and how proceed? How is Compound Addition proved? *Obs.* Does Compound Addition differ in principle from Simple Addition? From what does the apparent difference arise?

always carry for 10; but in compound numbers, the *ratio* of increase being *irregular*, we carry for *different* numbers. In each, however, we always carry for that number which it takes of the order or denomination *added*, to make *one* in the next higher order or denomination.

2. What is the sum of £10, 6s. 7d.; £18, 12s. 10d.; £5, 8s. 4d.? *Ans.* £34, 2s. 9d.

(3.)				(4.)			(5.)		
£	*s.*	*d.*	*far.*	£	*s.*	*d.*	£	*s.*	*d.*
6	7	4	2	10	15	8	21	18	10
0	6	7	1	16	11	0	1	6	11
12	15	6	0	25	18	9	35	12	7

(6.)				(7.)			(8.)		
lb.	*oz.*	*pwt.*	*gr.*	*oz.*	*pwt.*	*gr.*	*lb.*	*oz.*	*pwt.*
5	8	16	7	15	12	8	12	6	15
7	9	6	12	11	6	7	19	0	7
10	6	15	10	10	18	8	1	8	16
21	8	4	5	6	0	1	28	3	11

9. Add 7 lbs. 9 oz. 16 pwts. 10 grs.; 3 lbs. 10 oz. 8 pwts. 9 grs.; 8 lbs. 8 oz. 1 pwt. 4 grs.

10. A man bought a coach for £85, 12s.; a horse for £27, 8s. 10d.; a harness for £7, 16s. 11d.: what did the whole cost?

11. A merchant bought of one dairy-man 5 cwt. 11 lbs. 6 ounces of butter; of another, 8 cwt. 15 lbs. 9 oz.; of another, 7 cwt. 6 lbs. 10 oz.: how much did he buy of all?

12. Bought of one man 78 lbs. of wool; of another 96 lbs. 6 oz.; of another, 185 lbs. 11 oz.; of another, 320 lbs. 9 oz.; of another, 642 lbs. 8 oz.: how much was bought in all?

13. A man sold to one customer 2 tons, 62 lbs. 10 oz. of hay; to another, 5 tons, 40 lbs. 12 oz.; to another, 8 tons, 75 lbs. 6 oz.: how much did he sell to all?

14. A man wove 7 yds. 8 qrs. 2 na. of cloth in 1 day; the next day, 6 yds. 1 qr. 8 na.; the next, 8 yds. 3 qrs. 1 na.; the next, 5 yds. 2 qrs. 3 na.: how much did he weave in all?

15. Bought several pieces of cotton; one contained 26 yds. 1 qr. 2 na.; another, 30 yds. 2 qrs.; another, $20\frac{1}{4}$ yds. 3 na.; another, $32\frac{1}{4}$ yds. 1 na.: how many yards did they all contain?

16. A hotel-keeper bought at one time, 15 bu. 2 pks. 3 qts. of oats; at another, 10 bu. 1 pk. 2 qts.; at another, $20\frac{1}{2}$ bu. 6 qts.; then, $18\frac{1}{2}$ bu. 5 qts.: how much did he buy in all?

17. Bought 4 loads of wheat; the first containing 23 bu. 3 pks. 5 qts.; the second, $20\frac{1}{2}$ bu. 6 qts.; the third, $26\frac{1}{4}$ bu.; the fourth, $21\frac{3}{4}$ bu. 7 qts.: how many bushels did they all contain?

18. What is the sum of 16 m. 3 fur. 16 r.; 26 m. 1 fur. 33 r.; 10 m. 3 fur. 22 r.; 45 m. 7 fur. 20 r.?

19. A merchant bought 3 casks of oil; one held 2 hhds. 30 gals. 2 qts.; another, 3 hhds. 10 gals.; another, 1 hhd. 13 gals. 1 qt.: how much did they all hold?

20. Sold several lots of wine, in the following quantities; 1 pipe, 1 hhd. 21 gals. 2 qts. 1 pt.; 2 pipes, 11 gals. 3 qts. 1 pt.; 3 hhds. 15 gals. 2 qts.; 3 pipes, 10 gals. 2 qts. 1 pt.: how much was sold in all?

21. A mason plastered one room containing 45 square yards, 7 ft. 6 in.; another, 25 yds. 6 ft. 95 in.; another, 38 yds. 4 ft. 41 in.: what was the amount of his plastering?

22. Sold 10 A. 35 r. 10 sq. ft. of land at one time; at another, 3 A. 10 r. 15 ft.; at another, 18 A. 16 r. 23 ft.: what was the amount of land sold?

23. A merchant received several boxes of goods; one contained 16 cu. ft. 61 in.; another, 25 ft. 81 in.; another, 20 ft. 18 in.; another, 38 ft. 72 in.: how many cubic feet and inches did they all contain?

24. One pile of wood contains 10 c. 88 ft. 39 in.; another, 15 c. 56 ft. 73 in.; another, 30 c. 19 ft. 44 in.; another, 17 c. 84 ft. 21 in.: how much do they all contain?

25. Find the sum of 16 lbs. 6 oz. 5 drs. 2 sc. 9 grs.; 25 lbs. 8 oz. 7 drs. 1 sc.; and 45 lbs. 3 oz. 2 drs. 2 sc. Apothecaries' weight.

26. Find the sum of 45 m. $2\frac{1}{2}$ fur. 17 r. 5 yds. 2 ft. 9 in.; 43 m. $5\frac{1}{4}$ fur. 4 yds. 1 ft. 8 in.; 89 m. 16 r. 3 yds. 2 ft. 5 in.

27. Add together 17 leagues, 2 m. $3\frac{3}{4}$ fur. 35 r. $11\frac{1}{2}$ ft.; 19 l. 1 m. $7\frac{1}{4}$ fur. 28 r. $15\frac{1}{2}$ ft.; 26 l. 2 m. 3 fur. 2 r. 14 ft.

28. Add together 23 years, 2 mos. 3 wks. 5 d.; 68 yrs. 3 mos. 2 wks. 3 d.; 60 yrs. 4 mos. 1 wk. 6 d.; 49 yrs. and 4 d.

29. Add together 145 acres, 35 sq. r. 25 sq. yds. $7\frac{1}{2}$ sq. ft.; 123 A. 65 sq. r. 28 sq. yds. 8 sq. ft.; 84 A. 110 sq. r. 16 sq. yds. $6\frac{1}{4}$ sq. ft.

30. Add together 7 circles, 8 s. $17^\circ, 18', 48''$; 4 cir. 3 s. $21^\circ, 32', 54''$; 18 cir. 9 s. $11^\circ, 17', 39''$.

ADDITION OF FRACTIONAL COMPOUND NUMBERS.

1. What is the sum of £$\frac{1}{6}$, $\frac{1}{8}$s. and $\frac{1}{3}$d?

Suggestion.—We first reduce the fractions to whole numbers of lower denominations, (Art. 165,) then adding them as in the preceding rule, the result is 3s. 5d. $3\frac{1}{3}$ far. which is the answer required.

First Method.

£$\frac{1}{6}$=3s. 4d. 0 far.
$\frac{1}{8}$s.=0s. 1d. 2 far.
$\frac{1}{3}$d.=0s. 0d. $1\frac{1}{3}$ far.
Ans. 3s. 5d. $3\frac{1}{3}$ far.

Or, we may reduce the given fractions to the same denomination, viz: fractions of a penny; £$\frac{1}{6}=\frac{240}{6}$d.; $\frac{1}{8}$s.$=\frac{12}{8}$d. $\frac{1}{3}$d.$=\frac{1}{3}$d. (Art. 166.) Then, reducing these fractions to a common denominator as in the margin, and adding them, the sum is $\frac{6024}{144}$d., which being reduced to whole numbers, gives the same result as before. Hence,

Second Method.

$\frac{240}{6}=\frac{5760}{144}$.
$\frac{12}{8}=\frac{216}{144}$.
$\frac{1}{3}=\frac{48}{144}$.
Sum$=\frac{6024}{144}$.
Ans. 3s. 5d. $3\frac{1}{3}$ far.

168. *a.* To add *fractional* compound numbers.

Reduce the given fractions to whole numbers of lower denominations, then proceed as in compound addition.

Or, reduce the given fractions to the same denomination, then proceed as in adding common fractions. (Art. 127.)

OBS. The result will be the same, whether the given fractions are reduced to fractions of *lower* denominations, or to *higher*.

2. Add £$\frac{4}{5}$, $\frac{2}{3}$s. $\frac{7}{8}$d. £$\frac{3}{8}$, $\frac{1}{6}$s. *Ans.* £1, 4s. 4d. $3\frac{1}{2}$ f.
3. Add $\frac{5}{8}$ lb. to $\frac{2}{3}$ oz. $\frac{3}{5}$ pwt.
4. Add $\frac{1}{6}$ oz. $\frac{3}{10}$ pwt. $\frac{3}{8}$ gr.
5. Add $\frac{3}{5}$ ton, $\frac{3}{4}$ cwt. $\frac{1}{2}$ lb.
6. Add $\frac{3}{7}$ cwt. $\frac{2}{3}$ lb. $\frac{1}{4}$ oz.
7. Add $\frac{2}{3}$ m. to $\frac{1}{2}$ of $5\frac{1}{3}$ fur.
8. Add $7\frac{3}{4}$ in. $2\frac{2}{3}$ ft. $6\frac{5}{11}$ r.
9. Add $\frac{3}{8}$ yard $\frac{1}{4}$ na. $\frac{7}{8}$ in.
10. Add $\frac{3}{4}$ in. $\frac{5}{7}$ na. $\frac{5}{9}$ yd.
11. Add $\frac{3}{5}$ acre $\frac{5}{8}$ rood $\frac{7}{12}$ r.
12. Add $\frac{7}{12}$ sq. r. $\frac{2}{9}$ yd. $\frac{4}{7}$ ft.
13. Add $4\frac{3}{5}$ cord to $\frac{7}{8}$ cu. ft.
14. Add $\frac{5}{9}$ cu. yd. to $2\frac{11}{12}$ cu. ft.
15. Add $\frac{1}{7}$ hhd. wine, $8\frac{3}{4}$ gals. $1\frac{1}{2}$ qt., and $\frac{1}{9}$ hhd. $\frac{1}{2}$ of $6\frac{2}{3}$ gals.
16. Add $\frac{5}{8}$ bu. $\frac{11}{16}$ pk. $\frac{3}{8}$ qt. $\frac{1}{4}$ pt.; $\frac{7}{12}$ bu. $\frac{1}{2}$ pk. $\frac{2}{3}$ qt. $\frac{1}{4}$ pt.
17. Add $\frac{2}{3}$ of $\frac{7}{12}$ day, $\frac{3}{5}$ of $\frac{5}{7}$ hr. $\frac{4}{15}$ of $\frac{19}{20}$ min. and $\frac{3}{4}$ of $2\frac{4}{5}$ sec.
18. Bought two remnants of silk, one containing $\frac{5}{8}$ yd. $\frac{1}{2}$ qr. $\frac{3}{4}$ na., and the other $\frac{3}{4}$ yd. $\frac{5}{8}$ qr. $\frac{1}{2}$ na.: how much did both contain?
19. How many pounds in a load of hay which weighs $\frac{3}{4}$ ton $2\frac{1}{2}$ qrs. and $17\frac{3}{4}$ lbs.?

QUEST.—168.*a.* How add fractional compound numbers?

COMPOUND SUBTRACTION.

168.b. *Compound Subtraction* is the process of finding the *difference* between two *compound* numbers.

Ex. 1. From £15, 7s. 6d. 3 far., subtract £6, 4s. 8d. 2 far.

Suggestion.—We write the less number under the greater, pounds under pounds, shillings under shillings, &c., and beginning with the lowest denomination proceed thus: 2 far. from 3 far. leave 1 far.; set the 1 far. under the column of farthings. Next, 8d. cannot be taken from 6d.; we therefore borrow as many pence, as it takes to make *one* of the next higher denomination which is shillings; and 12d. added to 6d., make 18d. Now 8d. from 18d. leave 10d. But since we borrowed we must carry 1 to the 4s. which makes 5s., and 5s. from 7s. leave 2s. Finally, £6 from £15 leave £9. The answer therefore is £9, 2s. 10d. 1 far.

Operation.

	£	s.	d.	far.
	15 "	7 "	6 "	3
	6 "	4 "	8 "	2
Ans.	9 "	2 "	10 "	1

169. Hence, we derive the following general

RULE FOR COMPOUND SUBTRACTION.

I. *Write the less number under the greater, so that the same denominations may stand under each other.*

II. *Beginning at the right hand, subtract each lower number from the number above it, and set the remainder under the number subtracted.*

III. *When a number in the lower line is larger than that above it, add as many units to the upper number as it takes to make* ONE *of the next higher denomination; then subtract as before, and adding* 1 *to the next number in the lower line, proceed as in Simple Subtraction.*

PROOF.—*The proof is the same as in Simple Subtraction.*

OBS. *Compound* Subtraction is the same in principle as *Simple* Subtraction, and the *reasons* of the rule are the same. In both cases we begin to subtract at the right hand, and when the number in the lower line is larger than that above it,

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QUEST.—168.*b.* What is Compound Subtraction? 169. How do you write compound numbers for subtraction? Where begin to subtract, and how proceed? When a number in the lower line is larger than that above it, what is to be done? How is Compound Subtraction proved?

we borrow as many units as it takes of the order or denomination we are subtracting to make *one* of the next higher order or denomination; and in both, we carry 1 to the next figure in the lower number.

2. From £10, 7s. 4d. 3 far. take £2, 6s. 9d. 2 far.
Ans. £8, 0s. 7d. 1 far.

3. From £15, 16s. 10d. 3 far., take £7, 8s. 11d. 1 far.

4. From £56, 7s. 6d. 1 far., take £20, 3s. 10d. 3 far.

	(5.)	(6.)
From	16T. 10 cwt. 3 qrs. 7 lbs.	125T. 7 cwt. 2 qrs. 20 lbs.
Take	8T. 5 cwt. 1 qr. 2 lbs.	96T. 9 cwt. 3 qrs. 12 lbs.

	(7.)	(8.)
From	16 gals. 3 qts. 1 pt. 2 gi.	121 hhds. 28 gals. 1 qt.
Take	7 gals. 2 qts. 0 pt. 3 gi.	63 hhds. 21 gals. 3 qts.

9. Bought 2 silver pitchers, one weighing 2 lbs. 10 oz. 10 pwts. 7 grs.; the other 2 lbs. 3 oz. 12 pwts. 5 grs.: what is the difference in their weight?

10. A merchant had 28 yds. 3 qrs. 2 na. of cloth, and sold 15 yds. 1 qr. 3 na.: how much had he left?

11. A lady bought 2 pieces of silk, one of which contained 19 yds. 2 qrs. 1 na.; the other 15 yds. 3 qrs. 3 na.: what is the difference in the length?

12. From 25 m. 7 fur. 8 r. 12 ft. 6 in., take 16 m. 6 fur. 30 r. 4 ft. 8 in.

13. A man owning 95 A. 75 r. 67 sq. ft. of land, sold 40 A. 86 r. 29 ft.: how much had he left?

14. A farmer having bought 120 A. 3 R. 28 r. of land, divided it into two pastures, one of which contained 50 A. 2 R. 35 r.: how much did the other contain?

15. A tanner built two cubical vats, one containing 116 ft. 149 in., the other 245 ft. 73 in.: what is the difference between them?

16. A man having 65 C. 95 ft. 123 in. of wood in his shed, sold 16 C. 117 ft. 65 in.: how much had he left?

17. From 27 yrs. 8 mos. 3 wks. 4 ds. 13 hrs. 35 min.,
Take 19 yrs. 5 mos. 6 wks. 5 ds. 21 hrs. 20 min.

QUEST.—*Obs.* Does Compound Subtraction differ in principle from Simple Subtraction?

18. What is the time from July 4th, 1840, to March 1st, 1845?

Suggestion.—March is the 3d month, and July the 7th. Since 4 ds. cannot be taken from 1 d., we borrow 1 mo..(30 ds.) then 4 from 31 leaves 27. 1 to carry to 7 makes 8, but 8 from 3 is impossible; we therefore borrow 1 yr. (12 mos.) then, 8 from 15 leaves 7. 1 to carry to 0 is 1, and 1 from 5 leaves 4. Hence,	*Operation.*
	Yr. *mo.* *d.*
	1845 " 3 " 1
	1840 " 7 " 4
	Ans. 4 " 7 " 27

170. To find the time between two dates.

Write the earlier date under the later, placing the years on the left, the number of the month next, and the day of the month on the right, then subtract as in the preceding rule. (Art. 169.)

OBS. 1. The number of the month is easily determined by reckoning from January, the 1st month, Feb. the 2d, &c. (Art. 158. Obs.)

2. In finding the time between two dates, and in casting interest, 30 days are considered a month, and 12 months a year.

19. What is the time from Oct. 15th, 1835, to March 10th, 1842?

20. The Independence of the United States was declared July 4th, 1776. How much time had elapsed on the 25th of Aug. 1845?

21. A note dated Oct. 2d, 1840, was paid Dec. 25th 1843: how long was it from its date to its payment?

22. A ship sailed on a whaling voyage, Aug. 25th, 1840, and returned April 15th, 1844: how long was she gone?

23. From 268 m. 3 fur. 2 r. 10 ft. 3 in., take 149 m. 6 fur. 7 r. 12 ft. 5 in.

24. From 160 deg. 18 statute m. 210 r. 3 yds. 1 ft., take 63 deg. 25 m. 305 r. 4 yds. 2 ft.

25. From 275 A. 21 r. 18 yds. 4 ft. 81 in., take 112 A. 65 r. 28 yds. 5 ft. 130 in.

26. From 367 A. 2 roods, 8 r. 25 ft., take 175 A. 3 roods, 25 r. 210 ft.

QUEST.—170. How do you find the time between two dates? *Obs.* In finding time between two dates, and in casting interest, how many days are considered a month? How many months a year?

SUBTRACTION OF FRACTIONAL COMPOUND NUMBERS.

1. From $\frac{2}{3}$ of a pound sterling, take $\frac{3}{4}$ of a shilling.

Suggestion.—We first reduce the fractions to whole numbers of lower denominations, then subtracting as in the last rule, the result is 12s. 7d., which is the answer.

First Method.

£$\frac{2}{3}$=13s. 4d.
$\frac{3}{4}$s.= 0s. 9d.
Ans. 12s. 7d.

Or, we may reduce the given fractions to the same denomination, then to a common denominator, as in the margin, and subtracting the less numerator from the greater, the result is $\frac{151}{12}$s. whose value is $12\frac{7}{12}$s. or 12s. 7d. the same as above. Hence,

Second Method.

£$\frac{2}{3}=\frac{40}{3}$s.$=\frac{160}{12}$s.
and $\frac{3}{4}$s.$=\frac{9}{12}$s.
$\frac{160}{12}-\frac{9}{12}=\frac{151}{12}$s. or $12\frac{7}{12}$s.
Ans. $12\frac{7}{12}$s. or 12s. 7d.

170.*a*. To subtract *Fractional* Compound Numbers.

Reduce the given fractions to whole numbers of lower denominations, then proceed as in Compound Subtraction.

Or, reduce the given fractions to the same denomination, then proceed as in subtracting common fractions. (Art. 129.)

2. From £$\frac{4}{5}$, $\frac{3}{4}$s., take £$\frac{3}{8}$; $\frac{1}{6}$s. *Ans.* 9s. 1d.
3. From $\frac{7}{8}$s. take $\frac{1}{4}$d.
4. From $\frac{2}{3}$s. take $1\frac{1}{4}$d.
5. From $\frac{5}{7}$ ton take $\frac{7}{15}$ cwt.
6. From $\frac{8}{9}$ cwt. take $\frac{3}{5}$ oz.
7. From $\frac{5}{8}$ mile take $\frac{6}{7}$ rod.
8. From $\frac{3}{11}$ yd. take $\frac{4}{7}$ ft.
9. From $\frac{9}{10}$ acre take $\frac{2}{20}$ rod.
10. From $\frac{3}{5}$ sq. r. take $\frac{2}{3}$ sq. y.
11. From $\frac{3}{5}$ c. take $\frac{19}{20}$ cu. ft.
12. From $\frac{11}{12}$ cu. y. take $\frac{25}{40}$ in.
13. From $\frac{16}{23}$ hhd. take $\frac{17}{34}$ gal.
14. From $\frac{7}{8}$ gal. take $\frac{2}{3}$ pt.
15. From $\frac{3}{5}$ of $\frac{7}{9}$ hhd. wine take $3\frac{1}{2}$ gallons.
16. From $\frac{3}{4}$ of $\frac{5}{8}$ ton of hay take $12\frac{3}{5}$ lbs.
17. From $\frac{7}{8}$ of $\frac{4}{5}$ of a week take 1 day and $2\frac{1}{2}$ hours.
18. A merchant having a piece of cloth containing $27\frac{1}{4}$ yds. $2\frac{1}{2}$ qrs. $2\frac{1}{3}$ na. sold $16\frac{3}{4}$ yds. $1\frac{1}{2}$ na.: how much did he have left?
19. A man having $45\frac{1}{2}$ acres $39\frac{1}{4}$ sq. rods of land, sold $19\frac{1}{4}$ acres, $13\frac{1}{5}$ rods: how much did he have left?
20. A grocer bought a hogshead of oil containing only $51\frac{1}{4}$ gals. $2\frac{1}{2}$ qts. and $1\frac{1}{4}$ pts.: how much had leaked out?

QUEST.—170.*a*. How subtract fractional compound numbers?

COMPOUND MULTIPLICATION.

171. *Compound Multiplication* is the process of finding the amount of a *compound* number repeated or added to itself, a given number of times.

Ex. 1. What will 5 yards of broadcloth cost, at £2, 3s. 6d. 3 far. per yard?

Suggestion.—Writing the multiplier under the lowest denomination of the multiplicand, we proceed thus: 5 times 3 far. are 15 far., which are equal to 3d. and 3 far. over. Write the 3 far. under the denomination multiplied, and carry the 3d. to the next product. 5 times 6d. are 30d. and 3d. make 33d., equal to 2s. and 9d. Set the 9d. under the pence, and carry the 2s. to the next product. 5 times 3s. are 15s. and 2s. make 17s. Set the 17s. under the shillings. Finally, 5 times £2 are £10. *Ans.* £10, 17s. 9d. 3 far.

Operation.

	£	s.	d.	far.
	2 ″	3 ″	6 ″	3
				5
Ans.	10 ″	17 ″	9 ″	3

172. Hence, we derive the following general

RULE FOR COMPOUND MULTIPLICATION.

Beginning at the right hand, multiply each denomination of the multiplicand by the multiplier separately, and divide its product by the number required to make ONE *of the next higher denomination, setting down the remainder and carrying the quotient as in Compound Addition.*

OBS. When the multiplier is a *composite number*, we may first multiply by one of its factors, then this product by another, and so on, till we have multiplied by all its factors; the last product will be the answer.

2. Multiply £5, 7s. 8d. 2 far. by 18, using its factors.

Suggestion.—The factors of 18 are 6 and 3; we therefore first multiply by 6, and that product by 3.

	£	s.	d.	far.
	5 ″	7 ″	8 ″	2
				6
	32 ″	6 ″	3 ″	0
				3
Ans.	96 ″	18 ″	9 ″	0

QUEST.—171. What is Compound Multiplication? 172. What is the rule for Compound Multiplication? *Obs.* How proceed when the multiplier is a composite number?

3. What will 5 horses cost, at £25, 10s. 6d. apiece?

4. A company of 6 persons agreed to pay £81, 5s. 8d. apiece for their passage from Hamburg to New York: what was the expense of their passage?

5. What cost 9 yards of cloth, at 18s. 9¾d. per yard?

6. What cost 6 pipes of wine, at £9, 7s. 8½d. apiece?

7. What cost 8 cows, at £5, 10s. 6d. apiece?

8. In a solar year there are 365 days, 5 hrs. 48 min. 48 sec.: how many days, hours, &c., has a person lived who is 21 years old?

9. Bought 10 silver cups, each weighing 3 oz. 15 pwts. 10 grs.: what is the weight of the whole?

10. What is the weight of 72 silver dollars, each weighing 17 pwts. 8 grs.?

11. Bought 7 loads of hay, each weighing 1 T. 3 cwt. 3 qrs. 12 lbs.: what is the weight of the whole?

12. What is the weight of 20 hogsheads of molasses, each weighing 5 cwt. 3 qrs. 17 lbs. 10 oz.?

13. A man bought 9 oxen, weighing 1123 lbs. 15 oz. apiece: what was the weight of the whole?

14. A grocer bought 11 casks of brandy, each containing 54 gals. 3 qts. 1 pt. 2 gills: how much did they all contain?

15. If a stage-coach goes at the rate of 5 m. 2 fur. 30 r. per hour, how far will it go in 10 hours?

16. If a railroad car goes 21 m. 2 fur. 10 r. per hour, how far will it go in 15 hours?

17. Bought 12 pieces of broadcloth, each containing 27 yds. 1 qr. 2 na.: how many yards did all contain?

18. If a man mows 3 A. 35 sq. r. per day, how many acres can he mow in 30 days?

19. How many square yards of plastering will a house which has 9 rooms require, allowing 75 yds. 18 ft. to a room?

20. A man bought 15 loads of wood, each containing 1 C. 33 ft.: how many cords did he buy?

21. A miller constructed 7 cubical bins for grain, each containing 216 feet 152 in.: what was the contents of the whole?

22. If a ship sails 2° 25′ 10″ per day, how far will she sail in 20 days?

23. Multiply 50° 42′ 11″ by 32.

24. If a brewer sells 33 gals. 2 qts. 1 pt. of beer a day, how much will he sell in 24 days?
25. Multiply 40 gals. 3 qts. 1 pt. by 60.
26. What cost 82 tons of iron, at £4, 15s. $6\frac{1}{2}$ pence per ton?
27. Multiply 33 bu. 2 pks. 5 qts. by 100.
28. Multiply 9 yds. 3 qrs. 2 na., by 500.
29. Multiply 60 T. 5 cwt. 9 lbs. by 586.
30. Multiply 4 bu. 2 pks. 6 qts. by 1000.

COMPOUND DIVISION.

173. *Compound Division* is the process of dividing *compound* numbers.

Ex. 1. A man bought 4 boxes of sugar for £17, 6s. 9d.: how much was that a box?

Operation.

	£	*s.*	*d.*	*far.*
4)	17 ″	6 ″	9 ″	0
Ans.	4 ″	6 ″	8 ″	1

Suggestion.—Writing the divisor on the left of the dividend we proceed thus: 4 is contained in £17, 4 times and 1 over. Write the 4 under the pounds, and reducing the remainder £1 to shillings, add the given shillings 6, and we have 26s. Now 4 is in 26s., 6 times and 2s. over. Set the 6 under the shillings, and reduce the remainder 2s. to pence, to which add the given pence 9, and we have 33d. Again, 4 is in 33d., 8 times and 1d. over. Set the 8 under the pence, reduce the 1d. to farthings, and divide as before. *Ans.* £4, 6s. 8d. 1 far.

174. Hence we derive the following general

RULE FOR COMPOUND DIVISION.

I. *Beginning at the left hand, divide each denomination of the dividend by the divisor, and write the quotient figures under the figures divided.*

II. *If there is a remainder, reduce it to the next lower denomination, and adding it to the figures of the corresponding denomination of the dividend, divide this number as before. Thus proceed through all the denominations, and the several quotients will be the answer required.*

QUEST.—173. What is Compound Division? 174. What is the rule for Compound Division?

OBS. 1. Each quotient figure is of the same denomination as that part of the dividend from which it arose.

2. When the divisor is a *composite* number, we may divide first by one factor and this quotient by another, and so on till all the factors are used; the last quotient will be the answer. (Art. 78.)

If the divisor exceeds 12, but is not a composite number, long division may be employed. (Art. 77.)

2. Divide £274, 4s. 6d. by 21, using its factors.

Operation.

£	s.	d.
3)274	4	6
7)91	8	2
Ans. 13	1	2

Suggestion.—The factors of 21 are 3 and 7; we therefore divide by 3, then this quotient by 7.

3. Divide £635, 17s. by 31, using long division.

Suggestion.—We reduce the remainder £15, to shillings, to which we add the given shillings, making 317, and divide as before. The remainder 7s. may be reduced to pence and divided again, if necessary.

```
    £   s.   £   s.
31)635, 17 (20, 10 7/31.
   620
    15 rem.
    20
   317
   310
     7 rem.
```

4. Divide £7, 8s. 2d. by 3.

5. Divide £35, 10s. 8d. 3 far. by 6.

6. Divide £42, 17s. 3d. 2 far. by 8.

7. A man bought 5 cows for £23, 16s. 8d.: how much did they cost apiece?

8. A merchant sold 10 rolls of carpeting for £62, 12s. 9d.: how much was that per roll?

9. Paid £25, 10s. $6\frac{1}{4}$d. for 12 yards of broadcloth: what was that per yard?

10. A silversmith melted up 2 lbs. 8 oz. 10 pwts. of silver, which he made into 6 spoons: what was the weight of each?

11. The weight of 8 silver tankards is 10 lbs. 5 oz. 7 pwts. 6 grs.: what is the weight of each?

12. If 8 persons consume 85 lbs. 12 oz. of meat in a month, how much is that apiece?

QUEST.—*Obs.* Of what denomination is each quotient figure? When the divisor is a composite number, how proceed?

13. A dairy-woman packed 95 lbs. 8 oz. of butter in 10 boxes: how much did each box contain?

14. A tailor had 76 yds. 2 qrs. 3 na. of cloth, out of which he made 8 cloaks: how much did each cloak contain?

15. A man traveled 50 m. and 32 r. in 11 hours: at what rate did he travel per hour?

16. A man had 285 bu. 3 pks. 6 qts. of grain, which he wished to carry to market in 15 equal loads: how much must he carry at a load?

17. A man had 80 A. 45 r. of land, which he laid out into 36 equal lots: how much did each lot contain?

18. Divide 685 bu. 2 pks. 4 qts. by 45.

19. If £85, 7s. 7d. 3 far. are divided equally among 81 persons, how much will each receive?

APPLICATIONS OF THE COMPOUND RULES.

175. A *Bill*, in business operations, is a written statement of items, with the price of each, and the amount of the whole.

Required the amount of each of the following bills:

LONDON, Aug. 27th, 1852.

1. *John Porter, Esq.*,

Bought of H. Packard & Co.

20	Bibles, gilt,	at	16s. $8\frac{1}{2}$d.	. . .
12	" extra gilt,	"	18s. 9d.	. . .
8	Paradise Lost,	"	9s. $6\frac{1}{4}$d.	. . .
18	Homer, 2 vols.,	"	12s. 8d.	. . .
6	Virgil, 2 vols.	"	13s. $3\frac{3}{4}$d.	. . .

Received Payment,

H. PACKARD & CO.

NEW YORK, Jan. 3d, 1853.

2. *William Hall & Co.*,

To J. C. Carter, Dr.

For 20	pieces	silk,	at	£5, 7s. $6\frac{1}{4}$d.	. . .
" 7	"	"	"	£7, 3s. $8\frac{3}{4}$d.	. . .
" 8	"	linen,	"	£3, 17s. 5d.	. . .
" 16	"	merino,	"	£4, 10s. $9\frac{1}{2}$d.	. . .
" 14	"	velvet,	"	£9, 18s. $7\frac{2}{3}$d.	. . .

Received Payment,

J. C. CARTER.

Put the following memoranda into the form of bills, and find the amount of each:

3. James Henry bought, July 1st, 1852, of C. B. Lawrence, 25 lbs. gunpowder, at 4s. 6d.; 36 guns, at £1, 12s. 6d.; 12 rifles, at £2, 8s.; and 45 knapsacks, at 12s. 6d. What was the amount of his bill?

4. If you buy 27 lbs. sugar, at 7d. a pound; 36 drums of figs, at 4s. 6½d. a drum; 17 boxes of raisins, at 6s. 7d. a box, what will be the amount of your bill?

5. J. Dill bought 10 doz. pair silk hose, at 4s. 8d. a pair; 16 doz. thread ditto, at 3s. 4½d.; 21 doz. worsted ditto, at 4s. 6¼d.: what was the amount of his bill?

6. James Gordon sold 15 acres, 2 roods, and 15 rods of land, at £3, 15s. 7d. per rod: what amount did he receive?

7. Bought a piece of land 68 rods long, and 25½ rods wide, at £6, 4s. 6d. per acre: what did it amount to?

8. Elisha Fanning sold a customer a quarter of veal weighing 18 lbs. 4 oz., at 8¾d. per pound; a quarter of mutton weighing 16 lbs. 8 oz. at 7½d.; and a saddle of venison weighing 28 lbs. 4 oz. at 1s. 7d. per pound: what was the amount of the bill?

9. A drover bought 10 oxen each weighing 9 cwt. 15 lbs., at 8½d. per pound: what was the amount of his bill?

10. A hardware merchant bought 43 tons, 2 qrs. 17 lbs. of iron, at 1s. 7d. per pound: what was the amount of his bill?

11. A laborer dug a cellar 62 feet long, 25 feet wide, and 8½ ft. deep, at 5½d. per cu. yard: what was the amount of his bill

12. Bought 50 casks of molasses each containing 58 gals. 3 qts., at 2s. 6d. per gal.; afterwards 215 gals. 2 qts. leaked out, and the remainder was sold at 3s. 4d. per gal.: what was the result of the operation?

13. Bought 2 cwt. 3 qrs. 10 lbs. of saltpetre, at 3s. 7d. a pound; 16 cwt. 2 qrs. 17 lbs. dyewood, at 4s. 6d. a pound; 5 cwt. 1 qr. 11 lbs. indigo, at 15s. 8d. a pound: what was the amount of the bill?

14. George Spencer bought of Henry Brown, 75 yards of broadcloth, at 15s. 6d. per yard; 115 yards of silk, at 7s. 6d. per yard; 263 yards of bombazine, at 4s. 7d. per yard; 325 yards of cassimere, at 11s. 8d.: what was the amount of his bill?

SECTION VIII.

DECIMAL FRACTIONS.

ART. 176. *Decimal Fractions* are those which arise from dividing an *integer* into *ten* equal parts; then subdividing *one* of these parts into *ten* others, and so on, each succeeding part regularly decreasing in a *ten fold ratio.* Thus, if a unit is divided into 10 equal parts, 1 of these parts is a *tenth.* (Art. 103.) Now if 1 *tenth* is divided into 10 equal parts, 1 of these parts will be a *hundredth;* for $\frac{1}{10}\div 10=\frac{1}{100}$. (Art. 138.) Again, if 1 *hundredth* is divided into 10 equal parts, 1 of these parts will be a *thousandth;* for $\frac{1}{100}\div 10=\frac{1}{1000}$, &c.

OBS. These fractions are called *decimals*, from the Latin numeral *decem, ten,* which indicates both their *origin* and *ratio* of *decrease.*

177. Each order of whole numbers, we have seen, *increases* in value from units towards the left in a ten-fold ratio; and, conversely, each order must *decrease* from left to right in the same ratio, till we come to units' place again. (Art. 9.)

178. By extending this scale of notation below units towards the right hand, it is manifest that the *first* place on the right of units, will be *ten times* less in value than *units'* place; that the *second* will be ten times less than the *first;* the *third* ten times less than the *second,* &c.

Thus we have a series of *orders* below units, which decrease in a ten-fold ratio, and exactly correspond in value with *tenths, hundredths, thousandths,* &c., when expressed by common fractions. Hence,

179. *Decimal Fractions* are commonly expressed by writing the numerator with a point (.) before it, called the *separatrix.* Thus, $\frac{1}{10}$ is written .1; $\frac{2}{10}$ thus .2; $\frac{3}{10}$ thus .3, &c. $\frac{1}{100}$ is writ-

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QUEST.—176. What are decimal fractions? *Obs.* Why called decimals? 177. In what manner do whole numbers increase and decrease? 178. By extending this scale below units, what would be the value of the first place on the right of units? The second? The third? With what do these orders correspond? 179. How are decimal fractions expressed?

ten .01, putting the one in hundredths place; $\frac{5}{100}$ thus .05, &c. That is, tenths are written in the *first* place on the right of units; hundredths in the *second* place; thousandths in the *third* place, &c.

OBS. 1. If the numerator does not contain so many figures as there are ciphers in the denominator, the deficiency must be supplied by prefixing ciphers to it.

2. The *object* of the *decimal point* or *separatrix*, is to distinguish the fractional parts from whole numbers. To prevent it from being mistaken for the point used in *numeration*, the decimal point should be a period (.), and the other a comma (,).

180. *The denominator of a decimal fraction is always* 1, *with as many ciphers annexed to it as there are decimal figures in the given numerator.* (Art. 176.)

181. The names of the different *orders of decimals* or places below units, may be easily learned from the following

DECIMAL TABLES.

Hundreds.	Tens.	*Units.*	(Decimal Point.)	Tenths.	Hundredths.	*Thousandths.*	Ten thousandths.	Hundred thousandths.	*Millionths.*	Ten Millionths.	Hundred millionths.	*Billionths.*	Ten billionths.	Hundred billionths.	*Trillionths*, &c.
4	2	3	.	2	6	7	1	4	5	9	8	6	2	7	4

182. It will be seen from this table that the *value* of each figure in *decimals*, as well as in whole numbers, depends upon the *place* it occupies, reckoning from units. Thus, if a figure stands in the *first* place on the right of units, it expresses *tenths*; if in the *second*, *hundredths*, &c. Hence,

183. *Each removal of a decimal figure one place from units towards the right, diminishes its value ten times.*

Prefixing a cipher, therefore, to a decimal diminishes its

QUEST.—*Obs.* If the numerator does not contain so many figures as there are ciphers in the denominator, what must be done? What is the object of the decimal point? 180. What is the denominator of a decimal fraction? 181. Repeat the Decimal Table, beginning units, tenths, &c. 182. Upon what does the value of a decimal depend? 183. What is the effect of removing a decimal figure one place to the right?

value *ten times;* for it removes the decimal one place farther from units' place. Thus $.4=\frac{4}{10}$; but $.04=\frac{4}{100}$, and $.004=\frac{4}{1000}$.

Annexing ciphers to decimals does not alter their value; for, each significant figure continues to occupy the same place from units as before. Thus, $.5=\frac{5}{10}$; so $.50=\frac{50}{100}$, or $\frac{5}{10}$, and $.500=\frac{500}{1000}$, or $\frac{5}{10}$, &c. (Art. 116.)

OBS. 1. It should be remembered that the *units'* place is always the *right hand* place of a whole number. The effect of annexing and prefixing ciphers to decimals, is the *reverse* of annexing and prefixing them to whole numbers. (Art. 58.)

2. A whole number and a decimal written together, is called a *mixed number.* (Art. 108.)

184. To read *Decimal* Fractions.

Beginning at the left hand, read the figures as if they were whole numbers, and to the last one add the name of its order. Thus,

.5	is read	5 tenths.
.25	" "	25 hundredths.
.324	" "	324 thousandths.
.5267	" "	5267 ten thousandths.
.43725	" "	43725 hundred thousandths.
.735168,	" "	735168 millionths.

OBS. 1. In reading decimals as well as whole numbers, the *units'* place should always be made the *starting* point. It is advisable for young pupils to apply to every figure the name of its order, or the place which it occupies, before attempting to read them. Thus beginning at units' place—*units, tenths, hundredths, thousandths,* &c., pointing to each figure as he pronounces the name of its order.

2. Sometimes we pronounce the word *decimal* when we come to the separatrix, and then read the figures as if they were whole numbers; or, simply repeat them one after another. Thus, 125.427 is read, one hundred twenty-five, *decimal* four hundred twenty seven; or, one hundred twenty-five, *decimal* four, two, seven.

Read the following numbers:

(1.)	(2.)	(3.)	(4.)
.25	.5317	3.245	9.14712
.362	.1056	7.6071	1.06231
.451	.4308	4.3159	2.00729
.5675	.0105	3.87816	9.14051
.8432	.0007	5.91432	8.06705

QUEST.—What then is the effect of prefixing ciphers to decimals? What of annexing them? *Obs.* Which is the units' place? What is a whole number and a decimal written together, called? 184. How are decimals read? *Obs.* In reading decimals, what should be made the starting point? What other method *of reading* decimals is mentioned?

(5.)	(6.)	(7.)	(8.)
25.02	56.78417	1.253456	2.000008
36.032	21.05671	0.034689	0.500072
45.7056	42.05063	7.035042	8.305001
12.07067	95.10051	9.103005	9.000001

184.a. To write *Decimal* Fractions.

Beginning with the highest order, write each figure in the place indicated by its name, and to the result prefix the decimal point. If any order is omitted in the given decimal, write a cipher in its place.

Write the following fractions in decimals:

(9.)	(10.)	(11.)	(12.)
$2\frac{5}{10}$	$3\frac{5}{100}$	$15\frac{6435}{10000}$	$3\frac{12567}{100000}$
$4\frac{52}{100}$	$45\frac{62}{100}$	$10\frac{534}{1000}$	$4\frac{2005}{100000}$
$28\frac{6}{100}$	$5\frac{231}{1000}$	$2\frac{45}{1000}$	$17\frac{201}{100000}$
$6\frac{29}{100}$	$6\frac{23}{1000}$	$300\frac{701}{10000}$	$13\frac{123567}{1000000}$

13. Write 49 hundredths; 3 tenths; 445 ten thousandths; 7 hundredths; 5 thousandths.

14. Write 36 thousandths; 25 hundred thousandths; 1 millionth; 703 thousandths.

15. Write 7 hundredths; 8 thousandths; 95 ten thousandths; 63 millionths; 26 ten millionths.

16. Write forty-six and five thousandths; seventy-two and seven millionths; three thousand two hundred and sixty-four millionths; 64 and nine thousandths; 93 and sixteen millionths.

185. *Decimals* differ from *Common* Fractions both in their *origin*, and in the *manner of expressing them.*

Common Fractions arise from dividing a *unit* into *any* number of *equal* parts; consequently, the *denominator* may be *any number whatever.* (Art. 107.) Decimals arise from dividing a *unit* into *ten equal* parts, then subdividing one of those parts into *ten other equal* parts, and so on; consequently, the denominator is always 10, 100, 1000, &c. (Arts. 176, 180.)

Again, *Common Fractions* are expressed by writing the *numerator* over the *denominator; Decimals* are expressed by writing the *numerator only*, with a point before it, while the denominator is understood. (Arts. 107, 179.)

QUEST.—184.a. How do you write decimals? 185. How do decimals differ from common fractions? From what do common fractions arise? From what do decimals arise? How are common fractions expressed? How are decimals?

ADDITION OF DECIMAL FRACTIONS.

Ex. 1. What is the sum of 2.5; 24.457; 123.4 and 2.369?

Suggestion.—Having written the given numbers so that *tenths* may stand under *tenths*, *hundredths* under *hundredths*, &c., begin at the right hand or lowest order, and proceed as in adding simple numbers. Finally, place the *decimal point* in the amount, under those in the numbers added, and the result 152.726, is the answer.

Operation.

```
  2.5
 24.457
123.4
  2.369
-------
152.726
```

187. Hence, we deduce the following general

RULE FOR ADDITION OF DECIMALS.

Write the numbers so that the same orders may stand under each other, and beginning at the right hand or lowest order, proceed as in Simple Addition. (Art. 29.)

From the right of the amount, point off as many figures for decimals, as there are decimal places in either of the given numbers.

OBS. The *Proof* and the *reasons* for the rule, are the same as in addition of *simple* numbers.

(2.)	(3.)	(4.)
31.25	15.263	20.13
1.059	7.0003	117.056
126.05	213.0507	43.5
1235.6151	0.05	2185.05813
1393.9741 *Ans.*	85.306	620.30597

5. What is the sum of 2.5; 33.65 and 45.121?
6. What is the sum of 65.7; 43.09; 1.026 and 2.1765?
7. Add 6.15768; 1.713458 and .6573128?
8. Add .0256; 15.6941; 3.856 and .00035?
9. Add 256.31; 29.7; 468.213; 5.6 and .75.
10. Add 25.61; 78.003; 951.072 and 256.3052.
11. Add .567; 37.05; 63.501; 76.25 and .63.
12. Add .005; 1.25; 6.456; 10.2563 and 15.434.

QUEST.—187. What is the rule for addition of decimals? How point off the answer? *Obs.* How is addition of decimals proved?

13. Add 256.1; 10.15; 27.09; 85.560 and 2.067.

14. Add 5.00257; 3.600701 and 2.10607.

15. Add together 5 tenths, 25 hundredths, 566 thousandths, and 7568 ten thousandths.

16. Add together 34 hundredths, 67 thousandths, 13 ten thousandths, and 463 millionths.

17. Add together 7 thousandths, 68 hundred thousandths, 47 millionths, and 6 tenths.

18. Add together 423 ten millionths, 68 thousandths, 25 hundredths, 4 tenths, and 56 ten thousandths.

19. What is the sum of four hundred three and twenty-six hundredths; forty-seven and six tenths; ninety-four and eighteen thousandths; two hundred, and five ten thousandths?

20. What is the sum of eighteen and forty-five ten thousandths; sixty and one hundred twenty-three millionths; forty-nine and sixty three ten millionths?

SUBTRACTION OF DECIMAL FRACTIONS.

188. Ex. 1. From 25.367 subtract 13.18.

Suggestion.—Having written the less number under the greater, so that *tenths* may stand under *tenths*, *hundredths* under *hundredths*, we begin at the right hand or lowest order, and proceed as in Simple Subtraction. Finally, place the decimal point in the remainder under those in the given numbers, and the result 12.187, is the answer.

Operation.

```
       25.367
       13.18
Ans.   12.187
```

189. Hence, we deduce the following general

RULE FOR SUBTRACTION OF DECIMALS.

Write the less number under the greater, so that the same orders may stand under each other.

Beginning at the right hand or lowest order, subtract as in simple numbers, and from the right of the remainder, point off as many figures for decimals, as there are decimal places in either of the given numbers.

OBS. The *Proof* and the *reasons* for the rule, are the same as in subtraction of *simple* numbers.

QUEST.—189. What is the rule for subtraction of decimals? How point off the answer? *Obs.* How is subtraction of decimals proved?

2. From 15 take 1.5. *Ans.* 13.5.
3. From 256.0315 take 5.641.
4. From 15.7 take 1.156.
5. From 63.25 take 50.
6. From 201.001 take 56.04087.
7. From 1 take .125.
8. From 11.1 take .40005.
9. From .56078 take .325.
10. From 1.66 take .5589.
11. From 3.4001 take 2.000009.
12. From 1 take .000001.
13. From 256.31 take 125.4689301.
14. From 8960.320507 take 68.001.
15. From 57000.000001 take 1000.001.
16. From 75 hundredths take 75 thousandths.
17. From 6 thousandths take 6 millionths.
18. From 3252 ten thousandths take 3 thousandths.
19. From 589 take 22 thousandths.
20. From 7856 take 236 millionths.
21. From five tenths take five hundredths.
22. From six thousandths take seven ten thousandths.
23. From seven hundred thousandths take nine millionths.
24. From forty-seven and twenty-four hundredths take seven and sixty-three thousandths.

25. From five hundred six and ninety-nine millionths take two hundred forty-three and ninety-nine thousandths.

26. What is the difference between twenty-nine thousandths, and twenty-nine thousand?

27. What is the difference between forty-five hundredths, and forty-five thousandths?

28. What is the difference between five hundred sixty-nine thousandths, and five hundred sixty-nine millionths?

29. A man having nine-tenths of an acre of land, sold nineteen thousandths of an acre: how much did he have left?

30. A grocer having a hogshead of molasses, lost 215 thousandths of it by leakage: how much was left?

31. From a piece of cloth containing seventy-five and seventeen hundredths yards, thirty-six and seven thousandths yards were used: how many yards were left?

MULTIPLICATION OF DECIMAL FRACTIONS.

190. Multiplying by a *fraction*, we have seen, is taking a *part* of the multiplicand as many times, as there are *like parts* of a *unit* in the multiplier. (Art. 132.)

Ex. 1. What is the product of .48 multiplied by .5?

Suggestion.—The multiplier .5, is equal to $\frac{5}{10}$ or $\frac{1}{2}$, and .48 is equal to $\frac{48}{100}$. (Art. 180.) Now $\frac{48}{100} \times \frac{1}{2} = \frac{48}{200}$ or $\frac{24}{100}$, and $\frac{24}{100} = .24$, which is the answer required. (Art. 179.) In practice, we multiply as in whole numbers, and pointing off as many decimals in the product as there are decimal figures in both factors, we have .240. But since ciphers placed on the right of decimals do not affect their value, the 0 may be omitted, and the result is .24, the same as before. (Art. 183.)

Operation.

$$\begin{array}{r} .48 \\ .5 \\ \hline .240 \end{array} \text{ Ans.}$$

191. Hence, we deduce the following general

RULE FOR MULTIPLICATION OF DECIMALS.

Multiply as in whole numbers, and from the right of the product, point off as many figures for decimals, as there are decimal places in the multiplier and multiplicand.

If the product does not contain so many figures as there are decimal places in both factors, supply the deficiency by prefixing ciphers.

OBS. 1. The *Proof* and *reasons* for the rule, are the same as in multiplication of *simple* numbers.

2. The *reason* for pointing off as many decimal places in the product as there are decimals in both factors, may be illustrated thus:

Suppose it is required to multiply .25 by .5. Supplying the denominators .25 $= \frac{25}{100}$, and $.5 = \frac{5}{10}$. (Art. 180.) Now $\frac{25}{100} \times \frac{5}{10} = \frac{125}{1000}$; but $\frac{125}{1000} = .125$; (Art. 179;) that is, the product of .25 × .5, contains just as many decimals as the factors themselves.

In like manner it may be shown that the product of any two or more decimal numbers, must contain as many decimal figures as there are places of decimals in the given factors.

QUEST.—190. What is it to multiply by a fraction? 191. What is the rule for multiplication of decimals? How do you point off the product? When the product does not contain so many figures as there are decimals in both factors, what is to be done? *Obs.* How is multiplication of decimals proved?

	(2.)	(3.)	(4.)	(5.)
Multiply	8.45	96.071	456.03	.1236
By	.25	.0032	4.5	.027
	4225	192142	228015	8652
	1690	288213	182412	2472
Ans.	2.1125	.3074272	2052.135	.0033372

EXAMPLES FOR PRACTICE.

1. In 1 piece of cloth there are 31.7 yards: how many yards are there in 7.3 pieces?
2. In 1 barrel there are 31.5 gallons: how many gallons are there in 8.25 barrels?
3. In one rod there are 16.5 feet: how many feet are there in 35.75 rods?
4. How many cords of wood are there in 45 loads, allowing 8.25 of a cord to a load?
5. How many rods are there in a piece of land 25.35 rods long, and 20.5 rods wide?
6. If a man can travel 38.75 miles per day, how far can he travel in 12.25 days?
7. How many pounds of coffee are there in 68 sacks, allowing 961.25 pounds to a sack?
8. If a family consume .85 of a barrel of flour in a week, how much will they consume in 52.23 weeks?
9. What is the product of 10.001 into .05?
10. What is the product of 50.0065 into 1.003?

192. When the multiplier is 10, 100, 1000, &c.

Remove the decimal point in the multiplicand as many places towards the right, as there are ciphers in the multiplier, and the result will be the product. (Arts. 59, 191.)

11. Multiply 4.6051 by 100. *Ans.* 460.51.
12. Multiply 2.6501 by 1000.
13. Multiply .5678 by 10000.
14. Multiply .000781 by 2.40001.
15. Multiply 1.002003 by .0024.
16. Multiply .58001 by .0001003.
17. Multiply 8.001502 by .00005.

QUEST.—192. How proceed when the multiplier is 10, 100, 1000, &c.

18. Multiply 85689.31 by .000001.
19. Multiply .0000045 by 69.5.
20. Multiply .0340006 by .000067.
21. Multiply .5 by 5 millionths.
22. Multiply .15 by 28 ten thousandths.
23. Multiply 25 hundred thousandths by 7 and 3 tenths.
24. Multiply 225 millionths by 2 and 85 hundredths.
25. Multiply 2367 ten millionths by 3 and 2 ten thousandths.
26. Multiply .07506 by .253.
27. $2.00641 \times .0032$.
28. Multiply .003046 by .005.
29. 45.034×8.2401.
30. Mult. 2.406723 by .00081.
31. 9.3245×6.0532.
32. Mult. 75.00732 by .00005.
33. $623.0052 \times .00028$.
34. Mult. 823.0207 by .23006.
35. $720.3009 \times .24007$.
36. Multiply two thousandths by two ten thousandths.
37. Multiply five millionths by sixty-one thousandths.
38. Multiply two hundred sixty-three ten millionths by forty-eight ten thousandths.
39. Multiply seven hundred billionths by two thousand one hundred and fifty-six millionths.

DIVISION OF DECIMAL FRACTIONS.

193. Ex. 1. What is the quotient of .75 divided by .5?

Suggestion.—Divide as in whole numbers and point off as many decimal figures in the quotient as those in the dividend exceed those in the divisor, which is *one*; the result 1.5, is the answer.

Operation.

.5).75

Ans. 1.5

194. Hence, we derive the following general

RULE FOR DIVISION OF DECIMALS.

Divide as in whole numbers, and point off as many figures for decimals in the quotient, as the decimal places in the dividend exceed those in the divisor. If the quotient does not contain figures enough, supply the deficiency by prefixing ciphers.

OBS. 1. The *Proof* and the *reasons* for the rule, are the same as in division of *simple* numbers.

2. The *reason* for *pointing* off the quotient may be explained in the following manner: In multiplication of decimals, we have seen that the product has as many decimal figures, as the multiplier and multiplicand. Now since the dividend is equal

to the product of the divisor and quotient, (Art. 65,) it follows that the dividend must have as many decimals as the divisor and quotient together; consequently, we must point off as many decimals in the quotient as the decimal places in the dividend exceed those in the divisor.

3. When the number of decimals in the divisor is the *same* as that in the dividend, the quotient will be a *whole* number.

When there are *more* decimals in the divisor than in the dividend, annex as many ciphers to the dividend as are necessary to make its decimal places *equal* to those in the divisor. The quotient thence arising will be a whole number.

4. After all the figures of the dividend are divided, if there is a remainder, ciphers may be annexed to it, and the division continued at pleasure. The ciphers annexed must be regarded as decimal places belonging to the dividend.

For ordinary purposes, it will be sufficiently exact to carry the quotient to three or four places of decimals; but when great accuracy is required, it must be carried farther.

Note.—When there is a remainder at the close of the operation, the sign + should be annexed to the quotient to show that it is not complete.

2. Divide .289 by 2.4. *Quotient* .1204+.
3. Divide 1.345 by .5. *Quotient* 2.69.
4. Divide .063 by 9. *Quotient* .007.

EXAMPLES FOR PRACTICE.

1. If 1.7 of a yard of cloth will make a coat, how many coats will 10.2 yards make?

2. In 6.75 cords of wood, how many loads are there, allowin .75 of a cord to a load?

3. If a man mows 3.2 acres of grass per day, how long will it take him to mow 39.36 acres?

4. If 23.25 bushels of barley grow on an acre, how many acres will 556 bushels require?

5. In 74.25 feet how many rods?

6. In 99.225 gallons of wine, how many barrels?

7. If a man chops 3.75 cords of wood per day, how many days will it take him to chop 91.476 cords?

8. If a man can travel 35.4 miles per day, how long will it take him to travel 244.26 miles?

9. A dairy-man has 187.5 pounds of butter, which he wishes

QUEST.—194. What is the rule for division of decimals? How point off the quotient? *Obs.* How is division of decimals proved? When the number of decimal places in the divisor is equal to that in the dividend, what is the quotient? When there are more decimals in the divisor than in the dividend, how proceed? When there is a remainder, what may be done?

to pack in boxes containing 12.5 pounds apiece: how many boxes will it require?

10. In 3.575, how many times .25

195. When the divisor is 10, 100, 1000, &c.

Remove the decimal point in the dividend as many places towards the left, as there are ciphers in the divisor, and the result will be the quotient. (Arts. 80, 194.)

11. Divide 756.4 by 100. *Ans.* 7.564.
12. Divide 1268.2 by 1000. *Ans.* 1.2682.
13. Divide 1 by 1.25.
14. Divide 1 by 562.5.
15. Divide .012 by .005.
16. Divide 2 by .0002.
17. Divide 5 by .000001.
18. Divide 13.2 by .75
19. Divide .0248 by .04.
20. Divide 2071.31 by 65.3.
21. 245780.75÷10000.
22. 857603.4÷1000000.
23. 6370000÷.000007.
24. 792000÷.0000008.
25. 7843.5÷100000000.
26. 903.4÷1000000000.
27. Divide four hundred twenty-four millionths by fifteen ten thousandths.
28. Divide two hundred forty-eight and eight thousandths by seven and sixteen ten thousandths.
29. Divide six and one hundred and twenty-five ten millionths by one and nineteen millionths.
30. Divide sixty-three and eighty-one billionths by three and nine millionths.
31. Divide eight hundred fifty-six thousand nine hundred seventy-eight millionths by nine thousand two hundred and twenty-six ten thousandths.
32. Divide forty-five and four hundred seventy-six thousand two hundred ninety-nine ten millionths by twenty-nine and two hundred sixty-five ten thousandths.
33. Divide the product of twenty-five thousandths into seven hundredths by forty-six ten thousandths.
34. Divide the product of twenty-six and eighty-five ten thousandths into six hundredths by fifteen thousandths.
35. Divide the sum of thirty-eight ten thousandths and thirty-eight hundredths by thirty-eight.

QUEST.—195. When the divisor is 10, 100, 1000, &c., how proceed?

REDUCTION OF DECIMALS.

CASE I.—*Reducing Decimals to Common Fractions.*

Ex. 1. Reduce the decimal .25 to a common fraction.

Suggestion.—Since the denominator of a decimal fraction is always 1 with as many ciphers annexed to it as there are figures in the numerator, we erase the decimal point, and write the denominator under the numerator; the answer is $\frac{25}{100}$ or $\frac{1}{4}$. Hence,

Operation.
$.25=\frac{25}{100}$. *Ans.*

196. To reduce a *Decimal* to a *Common* Fraction.

Erase the decimal point, and write the denominator under the given numerator; the result will be a common fraction.

2. Reduce .125 to a com. fraction, then to its lowest terms.
3. Reduce .66 to a common fraction, &c.
4. Reduce .75 to a common fraction, &c.
5. Reduce .375 to a common fraction, &c.
6. Reduce .525 to a common fraction, &c.
7. Reduce .025 to a common fraction, &c.
8. Reduce .875 to a common fraction, &c.
9. Reduce .0625 to a common fraction, &c.

CASE II.—*Reducing Common Fractions to Decimals.*

Ex. 1. Reduce $\frac{3}{4}$ to a decimal fraction.

Suggestion.—We first reduce the numerator 3, to *tenths* by annexing a cipher to it, and it becomes 30 tenths. But the number whose value we wish to find, is not 3, but a *fourth* of 3; therefore the result 30 tenths, is *four* times too large. To correct this we divide it by the denominator 4, which gives 7 tenths (.7), and 2 tenths over. Again, we reduce this remainder 2 tenths, to *hundredths* by annexing another cipher, and the result is 20 hundredths, which is also 4 times too large; we therefore divide it by 4, and obtain 5 hundredths (.05). Now .7 and .05 are equal to .75, the answer required.

Operation.
4)300
.75 *Ans.*

PROOF.—$.75=\frac{75}{100}$, and $\frac{75}{100}$ reduced to its lowest terms, is equal to $\frac{3}{4}$. (Art. 120.) Hence,

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QUEST.—196. How reduce a decimal to a common fraction?

197. To reduce a *Common* Fraction to a *Decimal.*

Annex ciphers to the numerator and divide it by the denominator. Point off as many decimal figures in the quotient, as you have annexed ciphers to the numerator.

OBS. 1. If there are not so many figures in the quotient as you have annexed ciphers to the numerator, supply the deficiency by prefixing ciphers to the quotient.

2. The *reason* of this rule may also be illustrated in the following manner: Annexing a cipher to the numerator multiplies the fraction by 10. (Arts. 59, 133.) If, therefore, the numerator with a cipher annexed, is divided by the denominator, the quotient is obviously *ten* times too large. Hence, in order to obtain the true quotient, or a decimal equal to the given fraction, the quotient thus obtained must be divided by 10, which is done by pointing off *one* figure. (Art. 80.)

Again, annexing 2 ciphers to the numerator multiplies the fraction by 100; annexing 3 ciphers by 1000, &c., consequently, when two ciphers are annexed, the quotient will be 100 times too large, and must therefore be divided by 100; when three ciphers are annexed, the quotient will be 1000 times too large, and must be divided by 1000, &c. (Art. 80.)

3. Reduce $\frac{3}{2}$ to a decimal. *Ans.* 1.5

4. Reduce $\frac{3}{4}$, and $\frac{4}{5}$.

5. Reduce $\frac{3}{20}$, and $\frac{7}{35}$.

6. Reduce $\frac{3}{8}$, $\frac{1}{5}$, and $\frac{3}{5}$.

7. Reduce $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{2}{25}$.

8. Reduce $\frac{4}{25}$, $\frac{8}{20}$, $\frac{3}{75}$.

9. Reduce $\frac{5}{8}$, $\frac{2}{5}$, $\frac{1}{20}$.

10. Reduce $\frac{12}{480}$, $\frac{5}{1785}$.

11. Reduce $\frac{6}{240}$, $\frac{3}{1000}$.

12. Reduce $\frac{1}{3}$ to a decimal. .333333 + *Ans.*

13. Reduce $\frac{128}{999}$ to a decimal. .128128128 + *Ans.*

198. It will be seen in the last two examples there continues to be a remainder after each division, as long as we continue the operation.

In the 12th, the remainder is always 1; in the 13th, after obtaining three figures in the quotient, the remainder is the same as the given numerator, and the next three figures in the quotient are the same as the first three, when the same remainder recurs again.

199. Decimals which consist of the same figure or set of figures *continually repeated*, are called *Periodical* or *Circulating Decimals;* also, *Repeating Decimals* or *Repetends.*

Note.—For the method of finding the value of Circulating decimals, also of adding, subtracting, multiplying, and dividing them, see Higher Arithmetic.

QUEST.—197. How reduce a common fraction to a decimal? *Obs.* When there are not so many figures in the quotient as you have annexed ciphers, what is to be done? 199. What are Periodical or Repeating Decimals?

14. Reduce $\frac{1}{27}$ to its equivalent value in decimals.

15. Reduce $\frac{1}{7}$.

16. Reduce $\frac{1}{13}$.

17. Reduce $\frac{1}{40}$.

18. Reduce $\frac{37}{160}$.

19. Reduce $\frac{13}{16}$.

20. Reduce $\frac{43}{1600}$.

21. Reduce $\frac{1}{21}$.

22. Reduce $\frac{1}{14}$.

CASE III.—*Reducing compound numbers to decimals of higher denominations.*

Ex. 1. Reduce 7s. 6d. 3 far. to the decimal of a pound.

Suggestion.—Reducing 7s. 6d. 3 far. to a common fraction of a pound, we have £$\frac{363}{960}$. (Art. 164.) Next, reducing £$\frac{363}{960}$ to a decimal, it becomes £.378125, which is the answer required.

First Method.

7s. 6d. 3 f. = 363 far.

£1, = 960 far.

$\frac{363}{960}$ = £.378125

Or, writing the given numbers under each other, we may reduce each in succession to a decimal of the next higher denomination. Thus, since 3 far. is $\frac{3}{4}$d., if we annex a cipher to the 3 far. and divide by 4, the result is .75d. which annexed to the 6d., makes 6.75d. In like manner, the 6.75d. may be reduced to the decimal of a shilling by dividing by 12; and the result .5625, annexed to the given shillings, makes 7.5625s. Finally, the 7.5625s. may be reduced to the decimal of a pound by dividing by 20. Hence,

Second Method.

4	3.00
12	6.75
20	7.5625

Ans. £.378125

200. To reduce a *compound* number to the *decimal* of a *higher* denomination.

Reduce the given compound number to a common fraction; then reduce the common fraction to a decimal. (Arts. 164, 197.)

Or, write the given numbers under each other in their order, with the highest denomination at the bottom; then annexing ciphers to the lowest denomination, divide it by the number required of this denomination to make ONE *of the next higher, and place the quotient as a decimal, on the right of the number below.*

Proceed in this manner with each denomination, till you come to the one required, and the last quotient will be the answer.

QUEST.—200. How is a compound number reduced to the decimal of a higher denomination?

OBS. *Decimals* of *lower* denominations are reduced to decimals of *higher* denominations, in the same manner as whole numbers.

1. Reduce .75d. to the decimal of a £.

Suggestion.—We first divide by 12, then by 20, according to the rule above, and point off each quotient as in division of decimals. (Art. 194.)

Operation.

12	0.75
20	0.0625
Ans.	£.003125

2. Reduce 5s. 4d. to the decimal of £1. *Ans.* £.2666+.
3. Reduce 15s. 6d. to the decimal of £1.
4. Reduce 12s. 6d. 1 far. to the decimal of £1.
5. Reduce 9d. to the decimal of £1.
6. Reduce 2s. 7d. 2 far. to the decimal of a shilling.
7. Reduce 5 gals. 2 qts. 1 pt. to the decimal of a hogshead.
8. Reduce 18 hours 9 min. to the decimal of a day.
9. Reduce 5 cwt. 2 qrs. 15 lbs. to the decimal of a ton.
10. Reduce 2 ft. 6 in. to the decimal of a yard.
11. Reduce 6 furlongs 30 rods to the decimal of a mile.
12. Reduce 13 oz. 8 drs. to the decimal of a pound.
13. Reduce $9\frac{1}{4}$d. to the decimal of a shilling.
14. Reduce 5s. and 1 far. to the decimal of a £.
15. Reduce £31, 5s. $6\frac{3}{4}$d. to the decimal of a £.
16. Reduce .4 pint to the decimal of a hogshead.
17. Reduce .75 pound to the decimal of a ton.

CASE IV.—*Reducing decimals of higher denominations to whole numbers of lower denominations.*

Ex. 1. Reduce £.123 to shillings, pence, and farthings.

Suggestion.—We first multiply the given decimal by 20 to reduce it to shillings, and pointing off the product as in multiplication of decimals, the result is 2s. and .460s. over. Next we multiply this decimal by 12 to reduce it to pence, and pointing off the product as before, we have 5d. and .520d. over. Finally, multiplying this decimal by 4, and pointing off, we have 2 far. and .080 far. over, which is so small it may be disregarded. The numbers on the left of the decimal points, 2s. 5d. 2 far. are the answer. Hence,

Operation.

	£.123
	20
shil.	2.460
	12
pence	5.520
	4
far.	2.080
Ans.	2s. 5d. 2 f.

201. To reduce a *decimal* of a *higher* denomination to *whole* numbers of *lower* denominations.

Multiply the given decimal by that number which it takes of the next lower denomination to make ONE *of this higher, and point off the product, as in multiplication of decimal fractions.*

Proceed in this manner with the decimal figures of each succeeding product, and the numbers on the left of the decimal point in the several products, will be the answer required.

2. Reduce £.125 to shillings and pence. *Ans.* 2s. 6d.
3. Reduce .625s. to pence and farthings.
4. Reduce £.4625 to shillings and pence.
5. Reduce .756 gallons to quarts and pints.
6. Reduce .6254 days to hours, minutes, and seconds.
7. Reduce .856 cwt. to quarters, &c.
8. Reduce .6945 of a ton to hundreds, &c.
9. Reduce .7582 of a bushel to pecks, &c.
10. Reduce .8237 of a mile to furlongs, &c.
11. Reduce .45683 of an acre to roods and rods.
12. Reduce .75631 of a yard to quarters and nails.

EXERCISES IN DECIMAL COMPOUND NUMBERS.

202. *Decimals* of compound numbers, when reduced to *whole* numbers of lower denominations, may be *added* or *subtracted* like other compound numbers. (Arts. 168, 169.)

Or, if reduced to decimals of the *same denomination*, they may be *added* or *subtracted* like other decimals. (Arts. 187, '9.)

Note.—If the following exercises are found too difficult for beginners, they may be omitted till review.

1. What is the sum of £.25 and .5s.? *Ans.* 5s. 6d., or £.275.
2. Add £.125 to .4s.
3. Add £.625 to .25s. and .75d.
4. Add .275 ton and .08 cwt.
5. Add .6 acre and .4 rod.
6. From £.65 take 6.5s.
7. From .875s. take .25d.
8. From .281 t. take .75 cwt.
9. From .775 m. take 75 r.
10. Find the sum of £$\frac{1}{5}$ and $\frac{1}{6}$s. in the decimal of a £.
11. Find the difference between £$\frac{4}{5}$ and $\frac{3}{4}$d. in the decimal of a pound.

QUEST.—201. How reduce decimals of a higher denomination to whole numbers of lower denominations?

12. What cost .778125 ton of iron, at 2s. 6d. a pound?

13. What cost .94375 acre of land, at $2\frac{1}{2}$ dollars a rod?

14. Bought .75631 yard of satin, at 3d. a nail: how much did it come to?

15. What cost 17 cwt. 2 qrs. 16 lbs. of ginger, at 27 dollars per hundred?

16. What cost 18 cwt. 1 qr. 18 lbs. of tea, at 63 dollars a hundred?

17. What cost 56 hhds. 16 gals. 3 qts. of molasses, at £6, 17s. 6d. per hogshead?

18. How much will $15\frac{3}{4}$ cords of wood come to, at £.905 per cord?

19. What will 75 yds. 1 qr. 2 na. of silk come to, at £.25 per yard?

20. If .175 bushel of wheat cost $\frac{1}{4}$ dollar, what will a bushel cost?

21. Paid £$\frac{7}{8}$ for .625 yard of sarcenet: what was that a yard?

22. Paid $\frac{5}{8}$ dollar for .125 bbl. of flour: how much was that per barrel?

23. If you walk .965625 mile per hour, how far can you walk in a week?

24. What cost .778125 ton of butter, at 2s. per pound?

25. A Californian sold .815625 lb. of silver, at 5s. per grain: what did it come to?

26. Bought .45683 acre of land, at .5 dollar a foot: what did it come to?

27. A man gave £.775 for .125 cwt. of beeswax: how much was that a pound?

28. If you pay £.375 for .5 lb. of nutmegs, how much is that per ounce?

29. A man gave £.328 for .41 bbl. cider: what was that a gallon?

30. What cost $\frac{3}{4}$ cwt. of sugar, at .122 dollar per pound?

31. What will 28 lbs. 8 oz. beef come to, at £.928 per hundred?

32. A man paid £.825 for .33 cwt. of coffee: how much did his coffee cost a pound?

33. If you pay £.315 for .07875 of a hogshead of vinegar, how much will it cost you a quart?

FEDERAL MONEY.

203. *Federal Money*, we have seen, is the currency of the United States. Its denominations are *Eagles*, *dollars*, *dimes*, *cents*, and *mills*. (Art. 146.)

Note.—For the Table of Federal Money, the weight, and purity of its different coins, see Art. 146.

204. Federal Money is based upon the *Decimal* Notation; its denominations *increase* and *decrease* from right to left and left to right in a *tenfold ratio*, like simple numbers. It is therefore one of the most *convenient* and *comprehensive* systems of currency ever invented.

205. The dollar is regarded as the *unit*; *cents* and *mills* are fractional parts of the dollar, and are separated from it by a *decimal point* or *separatrix* (.), as decimals are separated from whole numbers. (Art. 179.) Thus, *Dollars* occupy *units'* place of simple numbers; *eagles*, or tens of dollars, tens' place; *dimes*, or tenths of a dollar, the place of *tenths*; *cents*, or hundredths of a dollar, the place of *hundredths*; *mills*, or thousandths of a dollar, the place of *thousandths*; *tenths* of a mill, or ten thousandths of a dollar, the place of *ten thousandths*, &c.

206. Accounts, in the United States, are kept in *dollars*, *cents*, and *mills*. Eagles are expressed in dollars, and dimes in cents. Thus, instead of five eagles, we say 50 dollars; instead of 6 dimes, we say, 60 cents, &c.

OBS. 1. Since *dimes* in business transactions, are expressed in *cents*, *two places* of decimals are assigned to cents. If therefore the number of cents is *less* than 10, *a cipher must always be placed on the left hand of them.* For example, 4 cents are written thus .04; 7 cents thus .07; 9 cents thus .09, &c.

2. Mills occupy the *third* place of decimals; therefore, when there are no cents in the given sum, *two ciphers* must be placed before the mills. Hence,

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QUEST. 203.—What is Federal Money? What are its denominations? Recite the Table. 204. Upon what is Federal Money based? 205. What is regarded as the unit in Federal Money? What are cents and mills considered? How are they distinguished from dollars? 206. How are accounts kept in the United States? How are Eagles expressed? Dimes? *Obs.* How many places are assigned to cents? When the number of cents is less than ten, what must be done? When no cents are mentioned, what do you do?

207. To read Federal Money.

Call all the figures on the left of the decimal point, dollars; the first two figures on the right of the point, cents; the third figure, mills; the other places on the right, decimals of a mill. Thus, \$3.25232 is read, 3 dollars, 25 cents, 2 mills, and 32 hundredths of a mill.

OBS. Sometimes all the figures after the point are read as decimals of a dollar. Thus, \$5.356 is read, "5 and 356 thousandths dollars."

Read the following sums of Federal Money.

1. \$250.56; \$105.863; \$200.057; \$506.507; \$850.071.
2. \$44.081; \$60.05; \$75.003; \$20.501; \$30.065.
3. \$3.7542; \$0.6054; \$4.0151; \$6.0057; \$8.0106.

Write the following sums in Federal Money:

4. 63 dollars, and 85 cents. *Ans.* \$63.85.
5. 150 dollars, and 73 cents.
6. 201 dollars, and 9 cents.
7. 300 dollars, 5 cents, and 3 mills.
8. 4 dollars, 6 cents, and 8 mills.
9. 100 dollars, 7 cents, 5 mills, and 3 tenths of a mill.
10. 1000 dollars, 6 mills, and 36 hundredths of a mill.

Note.—In business transactions, when dollars and cents are expressed together, the cents are frequently written in the form of a common fraction. Thus, \$76.45 are written $76\frac{45}{100}$ dollars.

REDUCTION OF FEDERAL MONEY.

CASE I.—*Reducing Dollars to Cents and Mills.*

EX. 1. Reduce 75 dollars to cents and mills.

Suggestion.—Since in 1 dollar there are 100 cents, in 75 dollars there are 75 times as many, or 7500 cents. Again, since in 1 cent there are 10 mills, in 7500 cents there are 7500 times as many, or 75000 mills. Now, to multiply by 10, 100, &c., we simply annex as many ciphers to the multiplicand as there are ciphers in the multiplier. (Art. 59.) Hence,

Operation.

$$\begin{array}{rl} 75 & \text{dolls.} \\ 100 & \\ \hline 7500 & \text{cts.} \\ 10 & \\ \hline \textit{Ans. } 75000 & \text{m.} \end{array}$$

QUEST.—207. How do you read Federal Money? *Obs.* What other mode of reading Federal Money is mentioned?

208. To reduce dollars to cents, *annex two ciphers.*

To reduce dollars to mills, *annex three ciphers.*

To reduce cents to mills, *annex one cipher.*

OBS. To reduce dollars and cents to cents, *erase the sign of dollars and the decimal point.* Thus, \$25.36 reduced to cents, becomes 2536 cents.

2. Reduce 9 cts. to mills.
3. Reduce \$25 to mills.
4. Reduce \$5 to cents.
5. Reduce \$364 to mills.
6. Reduce \$621 to mills.
7. Reduce \$6245 to cents.
8. Reduce \$75.26 to cents.
9. Reduce \$625.48 to cents.

CASE II.—*Reducing Cents and Mills to Dollars.*

10. Reduce 45000 mills to dollars and cents.

Operation.

1|0)4500|0 mills.

1|00)45|00 cents.

Ans. 45 dolls.

Suggestion.—Since 10 mills make 1 cent, 45000 mills will make as many cents as 10 is contained times in 45000, or 4500 cents. Again, since 100 cents make 1 dollar, 4500 cents will make as many dollars as 100 is contained times in 4500, or 45 dollars. Now, to divide by 10, 100, &c., we cut off as many figures from the right of the dividend as there are ciphers in the divisor. (Art. 80.) Hence,

209. To reduce cents to dollars, *point off two figures on the right.*

To reduce mills to dollars, *point off three figures on the right.*

To reduce mills to cents, *point off one figure on the right.*

OBS. The figures pointed off, are cents and mills.

11. Reduce 150 mills to cts.
12. Reduce 25000 mills to dolls.
13. Reduce 325 cts. to dolls.
14. Reduce 423 mills to cts.
15. Reduce 4320 m. to dolls.
16. Reduce 63500 cts. to dolls.
17. Reduce 4890 mills to cts.
18. Reduce 95673 mills to dolls.

210. Since *Federal Money* is based upon the *decimal system* of notation, it is evident that it may be subjected to the same operations and treated in the same manner as *Decimal Fractions.*

QUEST.—208. How are dollars reduced to cents? Dollars to mills? Cents to mills? *Obs.* Dollars and cents to cents? 209. How are cents reduced to dollars? Mills to dollars? Mills to cents? *Obs.* What are the figures pointed off?

ADDITION OF FEDERAL MONEY.

Ex. 1. A man bought a cow for \$15.75, a calf for \$2.375, a sheep for \$3.875, and a load of hay for \$8.68: how much did he pay for all?

Suggestion.—We write the dollars under dollars, cents under cents, &c., and proceed as in addition of decimals. From the right of the amount, we point off three figures for cents and mills.

Operation.

	\$15.75
	2.375
	3.875
	8.68
Ans.	\$30.680

211. Hence, we derive the following general

RULE FOR ADDING FEDERAL MONEY.

Write the given numbers under each other, so that dollars may stand under dollars, cents under cents, &c.

Begin at the right hand, and adding each column separately point off the amount as in addition of decimals. (Art. 187.)

OBS. If either of the given numbers have no cents expressed, supply their place by ciphers.

2. A farmer sold a firkin of butter for \$9.28, a cheese for \$1.17, a quarter of veal for 56 cents, and a bushel of wheat for \$1.12: how much did he receive for the whole?

3. A man bought a hat for \$5.375, a cloak for \$35.68, and a pair of boots for \$4.75: how much did he pay for all?

4. What is the sum of \$37.565, \$85.20, \$90.03, and \$150.638?

5. What is the sum of \$10.385, \$46.238, \$190.62, and \$23.036?

6. What is the sum of \$23.005, \$16.03, \$110.738, and \$131.26?

7. What is the sum of 63 dolls. and 4 cts., 86 dolls. and 10 cts., and 47 dolls. and 37 cts?

8. What is the sum of \$608.05, \$365.205, \$2.268, and \$47.006?

9. What is the amount of 11 dolls. 3 cts. and 5 mills, 16 dolls. and 8 mills, 49 dolls. 7 cts. and 8 mills?

QUEST.—211. What is the rule for Addition of Federal Money? How point off the amount? *Obs.* When any of the given numbers have no cents expressed, how is their place supplied?

10. What is the amount of 100 dolls. and 61 cts., 51 dolls. and 3 cts., 65 dolls. 8 cts. and 3 mills?

11. What is the amount of 95 dolls. 67 cts. and 8 mills, 120 dolls. 45 cts., 101 dolls. 7 cts. and 9 mills?

12. A lady bought a bonnet for \$6.67, a pair of gloves for \$0.625, a pair of shell combs for \$0.75, and a cap for \$2.50: what was the amount of her bill?

13. Add \$563.87½; \$19.18¾; \$960.37½; \$28.06¼; \$806.19¼.

14. Add 684.07¼; \$493.673; \$81.7358; \$65.409; \$85.0075.

15. Add \$8 three cents; 87½ cts.; \$96 six cents; \$9.81¼.

SUBTRACTION OF FEDERAL MONEY.

Ex. 1. A man bought a horse for \$56.50, and a cow for \$23.38: how much more did he pay for his horse than for his cow?

Suggestion.—We write the less number under the greater, placing dollars under dollars, &c., then subtract, and point off the answer as in subtraction of decimals.

Operation.

	\$56.50
	23.38
Ans.	\$33.12

212. Hence, we derive the following general

RULE FOR SUBTRACTING FEDERAL MONEY.

Write the less number under the greater, with dollars under dollars, cents under cents, &c.

Beginning at the right hand, subtract, and point off the remainder as in subtraction of decimals. (Art. 189.)

OBS. If either of the given numbers have no cents expressed, supply their place by ciphers.

2. A man owing \$57.35, paid \$17.93: how much does he still owe? *Ans.* \$39.42.

3. A grocer bought two hogsheads of molasses for \$68.90, and sold it for \$79.26: how much did he gain by the bargain?

4. A man owed a debt of \$105, and paid but \$23.67: how many dollars did he then owe?

QUEST.—212. What is the rule for Subtraction of Federal Money? How point off the remainder? *Obs.* When either of the given numbers has no cents expressed, how is their place supplied?

5. A merchant bought a quantity of silks for \$237.63, and sold it for \$196.03: how much did he lose?

6. A drover bought a flock of sheep for \$357, and sold them for \$17.33 less than he paid: how much did he sell them for?

7. From 365 dolls. 7 cts. take 208 dolls. 20 cts?

8. From 1 cent, subtract 6 mills.

9. From 1 doll. 6 cts. 7 mills, take 89 cts. 3 mills.

10. From 96 dolls. 6 cts., take 41 dolls. 63 cts. 8 mills.

11. From 100 dolls. 10 cts. 3 mills, take 1 cent 5 mills.

12. From 1000 dolls. 6 cts., take 100 dolls. and 5 mills.

13. From $6\frac{1}{4}$ cents, take $6\frac{1}{4}$ mills.

14. A young man deposited \$278.63 in a Savings Bank; at one time he drew out \$19 and 7 cents, at another \$21 and $37\frac{1}{2}$ cents, at another \$25 and 6 cents: how much had he left?

MULTIPLICATION OF FEDERAL MONEY.

213. Ex. 1. Multiply \$81.75 by 2.5.

Suggestion.—We multiply as in simple numbers, and since there are *three* decimal places in the multiplier and multiplicand, we point off the decimal places in the product as in multiplication of decimal fractions.

Operation.

```
 $81.75
    2.5
 ------
  40875
 16350
 -------
$204.375 Ans.
```

214. Hence, we derive the following general

RULE FOR MULTIPLYING FEDERAL MONEY.

Multiply as in simple numbers, and point off the product as in multiplication of decimals. (Art. 191.)

OBS. 1. In Multiplication of Federal Money, as well as in simple numbers, the multiplier must always be considered an *abstract number*. (Art. 45. Obs. 2.)

2. When the multiplier or multiplicand contains a common fraction, the fraction should be changed to a decimal. (Art. 197.)

3. In business operations, when the mills in the answer are 5, or over, it is customary to call them 1 cent; when under 5, they are disregarded.

2. Multiply \$15.80 by $12\frac{1}{2}$.

Solution.—$12\frac{1}{2}=12.5$, and $\$15.80\times12.5=\$197.50$.

QUEST.—214. What is the rule for Multiplication of Federal Money? *Obs.* In multiplication of Federal Money, what must the multiplier always be considered? When the multiplier or multiplicand contains a common fraction, how proceed?

3. Multiply $45.035 by 6.2. *Ans.* $279.217.

4. What cost 12 lbs. of beef at $9\frac{1}{2}$ cents a pound?

Analysis.—If 1 lb. cost $9\frac{1}{2}$ cts., 12 lbs. will cost 12 times as much. We therefore multiply the price of 1 lb. by the number of pounds, and point off the product as in the last article. Hence,

Operation.

$$\begin{array}{r} 9\tfrac{1}{2}\text{ cts.}=.095 \\ 12 \\ \hline \textit{Ans. }\$1.140 \end{array}$$

215. To find the *cost* of any *number* of articles, when the *price* of *one* is given.

Multiply the price of one by the number of articles, and the product will be the cost of the whole.

5. What cost 14 lbs. of starch, at $10\frac{1}{2}$ cts. per pound?
6. What cost $15\frac{1}{2}$ lbs. of sugar, at $9\frac{1}{2}$ cts. per pound?
7. What cost 25 gals. of molasses, at $18\frac{3}{4}$ cts. a gallon?
8. What cost $23\frac{1}{4}$ lbs. of raisins, at $28\frac{1}{2}$ cts. per pound?
9. What cost $33\frac{1}{2}$ lbs. of candles, at $12\frac{1}{2}$ cts. per pound?
10. What cost $16\frac{1}{4}$ lbs. of hyson tea, at $56\frac{1}{4}$ cts. a pound?
11. What will 83 lbs. of beef cost, at $\$4.62\frac{1}{2}$ per hund.?

Suggestion.—We multiply the price of 100 ($4.625) by 83, the given number of pounds, and the product $383.875, is the cost of 83 lbs. at $4.625 per *pound.* But the price is $4.625 per *hundred;* consequently, the product $383.875 is 100 times too large; we therefore divide it by 100, to give the true answer. Hence,

Operation.

$$\begin{array}{r} \$4.625 \\ 83 \\ \hline 13\ 875 \\ 3\ 70\ 00 \\ \hline \$3.83\ 875 \textit{ Ans.} \end{array}$$

216. To find the cost of articles bought and sold by the 100, or 1000.

Multiply the given price by the given number of articles, then if the price is for 100, *divide the product by* 100; *but if the price is for* 1000, *divide it by* 1000. (Art. 195.)

12. What will 825 feet of boards cost, at $6.75 per 1000?

13. At $4.50 per 1000, what will 1250 bricks cost?

14. A farmer sold a quarter of beef, weighing 256.5 lbs., at $\$5.37\frac{1}{2}$ per 100: how much did he receive for it?

15. At $\$4.62\frac{1}{2}$ per 100, what will 1675 lbs. of pork cost?

QUEST.—215. When the price of 1 article, pound, or yard, &c., is given, how is the cost of any number of articles found? 216. How do you find the cost of articles bought and sold by the 100, or 1000?

16. What cost 2129 feet of boards, at $18.25 per 1000?
17. What cost $456\frac{3}{4}$ yards of shirting, at $12\frac{1}{2}$ cts. per yard?
18. What cost 156 lbs. of chocolate, at $15\frac{1}{2}$ cts. a pound?
19. What cost 235 lbs. of cheese, at $16\frac{1}{4}$ cents a pound?
20. What cost 175 doz. eggs, at $10\frac{1}{2}$ cents per dozen?
21. At $47\frac{1}{2}$ cents per bushel, what will be the cost of 300 bushels of corn?
22. What cost 153 lbs. of sugar, at $8\frac{1}{2}$ cts. per pound?
23. What cost 1500 lbs. butter, at $8.50 per 100.
24. What cost 28500 ft. of timber, at $3.76 per 100?
25. What cost 8230 ft. of mahogany, at $70.20 per 1000?
26. What cost 7630 shingles, at $3.50 per 1000?
27. What cost 15024 pine shingles, at $8.37 per 1000?
28. At $16\frac{1}{2}$ cts. a pound, what cost $219\frac{1}{2}$ lbs. honey?
29. At $\$2.67\frac{3}{4}$ per yard, what will 400 yards of cloth cost?
30. At $\$5\frac{3}{4}$ per barrel, what will 1560 barrels of flour cost?

DIVISION OF FEDERAL MONEY.

Ex. 1. How many times are $4.25 contained in $\$27.62\frac{1}{2}$?

Suggestion.—We divide as in simple numbers, and from the right of the quotient, point off one figure for decimals, according to the rule for Division of Decimal Fractions. The answer is 6.5 times.

Operation.

```
$4.25)$27.625(6.5 Ans.
       2550
        2125
        2125
```

217. Hence, we derive the following general

RULE FOR DIVIDING FEDERAL MONEY.

Divide as in simple numbers, and point off the quotient as in division of decimals. (Art. 194.)

OBS. After all the figures of the dividend are divided, if there is a remainder, ciphers may be annexed to it, and the operation continued as in division of decimal fractions. The ciphers thus annexed must be regarded as decimal places of the dividend. (Art. 194. Obs. 3.)

2. Divide $71.91 by $7.65. *Ans.* 9.4.

QUEST.—217. What is the rule for Division of Federal Money? *Obs.* When there is a remainder after all the figures of the dividend are divided, how proceed?

3. Divide $149.625 by $2.375. *Ans.* 63.

4. If $75 are divided into 18 equal parts, what will be the value of each part? *Ans.* $4.166+.

5. A man bought 6 hats for $25.68: how much did they cost apiece?

Suggestion.—In this example, the *number* of articles is given with the *cost* of the *whole*, and it is required to find the price of *one* article. We divide the whole cost by the number of articles, and point off the quotient as in the rule above. Hence,

Operation.

$$\begin{array}{r} 6)\underline{25.68} \\ \textit{Ans.}\ \$4.28 \end{array}$$

218. To find the price of *one* article, when the *number* of articles, and *cost* of the *whole*, are given.

Divide the price of the whole by the number of articles, and the quotient will be the price of one article.

6. A man paid $27.82 for a quantity of indigo, which was $4.28 a pound: how many pounds did he buy?

Suggestion.—In this example the *cost* of the *whole* quantity and the *price* of 1 pound are given, to find the *number* of pounds. Now, $27.82, will obviously buy as many pounds as $4.28 are contained times in it, which is 6.5. He therefore bought 6.5 pounds. Hence,

Operation.

$$\begin{array}{l} \$4.28)\$27.82(6.5\text{ lbs.} \\ \quad\quad\ \ \underline{2568} \\ \quad\quad\quad\ 2140 \\ \quad\quad\quad\ \underline{2140} \end{array}$$

219. To find the *number* of articles, when the price of *one* and *cost* of the *whole*, are given.

Divide the cost of the whole, by the price of one, and the quotient will be the number of articles.

7. How many quarts of cherries at $7\frac{3}{4}$ cents a quart, can you buy for $1.24?

8. How many pounds of figs, at 14 cents a pound, can you buy for $3.57?

9. How many watermelons, at $12\frac{1}{2}$ cents apiece, can be bought for $3?

QUEST.—218. How find the price of one article, when the number of articles and cost of the whole are given? 219. How find the number of articles when the price of one, and the cost of the whole are given?

10. How many pen-knives, at 20 cts. apiece, can be bought for $7.20 ?

11. At $17\frac{1}{2}$ cts. a quart, how many quarts of molasses can be bought for $4.40 ?

12. A man bought 50 pair of thick boots for $175: how much did he give a pair?

13. A man paid $485.50 for 260 sheep: how much did he give per head?

14. At $2.50 a cord, how many cords of wood can I buy for $165 ?

15. At $4.75 per barrel, how many barrels of flour can I buy for $8.50 ?

16. If a man's income is $1.68 per day, how much is it per hour?

17. If a man pays $\$3.62\frac{1}{2}$ per week for board, how long can he board for $188.50 ?

18. Suppose a man's income is $500 a year, how much is that per day?

19. Suppose a man's interest money is $28.80 per day, how much is it per minute?

20. A mason received $94.375 for doing a job, which took him $75\frac{1}{2}$ days: how much did he receive per day?

21. At $\$1.12\frac{1}{2}$ per bushel, how many bushels of wheat can be bought for $523.75 ?

22. If $1285.20 were divided equally among 125 men, what would each receive.

23. If $1637.10 were divided equally among 150 men, what would each receive?

24. The salary of the President of the United States is $25000 a year: how much does he receive per day?

25. A man paid $66.51 for broadcloth, which was $7.39 per yard: how many yards did he buy?

26. If flour is $\$8.12\frac{1}{2}$ per barrel, how many barrels can be bought for $2047.50?

27. If 1563 lbs. of rice cost $117.225, how much will 1 pound cost?

28. If 556.25 lbs. of tobacco cost $69.532, how much will 1 pound cost?

29. At $47.184 per ton, how many tons of railroad iron can be bought for $28310.40 ?

APPLICATIONS OF FEDERAL MONEY.

220. Ex. 1. What is the cost of the several articles, and what the amount, of the following bill?

Boston, May 25th, 1845.

James Brown, Esq.,

Bought of Fairfield & Lincoln,

5 yds. broadcloth	at	$3.25	. . . . .
3 yds. cambric,	"	.12½	. . . . .
3 doz. buttons,	"	.15	. . . . .
6 skeins sewing silk,	"	.06¼	. . . . .
4 yds. wadding,	"	.08	. . . . .

Amount, $17.77.

Received Pay't,

FAIRFIELD & LINCOLN.

Ex. 2. NEW HAVEN, Sept. 2d, 1845.

Hon. R. S. Baldwin,

To Durrie & Peck, Dr.

4 Lovell's Young Speaker,	at	$ .62½	. . . .
5 Olmsted's Rudiments,	"	.58	. . . .
6 Morse's Geography,	"	.50	. . . .
8 Webster's Spelling Book,	"	.10	. . . .
8 Day's Algebra,	"	1.25	. . . .

What was the cost of the several articles, and the amount of his bill?

Put the following memoranda into the form of bills, and find the amount of each?

3. John Jacob Astor, Esq., of New York, bought, Aug. 18th, 1845, of G. W. Lewis & Co., 25 lbs. of sugar, at 9 cts. a pound; 50 lbs. of coffee, at 11 cts.; 12 lbs. of tea, at 75 cts.; 14 lbs. of raisins, at 14 cts.; 9 doz. eggs, at 10 cts.; and 15 lbs. butter, at 12½ cts. What was the cost of the several articles, and what the amount of his bill?

4. W. A. Sanford, Esq. of Philadelphia, bought, June 3d, 1845, of James Conrad, 28 yds. of silk, at $1.25 a yard; 22 yds. of muslin, at 56 cts.; 16 pair of cotton hose, at 37½ cts.; 35 pair of silk hose, $1.10; and 25 pair of shoes, at $1.25. What was the cost of the several articles, and how much is due on his account?

5. Messrs. Holmes & Homer of Cincinnati, bought, July 1st, 1845, of H. W. Morgan & Co., 100 bbls. flour, at $4.50 a barrel; 50 bbls. pork, at $8.25; 25 bbls. beef, at $9.75; 112 kegs of lard, at $3.25; and 25 bu. corn, at 84 cts. What was the cost of the several articles, and how much is due on his account?

6. F. C. Emerson, Esq. of New Orleans, bought, Aug. 12th, 1845, of W. H. Arnold & Co., 35 hhds. molasses, at $12.60 per hogshead; 2100 lbs. sugar, at $5\frac{1}{2}$ cts.; 14000 lbs. cotton, at $7\frac{1}{2}$ cts.; 1350 lbs. coffee, at $6\frac{1}{4}$ cts.; 31200 lbs. rice, at 8 cts.; 150 boxes of oranges, at \$4.12$\frac{1}{2}$ per box. They credited him 500 clocks, at $5.00 apiece, and his note to balance. What was the amount of charges, and what the amount of the note?

7. Bought 19 yds. broadcloth, at $5.63 per yard; 18 doz. pen-knives, at \$4.37$\frac{1}{2}$ per dozen; 78 lbs. of tea, at $.59; 245 lbs. butter, at $.13: what was the amount of the bill?

8. Bought 85 cwt. of beef, at $6.25 per hundred; 126 bbls. of pork, at \$18.62$\frac{1}{2}$ per barrel; 243 bbls. flour, at \$6.87$\frac{1}{2}$; 35 tons of hay, at \$19$\frac{1}{2}$; 25 cords of wood, at \$3.56$\frac{1}{4}$: what was the amount of the bill?

9. Bought 35 doz. gloves, at $4.50 per doz.; 95 yds. black silk, at \$.87$\frac{1}{2}$ per yard; 115 yds. colored ditto, at $.78; 36 crape shawls, at $32.50 apiece; 65 Broché ditto, at $17.83: what was the amount of the bill?

10. Bought 85 ploughs, at $9.63; 125 hoes, at 63 cents; 94 shovels, at 84 cents; 56 rakes, at 28 cents; 67 axes, at $1.13: what was the amount of the bill?

11. Bought 96 pair black silk hose, at 83 cents; 85 ditto white, at $87\frac{1}{2}$ cents; 135 ditto worsted, at $56\frac{1}{4}$ cents; 87 pair men's gloves, at 67 cents; 120 pair ladies' ditto, at 58 cents; 75 cravats, at 96 cents: what was the amount of the bill?

12. Bought 67 Latin Readers, at 63 cents; 60 Greek Readers, at $1.09; 84 Greek Grammars, at 68 cents; 95 Latin ditto, at $62\frac{1}{2}$ cents; 35 Virgil, at $2.13; 45 Sallust, at 78 cents; 52 Cicero's Orations, at 75 cents: what was the amount of the bill?

13. Bought 36 pair of boots, at $5.17; 216 pair thick shoes, at \$1.37$\frac{1}{2}$; 135 pair gaiters, at $1.38; 240 pair buskins, at 83 cents; 134 pair slippers, at 68 cents; 87 pair rubbers, at $1.13: what was the amount of the bill?

SECTION IX.

PERCENTAGE.

ART. 222. *Percentage* and *Per Cent.* signify a certain *allowance on a hundred*, or simply *hundredths*. Thus, the expressions 2 per cent., 4 per cent., 6 per cent., &c., of any number or sum of money, signify 2 hundredths ($\frac{2}{100}$,) 4 hundredths ($\frac{4}{100}$,) 6 hundredths ($\frac{6}{100}$,) of that number or sum.

Note.—The terms *Percentage* and *Per Cent.*, are derived from the Latin *per* and *centum*, signifying *by the hundred*.

223. Since *percentage* and *per cent.* signify *hundredths*, it is manifest any given per cent. may be expressed by decimals. For example, 1 per cent. is written thus .01; 2 per cent. thus .02; 3 per cent. thus .03; 103 per cent. thus 1.03; 125 per cent. thus 1.25; $\frac{1}{2}$ per cent. or $\frac{1}{2}$ of 1 per cent. thus .005; $\frac{1}{4}$ per cent. thus .0025, &c.

OBS. 1. When the given per cent. is *less* than 10, a cipher must always be prefixed to the figure expressing it, in the same manner as when the number of cents is less than 10. (Art. 206. Obs. 1.)

When the given per cent. is *more* than 99, it plainly requires a mixed number to express it. (Art. 183. Obs. 2.)

2. Parts of 1 per cent. may be expressed either by a *common* fraction, or by *decimals*. Thus, the expression 17$\frac{5}{8}$ per cent. is equivalent to .17625.

3. The *first two* decimal figures properly denote the *per cent.*, for they are *hundredths*; the other figures denote *parts of hundredths*, and therefore express *parts* of 1 per cent.

1. Write 1 per cent., 2 per cent., 3 per cent., 5 per cent., 8 per cent., and 9 per cent. in decimals.

2. Write 13 per cent.; 15; 30; 50; 75; 49; 73; 85.

3. Write $\frac{1}{2}$ per cent.; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{3}{10}$; $\frac{1}{8}$; $\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{7}$.

4. Write 3$\frac{1}{2}$ per cent.; 5$\frac{2}{3}$; 16$\frac{1}{8}$; 125; 331$\frac{1}{4}$; 462$\frac{1}{2}$.

QUEST.—222. What do the terms percentage and per cent. signify? What is meant by 2 per cent. of any sum? 4 per cent.? 6 per cent.? 7 per cent.? *Note.* From what are the terms percentage and per cent. derived? 223. How may any given percentage or per cent. be expressed? *Obs.* When the given per cent. is less than 10, how is it written? When more than 99, how? How are parts of 1 per cent. expressed?

EXERCISES IN PERCENTAGE.

5. A constable collected $400 for a merchant, and received 3 per cent. for his services: how much did he receive?

Suggestion.—Since 3 per cent. is $\frac{3}{100}$, or .03, we multiply the number of dollars collected by 3 per cent. written as a decimal, and pointing off the product as in multiplication of decimals, the result $12, is the answer required. Hence,

Operation.

$$\begin{array}{r} \$400 \\ .03 \\ \hline \textit{Ans. } \$12.00 \end{array}$$

225. To calculate percentage on any given number.

Multiply the given number by the given per cent. expressed in decimals, and point off the product as in multiplication of decimals.

OBS. 1. If the per cent. contains a common fraction which cannot be expressed decimally, first multiply by the decimal, then by the common fraction of the given per cent., and point off the sum of their products as above.

2. It is important for the learner to observe, that the *amount* of money *collected* is made the *basis* upon which the percentage is calculated. That is, the constable is entitled to 3 dollars as often as he *collects* 100 dollars, and *not* as often as he *pays over* 100 dollars, as is frequently supposed. For, in the latter case he would receive only $\frac{3}{103}$, instead of $\frac{3}{100}$ of the sum in question. This distinction is important, especially in calculating percentage on large sums.

6. What is 2 per cent. of $350? *Ans.* $7.
7. What is 4 per cent. of $863? *Ans.* $34.52.
8. What is 3 per cent. of $145.25? *Ans.* $4.3575.
9. What is $\frac{1}{2}$ per cent. of $180.42? (Art. 223. Obs. 2.)
10. What $\frac{1}{4}$ per cent. of $827.63?
11. What is $\frac{3}{4}$ per cent. of $128.632?
12. What is $\frac{1}{5}$ per cent. of $90.45?
13. What is 10 per cent. of $600.451?
14. What is 12 per cent. of $2500.63?
15. What is 20 per cent. of $2250.84?
16. What is $3\frac{1}{5}$ per cent. of $436?
17. What is $2\frac{1}{3}$ per cent. of $144?
18. What is $4\frac{1}{2}$ per cent. of $257?
19. What is $8\frac{3}{4}$ per cent. of $673?

QUEST.—225. What is the rule for calculating percentage? *Obs.* If the per cent. contains a common fraction which cannot be expressed decimally, how proceed? What is made the basis upon which the percentage for collecting money is calculated?

20. A merchant having deposited $200 in a bank, afterwards drew out $10\frac{1}{2}$ per cent. of it: how many dollars did he draw out?

21. A merchant makes a deposit of $1864 and draws out 25 per cent of it: how much has he left in the bank?

22. A merchant shipped 865 boxes of lemons; on the passage home, 15 per cent. of them were thrown overboard: how many boxes did he lose, and how many had he left?

23. How much is $6\frac{1}{2}$ per cent. of $1000?

24. How much is 7 per cent. of $1526.33?

25. How much is $8\frac{1}{4}$ per cent. of $16.825?

26. A young man worth $1500, lost $31\frac{1}{4}$ per cent. of it in gambling: how much did he lose, and how much had he left?

27. A merchant bought a cargo of flour for $1230, and paid $4\frac{1}{2}$ per cent. for bringing it home: what was the whole cost of his flour?

28. What is $87\frac{1}{2}$ per cent. of $100? Of $2537.50?

29. What is 112 per cent. of $150?

30. What is 125 per cent. of $635?

31. What is 250 per cent. of $17.35?

32. Which is the most, 7 per cent. of $1000, or 6 per cent. of $1100?

33. What is the difference between 6 per cent. and 7 per per cent. of $12000?

34. What is the difference between 9 per cent. of $2000, and 6 per cent. of $3000?

35. What is $17\frac{1}{2}$ per cent. of $10000?

36. What is $20\frac{1}{2}$ per cent. of $10500?

37. A man gave his two sons $10000 apiece; the elder added $15\frac{1}{2}$ per cent. to his the first year, and the younger spent $15\frac{1}{2}$ per cent. of his: what was the difference of their property at the end of the first year?

38. A laboring man earning $225 a year, laid up $23\frac{1}{2}$ per cent. of it: how much did he spend?

39. A man having deposited $856.25 in a savings bank, drew out $31\frac{1}{4}$ per cent. of it: how much had he left in the bank?

40. A farmer owning 3560 sheep, lost 50 per cent. of them by disease: how many had he left?

COMMISSION, BROKERAGE, AND STOCKS.

226. *Percentage* is applied to various calculations in the practical concerns of life. Among the most important of these are Commission, Brokerage, the Rise and Fall of Stocks, Interest, Discount, Insurance, Profit and Loss, Duties and Taxes.

227. *Commission* is the *per cent.* or *sum charged* by agents for their services in buying and selling goods, or transacting other business.

OBS. An *Agent* who buys and sells goods for another, is called a *Commission Merchant, a Factor, or Correspondent.*

228. *Brokerage* is the *per cent.* or *sum charged* by money dealers, called *Brokers*, for negotiating *Bills of Exchange*, and other monetary operations, and is similar to Commission.

229. By the term *Stocks*, is meant the *Capital* of moneyed institutions, as Banks, Manufactories, Railroad and Insurance Companies; the funds of Government, State Bonds, &c.

OBS. Stocks are usually divided into portions of $100 each, called *shares*; and the owners of these shares are called *Stockholders.*

230. The original *cost* or *valuation* of a share is called its *nominal*, or *par value*; the *sum* for which it can be sold, is its *real value.*

OBS. 1. The *rise* or *fall* of Stocks is reckoned at a certain per cent. of its *par value.* The term *par* is a Latin word, which signifies *equal*, or a *state of equality.*

2. When stocks sell for their original cost or valuation, they are said to be *at par*; when they sell for more than cost, they are said to be *above par*, or *at a premium*; when they do not sell at cost, they are said to be *below par*, or *at a discount.*

3. Persons who deal in Stocks are called *Stock Brokers*, or *Stock Jobbers.*

231. *Commission* and *brokerage* are reckoned at a *certain percentage* on the *amount* of money employed in the transaction; the rise and fall of stocks, on the *par value* of the given shares.

QUEST.—227. What is commission? *Obs.* What is an agent who buys and sells goods for another called? 228. What is brokerage? 229. What is meant by stocks? *Obs.* How are stocks usually divided? What are the owners of the shares called? 230. What is the par value of stocks? What the real value? *Obs.* What is the meaning of the term par? When are stocks at par? When above par? When below? 231. How are commission and brokerage reckoned? How the rise or fall of stocks?

Ex. 1. A commission merchant sold a quantity of corn for \$236, and charged 2 per cent. commission: how much did he receive for his services?

Suggestion.—We multiply the amount for which the corn was sold, by the given per cent. expressed in decimals as in percentage, and the result \$4.72, is the answer required. Hence,

Operation.
\$236
.02
Ans. \$4.72

232. To compute commission, brokerage, and the premium or discount on stocks.

Multiply the sum by the given per cent. expressed in decimals, and point off the product as in multiplication of decimals.

2. A tax-gatherer collected \$533.56, for which he was to have 3 per cent. commission: what did he receive?

3. An auctioneer sold \$860.45 worth of goods, at $2\frac{1}{2}$ per cent. commission: how much did he receive?

4. An agent sold a quantity of oil for \$265.35, and charged $2\frac{1}{4}$ per cent. commission: how much did the agent receive, and how much the owner?

5. Sold goods to the amount of \$356, at $4\frac{1}{3}$ per cent. commission: how much did I receive for my services?

6. Bought goods amounting to \$480, at $3\frac{1}{4}$ per cent. commission: what is the amount of my commission?

7. What is the commission on \$163.625, at $6\frac{1}{4}$ per cent.?

8. What is the commission, at $5\frac{1}{5}$ per cent. for purchasing flour to the amount of \$1365.25?

9. I send my agent \$1000 to be invested in cotton, after deducting his commission at 5 per cent.: what is his commission, and how many dollars worth of cotton shall I receive?

Suggestion.—Since \$1000 includes both the commission and the sum invested, it is plain that the agent ought not to take 5 per cent. of \$1000; for this would be charging commission on his commission. He is entitled to 5 per cent. on that part only which he invests. Now the sum invested is $\frac{100}{100}$ of itself, and the commission is $\frac{5}{100}$ of this sum; consequently, the commission added to the sum invested must be $\frac{105}{100}$ of the investment.

QUEST.—232. What is the rule for computing commission, brokerage, &c.?

The question then resolves itself into this: \$1000 is $\frac{105}{100}$ of what sum? Now the numerator 105, is equal to 100 cents increased by the 5 per cent. commission, or \$1.05. We therefore divide the given amount \$1000 by 1 dollar increased by the per cent. commission, or \$1.05, annexing ciphers to the remainder, and the quotient \$952.38 is the sum invested. Subtracting the part invested from the whole amount, \$1000—\$952.38 leaves \$47.62, the commission. Hence,

Operation.

```
$1.05)$1000($952.38
        945
        ---
         550
         525
         ---
          250
          210
          ---
           400
           315
           ---
            850
            840
            ---
             10
```

232.*a*. When the commission is to be deducted in advance from a specified sum and the balance invested.

Divide the given sum by \$1 increased by the per cent. commission, and the quotient will be the part to be invested.

Subtract the part to be invested from the given sum, and the remainder will be the commission required.

OBS. The commission may also be found by multiplying the sum invested by the given per cent. according to the preceding rule. (Art. 232.)

10. A man sent a broker \$10478.13 to lay out in stocks after deducting his brokerage, at $\frac{1}{2}$ per cent.: what was the brokerage, and how much stock did he receive?

11. A merchant negotiated a bill of exchange of \$5000 with a broker, and agreed to give him 7 per cent.: how much did the broker receive?

12. What is the brokerage on \$8265, at $5\frac{1}{2}$ per cent.?

13. What is the brokerage on \$6524.13, at 8 per cent.?

14. What is the commission on \$146.356, at 20 per cent.?

15. What is the commission on \$1625.75, at 25 per cent.?

16. What is the commission on \$25026.10, at 15 per cent.?

17. What is the brokerage on \$50265.95, at $3\frac{1}{2}$ per cent.?

18. What is the brokerage on \$38212.085, at $1\frac{1}{4}$ per cent.?

19. What is the brokerage on \$752600, at 1 per cent.?

20. What is the brokerage on an investment of \$1000000, at $\frac{1}{2}$ per cent.?

QUEST.—232.*a*. How proceed when the commission is to be deducted in advance?

21. Bought 10 shares of bank stock, for which I paid 4 per cent. premium: how much did the stock cost me?

Suggestion.—The stock manifestly cost me its par value, viz: \$1000, together with 4 per cent. of it. (Art. 229. Obs.) Now $\$1000 \times .04 = \$40$; and $\$1000 + \$40 = \$1040$. *Ans.*

22. A man bought 5 shares of the Boston and Providence Railroad stock, at $5\frac{1}{2}$ per cent. premium: what did his stock cost him?

23. A broker sold 15 shares of the New York and Erie Railroad stock, at 15 per cent. discount: what did they come to?

24. Sold 29 shares in the American Manufacturing Co., at 16 per cent. advance: what did they come to?

25. A stock jobber bought 45 shares of the Auburn and Rochester Railroad stock at 3 per cent. discount, which he sold at 7 per cent. advance: how much did he make by the transaction?

26. A widow invested \$9000 in the Commonwealth Bank stock at par, and finally sold it at 75 per cent. discount: how much did she lose?

27. A man owned 53 shares of the Long Island Railroad stock, which he sold at auction, at 18 per cent. premium: how much did they come to?

28. Bought 38 shares in the Union Gas Co. at 7 per cent. advance, and sold them at 5 per cent. discount: how much was my loss?

29. An agent received \$25265 to invest in land, after deducting 3 per cent. commission: what was his commission?

30. Received \$62470 to lay out in iron, after deducting $2\frac{1}{2}$ per cent. commission: what sum have I to lay out?

31. Sold 5 pieces of cloth, each containing $31\frac{1}{4}$ yds., at $\$3.87\frac{1}{2}$ per yard, and charged $2\frac{1}{2}$ per cent. commission, and 3 per cent. for guaranteeing the payment: how much shall I pay over?

32. Sold 65 bales of cotton, averaging $483\frac{1}{2}$ lbs. to a bale, at $4\frac{1}{2}$ cents a pound: what is my commission at $3\frac{1}{4}$ per cent.?

33. Suppose you consign $365\frac{1}{2}$ bushels of wheat to your agent; he sells it at $87\frac{1}{2}$ cents per bushel, and charges $4\frac{3}{4}$ per cent. commission and guaranty: how much will you receive for your wheat?

INTEREST.

233. INTEREST is the *sum* paid for the *use of money* by the borrower to the lender. It is reckoned at a given *per cent. per annum;* that is, so many dollars are paid for the use of $100 for *one year;* so many cents for 100 cents; so many pounds for £100, &c.

234. The *money lent*, is called the *principal.*

The *per cent.* paid per annum, is called the *rate.*

The *sum* of the principal and interest, is called the *amount.* Thus, suppose I lend a man $100 at 7 per cent., and at the end of the year he pays me $100 the sum lent, and $7 for the use of it: the *principal*, in this case, is $100; the *rate*, 7 per cent.; the *interest*, $7; and the *amount*, $107.

OBS. The learner should be careful to notice the distinction between *Commission* and *Interest.* The *former* is reckoned at a certain *per cent.* without regard to time; (Art. 231;) the latter is reckoned at a certain *per cent.* for *one year;* consequently, for *longer* or *shorter* periods than one year, *like proportions* of the percentage for one year are taken.

The term *per annum* is a Latin phrase, which signifies *by the year.*

235. The *rate* of interest is usually established by law. It varies in different countries and in different parts of our own country.

OBS. 1. The *legal rate* of interest in New England, New Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, Tennessee, Kentucky, Ohio, Indiana, Illinois, Missouri, and Arkansas, is 6 per cent.

In New York, South Carolina, Michigan, Wisconsin, and Iowa, it is 7 per cent.

In Georgia, Alabama, Mississippi, and Florida, it is 8 per cent.; in Louisiana, 5 per cent.; in Texas, 10 per cent.

On debts in favor of the *United States*, it is 6 per cent.

2. In *Canada* and *Nova Scotia*, the legal rate is 6 per cent.; in *England* and *France*, 5 per cent.; in *Ireland*, 6 per cent. In *Italy* about the commencement of the 13th century, it varied from 20 to 30 per cent.

236. Any rate of interest *higher* than the legal rate, is called *usury*, and the person exacting it is liable to a heavy penalty.

QUEST.—233. What is Interest? How is it reckoned? 234. What is the principal? The rate? The amount? *Obs.* What is meant by the term per annum? 235. How is the rate usually determined? Is it the same everywhere? 236. What is any rate higher than the legal rate called? What is the consequence of exacting usury? Is it safe to take less than legal interest? *Obs.* When no rate is mentioned, what rate is understood?

Any rate *less* than the legal rate may be taken, if the parties concerned so agree.

OBS. 1. When no rate is mentioned, the rate established by the laws of the State in which the transaction takes place, is always understood to be the one intended by the parties.

2. The term *per annum*, is seldom expressed but it is always understood; for the *rate* is the *per cent.* paid *per annum.* (Art. 234.)

237. In calculating interest, a month, whether it contains 30 or 31 days, or even but 28 or 29, as in the case of February, is assumed to be *one twelfth* of a year. Therefore, the interest for any number of months, is so many *twelfths* of *one year's* interest.

Again, in reckoning interest, 30 days are commonly considered a month; consequently the interest for any number of days, is so many *thirtieths* of *one month's* interest. (Art. 170. Obs. 2.)

OBS. This practice seems to have been originally adopted on account of its convenience. Though not strictly accurate, it is sanctioned by general usage.

238. Ex. 1. What is the interest of \$15 for 1 year, at 4 per cent.?

Analysis.—Since 4 per cent. is $\frac{4}{100}$, the interest of \$1 (100 cents) for a year, is 4 cents, consequently, the interest of \$15 for the same time, is 15 times as much.

We therefore multiply the principal by the given rate per cent. expressed in decimals, and point off the product as in multiplication of decimals.

Operation.

$$\begin{array}{rl} \$15 & \text{prin.} \\ .04 & \text{rate.} \\ \hline \$.60 & \text{int.} \end{array}$$

2. What is the interest of \$67.35, for 3 years, at $6\frac{1}{2}$ per cent.?

Suggestion.—The interest for three years is manifestly 3 times as much as for 1 year. We therefore first find the interest for 1 year, as before; then multiplying this by 3, the result is the answer required.

Operation.

$$\begin{array}{rl} \$67.35 & \text{prin.} \\ .065 & \text{rate.} \\ \hline 33675 & \\ 40410 & \\ \hline \$4.37775 & \text{int. 1 y.} \\ 3 & \text{yrs.} \\ \hline \$13.13325 & \text{int. 3 y.} \end{array}$$

QUEST.—237. In reckoning interest, what part of a year is a month considered? How many days are commonly considered a month? *Obs.* What is the origin of this practice? Is it accurate?

3. What is the interest of \$116.15 for 1 year, 1 month, and 15 days, at 7 per cent.? What is the amount?

Suggestion.—First find the interest for 1 year as above; then since 1 month is $\frac{1}{12}$ of a year, take $\frac{1}{12}$ of 1 year's interest for the given month. Again, since 15 days are $\frac{1}{2}$ of a month, take $\frac{1}{2}$ of 1 month's interest for the given days. Now adding the int. for 1 year, 1 m., and 15 days together, gives the interest for the whole time. Adding the principal to the interest gives the amount.

Operation.

	\$116.15	prin.
	.07	rate.
12)	\$8.1305	int. 1 y.
2)	.6775	" 1 m.
	.3387	" 15 d.
	\$9.1467	interest.
	\$116.15	prin. add.
	\$125.2967	amount.

241. From the preceding illustrations and principles, we derive the following general

RULE FOR COMPUTING INTEREST.

I. FOR ONE YEAR. *Multiply the principal by the given rate, and from the product point off as many figures for decimals, as there are decimal places in both factors.*

II. FOR TWO OR MORE YEARS. *Multiply the interest of* 1 *year by the given number of years.*

III. FOR MONTHS. *Take such a fractional part of* 1 *year's interest, as is denoted by the given number of months.*

IV. FOR DAYS. *Take such a fractional part of* 1 *month's interest, as is denoted by the given number of days.*

The AMOUNT *is found by adding the interest to the principal.*

OBS. 1. The *reason* of this rule may be seen from the analysis of the preceding examples.

2. When the rate per cent. is *less* than 10, a cipher must always be prefixed to the figure denoting it. (Art. 223. Obs. 1.)

3. For 1 month take $\frac{1}{12}$ of 1 year's interest; for 2 months, $\frac{1}{6}$; for 3 months, $\frac{1}{4}$; for 4 months, $\frac{1}{3}$; for 6 months, $\frac{1}{2}$; &c.

4. For 1 day take $\frac{1}{30}$ of 1 month's interest; for 2 days, $\frac{1}{15}$; for 3 days, $\frac{1}{10}$; for 10 days $\frac{1}{3}$; for 20 days, $\frac{2}{3}$; &c. (See Table of aliquot parts of Time, p. 149.)

4. What is the interest of \$75.21 for 1 year, at 6 per cent.?
\$4.5126. *Ans.*

QUEST.—241. What is the general rule for computing interest for a year? How find the interest for any number of years? How for months? How for days? How find the amount?

5. What is the interest of $100 for 1 year, at 5 per cent.? at 6 per cent.? at 4 per cent.? at 7 per cent.?

6. What is the interest of $35.31 for 1 year, at 6 per cent.?

7. What is the interest of $50.10 for 1 year, at 7 per cent.?

8. What is the interest of $63 for 1 year, at $5\frac{1}{2}$ per cent.?

9. What is the interest of $136.75 for 1 year, at $4\frac{1}{2}$ per cent.?

10. What is the interest of $260.61 for 1 year, at 6 per cent.? What is the amount? *Ans.* $15.636 int. $276.246 amount.

11. What is the interest of $140.25 for 1 year, at 7 per cent.? What is the amount?

12. What is the interest of $163.40 for 1 year, at 8 per cent.? What is the amount?

13. What is the interest of $400 for 1 year, at 6 per cent.? What is the amount?

14. What is the amount of $500 for 1 year, at 7 per cent.?

15. What is the amount of $1000 for 1 year, at 8 per cent.?

16. What is the interest of $100 for 3 years, at 6 per cent. per annum? *Ans.* $18.

17. At 5 per cent. per annum, what is the interest of $45 for 4 years? *Ans.* $9.

18. At 6 per cent., what is the interest of $200 for 5 years? What is the amount?

19. At 7 per cent., what is the interest of $250 for 10 years? What is the amount?

20. At 8 per cent., what is the interest of $340.50 for 3 years? What is the amount?

21. At 6 per cent. per annum, what is the interest of $100 for 1 month? *Ans.* $.50.

22. At 5 per cent., what is the interest of $600 for 6 months?

23. At 7 per cent., what is the interest of $250 for 4 months?

24. What is the interest of $375.31 for 3 mo. at 6 per cent.?

25. What is the interest of $60 for 7 months, at 8 per cent.? What is the amount?

26. What is the interest of $96 for 10 months, at 6 per cent.? What is the amount?

27. At 6 per cent., what is the interest of $600 for 1 day?

28. At 4 per cent., what is the interest of $470 for 10 days?

29. What is the interest of $1000 for 1 y. 1 m. and 1 d., at 6 per cent.?

30. What is the interest of \$42.50 for 2 years and 6 months, at 7 per cent.?

31. What is the interest of \$69.46 for 1 year and 8 months, at 8 per cent.?

32. What is the interest of \$45.23 for 1 year and 2 months, at 5 per cent.?

33. What is the interest of \$43.01 for $2\frac{1}{2}$ years, at 7 per cent.?

34. What is the interest of \$215.135 for 2 years and 8 months, at 6 per cent.?

35. At 8 per cent., what is the interest of \$75.98 for 3 years?

36. At $5\frac{1}{2}$ per cent., what is the interest of \$939 for 4 years?

37. At 6 per cent., what is the interest of \$137.50 for 6 months and 3 days.

38. At 7 per cent., what is the interest of \$1500 for 10 days?

39. At 20 per cent., what is the interest of \$3000 for 3 days?

40. At $12\frac{1}{2}$ per cent., what is the interest of \$1736.25 for 6 months?

41. What is the amount of \$8367.29 for 3 months and 17 days, at 7 per cent.?

42. At 6 per cent. what is the amount of \$4565.61 for 4 months and 7 days?

43. What is the interest of \$5625.43 for 4 months and 18 days, at $6\frac{1}{4}$ per cent.?

44. At $5\frac{1}{2}$ per cent. what is the interest of \$624.625 for 7 months 3 days?

45. At 8 per cent. what is the interest of \$11261.18$\frac{3}{4}$, for 3 months and 3 days?

46. At 7 per cent. what is the interest of \$9208.95 for 11 months and 5 days? The amount?

47. What is the interest of \$15206.843, at $7\frac{1}{2}$ per cent. for 1 year, 8 months, and 25 days? The amount?

48. What is the interest of \$10050.69, at $5\frac{1}{2}$ per cent. for 2 years, 9 months, and 5 days? The amount?

49. What is the interest of \$11607.858, at 7 per cent. for 3 years, 6 months, and 9 days? The amount?

50. What is the interest of \$41361.18, at 6 per cent. for 5 years, 7 months, and 3 days? The amount?

SECOND METHOD OF COMPUTING INTEREST.

242. Since the interest of $1 at 6 per cent., is 6 cents for 1 year, for 1 mo. it must be $\frac{1}{12}$ of 6 cents, 5 mills; for 2 mo. 10 mills or 1 cent, &c. Hence,

243. The interest of $1 at 6 per cent. is 1 cent for every 2 months, or *half* as many *cents* as there are months in the given time.

244. Again, 6 days are 1 *fifth* of 30 days; therefore the interest of $1 for 6 days must be $\frac{1}{5}$ of the interest for 30 days. But the interest of $1 for 30 days, (1 mo.) at 6 per cent., is 5 mills; consequently, the interest of $1 for 6 days, is $\frac{1}{5}$ of 5 mills, which is 1 mill; for 12 days, (2 times 6,) it is twice as much, or 2 mills, &c. Hence,

245. The interest of $1 at 6 per cent. is 1 mill for every 6 days, or one *sixth* as many *mills* as there are *days* in the time.

OBS. When the number of days is 6 and less than 60, in finding 1 sixth of them, write the first quotient figure in *thousandths'* place; if less than 6, in *ten thousandths'* place; if 60 and under 600, in *hundredths'* place.

1. What is the interest of $60 for 1 year 5 months and 3 days, at 6 per cent.?

		First Method.
Suggestion.—For 1 y. the int. of $1 is	$.06	$60.00
For 5 m: " "	$.025	.0855
For 3 d. " "	$.0005	30000
Hence, the int. of $1 for the time is,	$.0855	30000
Now if the interest of $1 for the given time is $.0855, the interest of $60 will be 60 times as much, which is $5.13.		48000
		$5.130000 *Ans.*

	Second Method.
Or, since the interest of $1 at 6 per cent. is *half* as many cents as there are months in the given time, we may multiply by half the number of months, which is $8\frac{11}{20}$, and pointing off two figures for decimals, the result is $5.13, the same as before.	$60
	$8\frac{11}{20}$
	480
	33
	$5.13 *Ans.*

QUEST.—243. What is the interest of $1 at 6 per cent. for any number of months? 245. For any number of days?

246. From the preceding illustrations we derive a

SECOND RULE FOR COMPUTING INTEREST.

Multiply the principal by the interest of $1 *for the given time, and point off the product as before.* (Art. 241.)

Or, multiply the principal by half the number of months, and point off two more decimals in the product than there are decimal figures in the multiplicand.

OBS. 1. When the latter method is employed, the years must be reduced to months, and the days to the fraction of a month, then take *half* of them.

Or, divide the days by 6, and add the quotient to half the number of months, regarding both as decimals. (Arts 243, 245.)

2. The interest at any other rate, *greater* or *less* than 6 per cent., may be found by adding to, or subtracting from the interest at 6 per cent., such a *fractional part of itself*, as the required rate exceeds or falls short of 6 per cent. Thus, if the required rate is 7 per cent., first find the interest at 6 per cent., then add $\frac{1}{6}$ of it to itself; if 5 per cent., subtract $\frac{1}{6}$ of it from itself, &c.

3. The *reason* of the first part of the above rule, is obvious from the fact that the interest of any sum for any specified time, must be as many times the interest of $1, as there are dollars in the given principal.

The *reason* that multiplying the principal by *half* the number of months will give the true interest, is because the interest of $1, at 6 per cent., is 1 cent for every *two months*.

2. What is the interest of $261.25, at 6 per cent. for 7 months and 12 days? *Ans.* $9.66625.

3. What is the interest of $268.40 at 7 per cent. for 8 months?

4. What is the interest of $217.375, at 5 per cent. for 6 months?

5. What is the interest of $169.25, at 6 per cent. for 7 months?

6. What is the interest of $350.192 for 8 months and 15 days, at 6 per cent.?

7. What is the interest of $68.29 at 7 per cent. for 7 months and 13 days?

8. What is the interest of $96.17 at 5 per cent. for 11 months and 20 days?

QUEST.—246. What is the second method of computing interest? *Obs.* When the rate is greater or less than 6 per cent., how proceed? How does it appear that multiplying the principal by the interest of $1 for the time, will give the true interest? How does it appear that multiplying by half the number of months will give the true interest?

9. What is the interest of $168 for 2 years, 7 months, and 20 days, at 6 per cent.?

10. What is the interest of $173 for 3 years, 1 month, and 15 days, at 6 per cent.?

11. What is the interest of $145 for 6 months and 24 days, at 6 per cent.?

12. What is the interest of $461.318 for 1 year, and 11 months, at 7 per cent.?

13. What is the interest of $300 for 4 months and 18 days, at 7 per cent.?

14. At 5 per cent., what is the interest of $256.25 for 9 months and 15 days?

15. What is the interest on a note of $450 from Jan. 1st, 1844, to March 13th, 1845, at 6 per cent.?

Note.—When it is required to compute the interest on a *note*, first find the *time* for which the note has been on interest, by subtracting the *earlier* from the *later* date, then proceed as before. (Art. 170.)

	Operation.	$450 principal.
	Yr. mo. d.	.072 int. of $1 for the time.
	1845 " 3 " 13	900
	1844 " 1 " 1	31 50
Time	1 " 2 " 12	$32.400 *Ans.*

16. What is the interest on a note of $753.28 from April 15th, 1853, to Oct. 13th, 1853, at 7 per cent.?

17. What is the interest on a note of $631.375 from Nov. 16th, 1851, to June 10th, 1852, at 7 per cent.?

18. What is the interest on a note of $729.93 from March 7th, 1850, to Dec. 14th, of the same year, at 5 per cent.?

COMPUTING INTEREST BY DAYS.

247. The preceding rules for computing interest for months and days, are based upon the supposition that 30 days make a month, and 12 months a year. This we have seen involves a slight inaccuracy; for some months have 31 days, while Feb. has only 28; and 12 times 30 days make but 360 days, whereas a year contains 365 days. (Art. 237.) Now as interest is reckoned at a certain per cent. *per annum* on the money lent, it is plain the exact interest for 1 day, is $\frac{1}{365}$ of 1 *year's interest*; for 2 days, $\frac{2}{365}$; for 3 days, $\frac{3}{365}$, &c. Hence,

247.*a*. To compute interest by *days*, at any given per cent.

Find the interest for one year at the given rate, then multiply this by the number of days, and divide the product by 365; *the result will be the interest required.*

Or, multiply the principal by the whole number of days, and divide the product by 6000; *the result will be the interest at* 6 *per cent.*

OBS. 1. If the rate is 7 per cent., *add* 1 sixth; if 5 per cent., *subtract* 1 sixth, &c. (Art. 246. Obs. 2.)

2. The latter method is the same as multiplying by 1 *sixth* of the days, and pointing off *three* more figures for decimals than there are decimal places in the principal. It is based on the supposition that 30 days make a month, and 12 months a year.

3. The difference between 12 months of 30 days each and a common year, is $\frac{5}{365}$ or $\frac{1}{73}$; therefore, reckoning interest by months of 30 days, gives $\frac{1}{73}$ too much. On small sums, the error is so trifling it is commonly disregarded.

19. Find the interest of $473 by days, from March 20th to the 8th of July following, at 7 per cent.

Suggestion.—We may find the number of days by the Table p. 146, or in the following manner:

In March	there	are	11	days.
In April	"	"	30	"
In May,	"	"	31	"
In June	"	"	30	"
In July	"	"	8	"
Whole number,			110	days.

The answer is $9.978.

Operation.

$$\begin{array}{r l}
\$473 & \text{prin.} \\
.07 & \text{rate.} \\
\hline
33.11 & \text{int. 1 y.} \\
110 & \text{days.} \\
\hline
365)3642.10(\$9.978 & \\
3285 & \\
\hline
3571 & \\
3285 & \\
\hline
2860 & \\
2555 & \\
\hline
\end{array}$$

20. What is the interest of $519.38, at 6 per cent. for 65 days?

21. What is the int. of $761.56 for 217 days, at 7 per cent.?

22. What is the int. of $850.395 for 318 days, at 5 per cent.?

23. What is the amt. of $781.13, at 7 per cent. for 35 days?

24. What is the amt. of $806.28, at 6 per cent. for 46 days?

25. What is the interest of $923.68, at 7 per cent. from June 16th to 21st of the following Sept.?

QUEST.—247.*a*. How compute interest by days, at any given per cent.? *Obs.* What is the difference between a common year, and 12 months of 30 days each?

26. At 6 per cent., what is the amount of $673.21 from April 3d to the 15th of Sept. following?

27. At 7 per cent., what is the amount of 709.81 from Nov. 17th to the 20th of the following Feb.?

28. What is the amount of $1563.875 from the 9th of Jan. to the 13th of the following Dec., at 7 per cent.?

29. What is the amount of $1604.17 from March 5th, 1852, to Jan. 7th, 1853, at 7 per cent.?

30. What is the amount of $2043.63, at 6 per cent. from March 4th, 1851, to June 19th, 1853?

EXAMPLES FOR PRACTICE.

1. What is the interest of $45.25 for 8 months, at 6 per cent.?

2. What is the interest of $167.375 for 6 months, at 6 per cent.?

3. What is the interest of $93.86 for 3 months and 15 days, at 6 per cent.?

4. What is the interest of $110 for 1 month and 20 days, at 6 per cent.?

5. At 7 per cent., what is the interest of $158.91 for 1 year and 3 months?

6. At 7 per cent., what is the amount of $217 for 1 year and 8 months?

7. At 6 per cent., what is the amount of $348.10 for 2 years and 1 month?

8. At 7 per cent., what is the interest of $400 for 1 year and 6 months?

9. At 7 per cent., what is the amount of $213.01 for 9 months?

10. At 5 per cent., what is the amount of $603 for 2 years and 5 months?

11. What is the amount of $861 for 8 months and 24 days, at 6 per cent.?

12. What is the amount of $1236 for 3 months and 14 days, at 7 per cent.?

13. What is the interest of $1400 for 1 year, 1 month, and 9 days, at 7 per cent.?

14. What is the interest of $469.20 for 27 days, at 8 per cent.?

15. What is the amount of \$705 for 5 years, at 9 per cent.?

16. What is the amount of \$1000 for 10 years, at 5 per cent.?

17. What is the amount of \$1650.06 for 20 years, at 7 per cent.?

18. What is the amount of \$2500 for 7 years, at 15 per cent.?

19. At $4\frac{1}{2}$ per cent., what is the interest of \$17000 for $1\frac{1}{2}$ years?

20. At $7\frac{1}{4}$ per cent., what is the interest of \$1625.81 for 45 days?

21. At $12\frac{1}{2}$ per cent., what is the amount of \$165.18 for 33 days?

22. At 7 per cent., what is the amount of \$8531 for 63 days?

23. At 6 per cent., what is the amount of \$16021 for 93 days?

24. What is the interest on a note of \$65, dated Jan. 10th, 1844, to May 16th, 1845, at 6 per cent.?

25. What is the interest of \$170 from June 19th, 1840, to July 1st, 1841, at 7 per cent.?

26. What is the interest of \$105.63 from Feb. 22d, 1839, to Aug. 10th, 1840, at 5 per cent.?

27. What is the interest of \$234 from April 10th, 1834, to Oct. 1st, 1835, at 6 per cent.?

28. What is the interest of \$195.22 from June 25th, 1838, to March 31st, 1840, at 6 per cent.?

29. What is the interest of \$391 from Sept. 1st, 1840, to Nov. 30th, 1841, at 8 per cent.?

30. What is the interest of \$510.83 from March 21st, 1842, to Dec. 30th, 1842, at 7 per cent.?

31. At 6 per cent., what is the interest of \$469.65 from August 10th, 1843, to Feb. 6th, 1844?

32. At 7 per cent., what is the amount due on a note of \$285, dated March 15th, 1844, and payable Sept. 18th, 1845?

33. At 6 per cent., what is the amount due on a note of \$391, dated Oct. 9th, 1844, and payable March 1st, 1845?

34. At 5 per cent., what is the amount of \$623 from Feb. 19th, 1844, to Aug. 10th, 1844?

35. At 4 per cent., what is the amount of \$589.20 from January 10th, 1844, to January 13th, 1845?

36. At 4 per cent. what is the amount of \$731.27 from July 1st, 1844, to April 4th, 1845?

+ 3 times

37. What is the interest of $849 from July 4th, 1841, to July 7th, 1845, at 6 per cent.?

38. What is the interest of $966 from Jan. 1st, 1842, to March 20th, 1844, at 7 per cent.?

39. What is the interest of $1539 from May 21st, 1842, to Aug. 19th, 1843, at 6 per cent.?

40. What is the amount of $1100 from June 15th, 1840, to Aug. 3d, 1845, at 5 per cent.?

41. What is the amount of $1 for 50 years, at 6 per cent.?

42. What is the amount of one cent for 500 years, at 7 per cent.?

PARTIAL PAYMENTS.

248. When *partial payments* are made and endorsed upon Notes and Bonds, the *Rule* for computing the interest adopted by the *Supreme Court of the United States*, is the following:

I. *Cast the interest on the principal to the time of the first payment; if the payment exceeds the interest, subtract the excess from the principal, and cast the interest on the balance to the time of the next payment.*

II. *If the payment is less than the interest, cast the interest on the former principal to the time of the next payment, or until the sum of the payments exceeds the interest due; then subtract the excess from the principal, and proceed as before.*

Note.—The above rule is adopted by *New York*, *Massachusetts*, and most of the other States of the Union.—*Johnson's Chancery Reports*, Vol. I. p. 17.

$850. WASHINGTON, Jan. 1st, 1841.

43. For value received, I promise to pay George Howland, or order, eight hundred and fifty dollars, on demand, with interest at 6 per cent.

JOHN HAMILTON.

The following payments were endorsed on this note:

July 1st, 1841, received $100.62.
Dec. 1st, 1841, received $15.28.
Aug. 13th, 1842, received $175.75.

What was due on taking up the note, Jan. 1st, 1843?

QUEST.—248. What is the general method of casting interest on Notes and Bonds, when partial payments have been made? When the payment is less than the interest, how proceed?

Operation.

Principal, ...		$850.00
Interest to first payment, July 1st, (6 months,)		25.50
Amount due on note July 1st, 1841, ...		$875.50
1st payment, (to be deducted from amount) . .		100.62
Balance due after 1st pay't July 1st, 1841, . .		$774.88
Int. on bal. to 2d pay't Dec. 1st, (5 mo.,) . . .		$19.37
2d pay't is less than int. then due, . . .	$15.28	
Int. continued on bal. from Dec. 1st, 1841, to 3d. pay't, (8 mo. 12 d.,) . . .		82.54
Amount, Aug. 13th, 1842, ...		$826.79
3d payment, Aug. 13th, 1842	175.75	
Sum of payments deducted from am. . . .		191.03
Balance due Aug. 13th, ...		$635.76
Int. on bal. to Jan. 1st, (4 mo., 18 d.,)		14.62
Bal. due on taking up the note, Jan. 1st, 1843, .		$650.38

$500. NEW YORK, May 10th, 1842.

44. For value received, I promise to pay James Monroe, or order, five hundred dollars on demand, with interest at 7 per cent.

HENRY SMITH.

The following sums were endorsed upon it:

Received, Nov. 10th, 1842, $75.
Received, March 22d, 1843, $100.

What was due on taking up the note, Sept. 28th, 1843?

$692.35. BOSTON, Aug. 15th, 1843.

45. Three months after date, I promise to pay John Warren, or order, six hundred and ninety-two dollars and thirty-five cents, with interest at 6 per cent., value received.

SAMUEL JOHNSON.

Endorsed Nov. 15th, 1843, $250.375.
" March 1st, 1844, $65.625.

How much was due on the note, July 4th, 1845?

$1000. PHILADELPHIA, June 20th, 1841.

46. Six months after date, I promise to pay Messrs. Carey, Hart & Co., or order, one thousand dollars, with interest at 5 per cent., value received.

HORACE PRESTON.

Endorsed Jan. 10th, 1844, $125.
" June 16th, 1844, $93.
" Feb. 20th, 1845, $200.

What was the balance due on the note, Aug. 1st, 1845?

CONNECTICUT RULE.

249. I. "Compute the interest on the principal to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after-payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner, from one payment to another, till all the payments are absorbed; *provided* the time between one payment and another be one year or more."

II. "If any payments be made before one year's interest has accrued, then compute the interest on the principal sum due on the obligation, for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above."

III. "If a year extends beyond the time of final settlement, find the amount of the principal remaining unpaid up to the time of settlement, likewise the amount of the endorsements from the time they were paid to the time of settlement, and deduct the sum of these several amounts from the amount of the principal."

"If any payment be made of a less sum than the interest accruing at the time of such payment, no interest is to be computed on this excess of interest, but the interest is to be continued on the principal until the next payment, and so on till the payments exceed the interest."—*Kirby's Reports.*

$1000. NEW HAVEN, Feb. 4th, 1846.

47. For value received, I promise to pay William Anderson, or order, one thousand dollars on demand, with interest at 6 per cent.

GEORGE PIERSON.

The following payments were endorsed on this note:

1st pay't March 19th, 1847, received $200.
2d " Nov. 19th, 1847, received $350.
3d " Sept. 19th, 1849, received $15.

How much was due on taking up the note, April 7th, 1850?

Operation.

Principal, ...	$1000.000
Int. to 1st pay't March 19th, 1847, (1 yr. 1 mo. 15d.)	67.500
Amt. due on note March 19th, 1847, ...	$1067.500
1st payment, (to be deducted from amount,) . .	200.000
Bal. due March 19th, 1847, ...	$867.500
Int. on bal. for 1 yr. (2d pay't being made before 1 year's int. had accrued,) . .	52.050
Amt. due March 19th, 1848, ...	$919.550
Amt. of 2d pay't to end of y. Mar. 19th, 1848, (4 m.)	357.000
Bal. due March 19th, 1848, ...	$562.550
Int. on bal. to 3d pay't Sept. 19th, 1849, (1 yr. 6 mo.)	50.630
Amt. due Sept. 19th, 1849, ...	$613.180
3d payment (less than int. then due,) deducted, .	15.000
Balance due Sept. 19th, 1849, ...	$598.180
Int. continued on former prin. to time of settlement, April 7th, 1850, (6 mo. 18 d.) .	18.564
Amount due on taking up the note, April 7th, 1850,	$616.744

THIRD RULE.

249.a. First find the amount of the given principal for the whole time; then find the amount of each of the several payments from the time it was endorsed to the time of settlement. Finally, subtract the amount of the several payments from the amount of the principal, and the remainder will be the sum due.

OBS. 1. For short periods and small sums, this rule is convenient, and perhaps sufficiently accurate for common purposes; but when the interest is to be paid annually, and the notes are large and for long periods, it leads to grave errors.

2. It will be an excellent exercise for the pupil to cast the interest on the preceding notes by each of the above rules, and compare the results.

$416. ALBANY, March 21st, 1850.

48. On demand, I promise to pay to the order of Henry Patton, four hundred and sixteen dollars, with interest at 7 per cent., value received.

JOHN MARSHALL.

Received on the above note the following sums:

June 15th, 1850, $35.00.
Oct. 9th, 1850, 23.00.
Jan. 12th, 1851, 68.00.

What was due on the note, Sept. 21st, 1851?

Operation.

Principal, ...		$416.000
Int. at 7 per cent. to Sept. 21st, 1851, (1 y. 6 mo.) .		43.680
Amt. of note to settlement Sept. 21st, 1851, . .		$459.680
Amt. 1st pay't (1 y. 3 mo. 6 d.) . .	$38.103	
Amt. 2d " (11 mo. 12 d.) . .	24.530	
Amt. 3d " (8 mo. 9 d.) . . .	71.292	$133.925
Balance due Sept. 21st, 1851, ...		$325.755

INTEREST ON STERLING MONEY.

47. What is the int. of £175, 10s. 6d. for 1 y., at 5 per cent.?

We first reduce the 10s. 6d. to the decimal of a pound, (Art. 200,) then multiply the principal by the rate, and point off the product as before. The 8 on the left of the point is pounds, the figures on the right are decimals of a pound, and must be reduced to shillings, pence, and farthings. (Art. 201.) Hence,

£175.525 prin.
.05 rate.
£8.77625 int. 1 year.
20
15.52500 s.
12
6.30000 d.
4
1.20000 far.
Ans. £8, 15s. $6\frac{1}{4}$d

250. To compute interest on pounds, shillings, pence, &c.

Reduce the given shillings, pence and farthings, to the decimal of a pound; then proceed as in Federal money. (Art. 200.) *The figures on the left of the decimal point, are pounds; those on the right are decimals of a pound, and must be reduced to shillings, pence, and farthings.* (Art. 201.)

48. What is the interest of £56, 15s. for 1 year and 6 months, at 6 per cent.? *Ans.* £5, 2s. $1\frac{3}{4}$d.

49. What is the interest of £75, 12s. 6d. for 1 year and 3 months, at 7 per cent.?

50. What is the interest of £96, 18s. for 2 years and 6 months, at $4\frac{1}{2}$ per cent.?

51. What is the amount of £100 for 2 years and 4 months, at 5 per cent.?

52. What is the amount of £430, 16s. 10d. for 1 year and 5 months, at 6 per cent.?

QUEST.—250. What is the rule for computing interest on sterling money?

PROBLEMS IN INTEREST.

251. Calculations in interest require particular attention to the following *parts* or *terms*, viz: *the principal, the rate per cent., the time, the interest, and the amount.* These *parts* or *terms* have such a relation to each other, that if any three of them are given, the *others* may be found. These operations give rise to the following *Problems:*

OBS. 1. A *Problem*, in its common acceptation, is a question to be solved.

2. A number or quantity is said to be *given*, when its value is stated, or may be directly *inferred* from something else which is given. Thus, when the principal and interest are given, the *amount* may be said to be *given*, because it is merely the *sum* of the principal and interest. So, if the principal and the amount are given, the *interest* may be said to be *given*, because it is the *difference* between the principal and the amount.

PROBLEM I.

252. *To find the* INTEREST, *the principal, rate per cent., and the time being given.*

This problem embraces the preceding examples in Interest, and has already been illustrated. (Arts. 241, 246.)

PROBLEM II.

To find the RATE PER CENT., *the principal, the interest, and the time being given.*

1. A man loaned $75 for 4 years, and received $24 interest: what was the rate per cent.?

Analysis.—The interest of $75 at 1 per cent. for 1 year, is $.75; consequently, for 4 years it is $.75×4=$3. Now since $3 is 1 per cent. interest on the principal for the given time, $24 must be $\frac{24}{3}$ of 1 per cent., which is equal to 8 per cent. (Art. 121.)

Or, we may reason thus: If $3 is 1 per cent. on the principal for the given time, $24 must be as many per cent. as $3 is contained times in $24; and $24÷3=8. The rate was therefore 8 per cent. Hence,

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QUEST.—251. What are the parts or terms which require attention in interest? Are all of these parts given? How many of them must be given to find the others? *Obs.* What is a problem? When is a number or quantity said to be given? 252. What terms are given when it is required to find the interest?

253. To find the *rate per cent.* when the principal, interest, and time are given.

Divide the given interest by the interest of the principal at 1 *per cent. for the given time, and the quotient will be the required per cent.*

Or, find the interest of the principal at 1 *per cent. for the given time; then make the interest thus found the denominator and the given interest the numerator of a common fraction; reduce this fraction to a whole or mixed number, and the result will be the per cent. required.* (Art. 121.)

2. If I borrow $300 for 2 years, and pay $42 interest, what rate per cent. do I pay?

Suggestion.—The interest of $300 for 2 years at 1 per cent., is $6. (Art. 238.)

Operation.
$6)$42
Ans. 7 per ct.

PROOF.—$300 × .07 × 2 = $42.

3. If I borrow $460 for 3 years, and pay $82.80 interest, what is the rate per cent.?

4. A man loaned $500 for 8 months, and received $40 interest: what was the rate per cent.?

5. At what rate per cent. must $450 be loaned, to gain $56.50 interest in 1 year and 6 months?

6. At what per cent. must $750 be loaned, to gain $225 in 4 years?

7. A man has $8000 which he wishes to loan for $600 per annum for his support: at what per cent. must he loan it?

8. A gentleman deposited $1250 in a savings bank, for which he received $31.25 every 6 months: what per cent. interest did he receive on his money?

9. A capitalist invested $9260 in railroad stock, and drew a semi-annual dividend of $416.70: what rate per cent. interest did he receive on his money?

10. A man built a hotel at an expense of $175000, and rented it for $8750 per year: what per cent. interest did his money yield him?

QUEST.—253. When the principal, interest, and time are given, how is the rate per cent. found?

PROBLEM III.

To find the PRINCIPAL, *the interest, the rate per cent., and the time being given.*

11. What sum must be put at interest, at 6 per cent., to gain \$30 in two years?

Analysis.—The interest of \$1 for two years at 6 per cent., (the given time and rate), is 12 cents. Now 12 cents interest is $\frac{12}{100}$ of its principal \$1; consequently, \$30 the given interest, must be $\frac{12}{100}$ of the principal required. The question therefore resolves itself into this: \$30 is $\frac{12}{100}$ of what number of dollars? If \$30 is $\frac{12}{100}$, $\frac{1}{100}$ is $\frac{1}{12}$ of \$30, which is $\$2\frac{1}{2}$; and $\frac{100}{100}=\$2\frac{1}{2}\times100$, which is \$250, the principal required.

Or, we may reason thus: Since 12 cents is the interest of 1 dollar for the given time and rate, 30 dollars must be the interest of as many dollars for the same time and rate, as 12 cents is contained times in 30 dollars. And $\$30\div.12=250$. The principal was therefore \$250. Hence,

254. To find the *principal*, when the interest, rate per cent., and time are given.

Divide the given interest by the interest of \$1 *for the given time and rate, expressed in decimals; and the quotient will be the principal required.*

Or, make the interest of \$1 *for the given time and rate, the numerator, and* 100 *the denominator of a common fraction; then divide the given interest by this fraction, and the quotient will be the principal required.* (Art. 141.)

12. What sum put at interest will produce \$13.30 in 6 months, at 7 per cent.?

Suggestion.—The interest of \$1 for 6 months at 7 per cent. is \$.035.

PROOF.—$\$380\times.07\times\frac{1}{2}=\$13.30$.

Operation.

$\$.035)\$13.300$

Ans. \$380

13. A father bequeaths his son \$500 a year: what sum must be invested, at 5 per cent. interest, to produce it?

QUEST.—254. When the interest, rate per cent., and time are given, how is the principal found?

14. What sum must be put at 6 per cent. interest, to gain $350 interest semi-annually?

15. A gentleman loaned his money at 7 per cent., and received $1200 interest a year: how much was he worth?

PROBLEM IV.

To find the TIME, *the principal, the interest, and the rate per cent. being given.*

16. A man loaned $80 at 5 per cent., and received $10 interest: how long was it loaned?

Analysis.—The interest of $80 at 5 per cent. for 1 year is $4. Now, since $4 interest requires the principal 1 year at the given per cent., $10 interest will require the same principal $\frac{10}{4}$ of 1 year, which is equal to $2\frac{1}{2}$ years. (Art. 121.)

Or, we may reason thus: If $4 interest requires the use of the given principal 1 year, $10 interest will require the same principal as many years as $4 are contained times in $10. And $10÷4=2.5. *Ans.* 2.5 years.

255. To find the *time* when the principal, interest, and rate per cent. are given.

Divide the given interest by the interest of the principal at the given rate for 1 *year, and the quotient will be the time.*

Or, make the given interest the numerator, and the interest of the principal for 1 *year at the given rate the denominator of a common fraction, which being reduced to a whole or mixed number, will give the time required.*

OBS. If the quotient contains a decimal of a year, it should be reduced to months and days. (Art. 201.)

17. How long will it take $100, at 5 per cent., to double itself; that is, to gain $100 interest?

Suggestion.—The interest of $100 for 1 year, at 5 per cent., is $5.

Operation.

$5)$100

Ans. 20 years.

PROOF.—$100 × .05 × 20=$100. (Art. 238.)

18. In what time will $500, at 6 per ct. gain $100 interest?

QUEST.—255. When the principal, interest, and rate per cent. are given, how is the time found? *Obs.* When the quotient contains a decimal of a year, what should be done with it?

TABLE.

Showing in what time any given principal will double itself at any rate, from 1 to 20 per cent. Simple Interest.

Per cent.	Years.	Per cent.	Years.	Per cent.	Years.	Per cent.	Years.
1	100	6	$16\frac{2}{3}$	11	$9\frac{1}{11}$	16	$6\frac{1}{4}$
2	50	7	$14\frac{2}{7}$	12	$7\frac{1}{3}$	17	$5\frac{15}{17}$
3	$33\frac{1}{3}$	8	$12\frac{1}{2}$	13	$7\frac{9}{13}$	18	$5\frac{5}{9}$
4	25	9	$11\frac{1}{9}$	14	$7\frac{1}{7}$	19	$5\frac{5}{19}$
5	20	10	10	15	$6\frac{2}{3}$	20	5

19. How long will it take $100, at 6 per ct., to double itself?
20. How long will it take $100, at 7 per ct., to double itself?
21. How long will it take $7250, at 10 per ct., to double itself?

Note.—For method of finding the *principal*, when the *rate per cent.*, the *time*, and the *amount*, are given, see Discount. (Art. 260.)

COMPOUND INTEREST.

256. *Interest* is of two kinds, *Simple* and *Compound*.

Simple Interest is that which is reckoned on the *principal* only, as in the preceding articles.

Compound Interest is reckoned not only on the *principal*, but also on the *interest* as it becomes due. It may therefore be called *interest* upon *interest*.

Ex. 1. What is the compound interest of $500 for 3 years, at 6 per cent.?

Operation.

Principal, ...	$500.00
Interest for 1st year $500 × .06 equals	30.00
Amount for 1 year, ...	$530.00
Interest for 2d year $530 × .06 equals	31.80
Amount for 2 years, ...	$561.80
Interest for 3d year $561.80 × .06 equals	33.70
Amount for 3 years, ...	$595.50
Principal deducted, ...	500.00
Compound interest for 3 years, ...	$95.50

QUEST.—256. Of how many kinds is interest? What is simple interest? What is compound interest?

257. Hence, to calculate *compound* interest.

Cast the interest on the given principal for 1 *year, or specified time, and add it to the principal; then cast the interest on this amount for the next year, or specified time, and add it to the principal as before. Proceed in this manner with each successive year, or period of the proposed time.*

Finally, subtract the given principal from the last amount, and the remainder will be the compound interest.

OBS. Interest for months and days must be cast on the last amount, and be added to it, before subtracting the principal.

2. What is the com. int. of $350 for 4 years, at 6 per cent.?

3. What is the com. int. of $865 for 5 years, at 7 per cent.?

4. What is the amount of $250 for 6 years, at 5 per cent. compound interest?

5. What is the amount of $1000 for 3 years, at 4 per cent. compound interest, payable semi-annually?

6. What is the amount of $1200 for 2 years, at 6 per cent. compound interest, payable quarterly?

7. What is the amount of $800 for 3 years, at 5 per cent. compound interest, payable semi-annually?

8. What is the amount of $1500 for 5 years, 3 months and 6 days at 7 per cent. compound interest?

258. To calculate compound interest by the Table.

Find the amount of $1, *or* £1 *for the given number of years by the table, multiply it by the given principal, and the product will be the amount required.*

Subtract the principal from the amount thus found, and the remainder will be the compound interest.

10. What is the compound interest of $200 for 10 years, at 6 per cent.? What is the amount?

Operation.

Amount of $1 for 10 years by table, . .	$1.790848
The given principal, ...	200
Amount required, ...	$358.169000
Principal to be subtracted, ...	$200
Compound interest required, ...	$158.1696

QUEST.—257. What is the rule for calculating compound interest? 258. How is compound interest computed by the Table?

TABLE,

Showing the amount of $1, *or* £1, *at* 3, 4, 5, 6, *and* 7 *per cent. compound interest, for any number of years from* 1 *to* 35.

Years.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1.	1.030,000	1.040,000	1.050,000	1.060,000	1.07,000
2.	1.060,900	1.081,600	1.102,500	1.123,600	1.14,490
3.	1.092,727	1.124,864	1.157,625	1.191,016	1.22,504
4.	1.125,509	1.169,859	1.215,506	1.262,477	1.31,079
5.	1.159,274	1.216,653	1.276,282	1.338,226	1.40,255
6.	1.194,052	1.265,319	1.340,096	1.418,519	1.50,073
7.	1.229,874	1.315,932	1.407,100	1.503,630	1.60,578
8.	1.266,770	1.368,569	1.477,455	1.593,848	1.71,818
9.	1.304,773	1.423,312	1.551,328	1.689,479	1.83,845
10.	1.343,916	1.480,244	1.628,895	1.790,848	1.96,715
11.	1.384,234	1.539,454	1.710,339	1.898,299	2.10,485
12.	1.425,761	1.601,032	1.795.856	2.012,196	2.25,219
13.	1.468,534	1.665,074	1.885,649	2.132,928	2.40,984
14.	1.512,590	1.731,676	1.979,932	2.260,904	2.57,853
15.	1.557,967	1.800,944	2.078,928	2.396,558	2.75,903
16.	1.604,706	1.872,981	2.182,875	2.540,352	2.95,216
17.	1.652,848	1.947,900	2.292,018	2.692,773	3.15,881
18.	1.702,433	2.025,817	2.406,619	2.854,339	3.37,993
19.	1.753,506	2.106,849	2.526,950	3.025,600	3.61,652
20.	1.806,111	2.191,123	2.653,298	3.207,135	3.86,968
21.	1.860,295	2.278,768	2.785,963	3.399,564	4.14,056
22.	1.916,103	2.369,919	2.925,261	3.603,537	4.43,040
23.	1.973,587	2.464,716	3.071,524	3.819,750	4.74,052
24.	2.032,794	2.563,304	3.225,100	4.048,935	5.07,236
25.	2.093,778	2.665,836	3.386,355	4.291,871	5.42,743
26.	2.156,592	2.772,470	3.555,673	4.549,383	5.80,735
27.	2.221,289	2.883,369	3.733,456	4.822,346	6.21,386
28.	2.287,928	2.998,703	3.920,129	5.111,687	6.64,883
29.	2.356,566	3.118,651	4.116,136	5.418,388	7.11,425
30.	2.427,262	3,243,398	4.321,942	5.743,491	7.61,225
31.	2.500,080	3.373,133	4.538,039	6.088,101	8.14,571
32.	2.575,083	3.508,059	4.764,941	6.453,386	8.71,527
33.	2.652,335	3.648,381	5.003,189	6.840,590	9.32,533
34.	2.731,905	3.794,316	5.253,348	7.251,025	9.97,811
35.	2.813,862	3.946,089	5.516,015	7.686,087	10.6,765

Note.—The following examples may be solved by the Rule, or by the Table.

11. What is the amount of $1860 for 8 years, at 7 per cent. compound interest?

12. What is the amount of $20000 for 10 years, at 3 per cent. compound interest?

13. What is the amount of $3500 for 6 years, at 6 per cent. compound interest?

14. What is the amount of $350 for 12 years, at 4 per cent.?

15. What is the interest of $469 for 15 years, at 3 per cent.?

16. What is the interest of $500 for 24 years, at 6 per cent.?

17. What is the interest of $650 for 30 years, at 7 per cent.?

DISCOUNT.

259. DISCOUNT is an *allowance* made for the payment of money before it is *due.*

The *present worth* of a sum or debt, payable at a future time without interest, is the sum, which, if put at legal interest until it becomes due, *will amount to the debt.*

Ex. 1. What is the present worth of a debt of $583.15, payable in 1 year and 6 months without interest, when money is worth 6 per cent. per annum? What is the discount?

Analysis.—The amount of $1 for 1 year and 6 months, at 6 per cent. interest, is $1.09; that is, the amount is $\frac{109}{100}$ of the principal. The question then resolves itself into this: $583.15 is $\frac{109}{100}$ of what principal? If $583.15 is $\frac{109}{100}$, $\frac{1}{100}$ is 583.15÷109, or $5.35; and $\frac{100}{100}$=$5.35×100, which is $535.

Or, we may reason thus: Since $1.09 (amount) requires $1 principal for the given time, $583.15 (amount) will require as many dollars as $1.09 is contained times in $583.15. But $1.09 is the amount of $1 for the given time and rate. We therefore divide the debt by the amount of $1 for the time, and the quotient $535, is the present worth.

Operation.

```
$1.09)$583.15($535
       545
        381
        327
         545
         545

$583.15 the debt.
 535.00 pres. worth.
 $48.15 discount.
```

We then subtract the present worth from the debt, and it gives the discount required. Hence,

QUEST.—259. What is discount? What is the present worth of a debt, payable at some future time, without interest?

260. To find the *present worth* and the *discount* of a sum or debt, for any given time or rate.

Divide the given sum by the amount of $1, *for the given time and rate, and the quotient will be the present worth of the debt.*

Subtract the present worth from the given sum or debt, and the remainder will be the discount.

OBS. This process is often classed among the Problems of Interest. It is the same as if the *amount*, which answers to the given debt, the *rate per cent.*, and the *time* were given, to find the *principal*, which answers to the *present worth.*

2. What is the present worth of $250.38 payable in 8 months, when money is worth 6 per cent. per annum? What is the discount? *Ans.* $240.75 pres. worth; $9.63 dis.

PROOF.—$240.75 × .04=$9.63, the same as the answer.

3. What is the present worth of $475, payable in 1 year, when money is worth 7 per cent. per annum?

4. What is the present worth of $175, payable in 2 years, when money is worth 7 per cent. per annum?

5. What is the present worth of $1000, payable in 4 months, when the rate of interest is 6 per cent.?

6. What is the discount on $750, due 6 months hence, when interest is 5 per cent. per annum?

7. A man sold a farm for $1800, payable in 15 months: what is the present worth of the debt, allowing the rate to be 6 per cent.?

8. I have a note of $1150.33, payable in 9 months: what is its present worth, at 7 per cent. interest per annum?

9. A merchant sold goods amounting to $840.75, payable in 6 months: how much discount should he make for cash down, when money is worth 7 per cent.?

10. What is the discount on a draft of $2500, payable in 3 months, at $4\frac{1}{2}$ per cent. per annum?

11. What is the present worth of $5000, payable in 2 months, at 6 per cent. per annum?

12. What is the difference between the discount on $500 for 1 year, and the interest of $500 for 1 year, at 6 per cent.?

QUEST.—260. How do you find the present worth of a debt? How find the discount?

BANK DISCOUNT.

261. It is customary for *Banks* in discounting notes or drafts, to deduct *in advance* the *legal interest* on the given sum from the time it is discounted to the time when it becomes due. Hence,

Bank discount is the same as simple interest paid *in advance*.

OBS. 1. The difference between *bank discount* and *true discount*, is the interest of the true discount for the given time. On small sums, for a short period, this difference is trifling, but when the sum is large, and the time for which it is discounted is long, the difference is considerable.

Thus, the *bank* discount on a note of $106, payable in 1 year at 6 per cent., is $6 36; the *true* discount is $6. The *bank* discount on $106000 for the same time and date, is $6360; the *true* discount is only $6000.

2. Taking *legal interest in advance*, according to the general rule of law, is *usury*. An exception is generally allowed, however, in favor of notes, drafts, &c., which are payable in *less* than a year.

The Safety Fund Banks of the State of New York, though the legal rate of interest is 7 per cent., are not allowed by their charters to take over 6 per cent. discount in advance on notes and drafts which mature within 63 days from the time they are discounted.*

262. According to custom, a note or draft is not presented for collection until *three* days after the time specified for its payment. These three days are called *days of grace*. It is customary to charge interest for them.

After subtracting the discount from the note, the remainder is called the *present worth*, *avails* or *proceeds* of the note.

13. What is the bank discount on a note of $500, payable in 1 year, at 6 per cent.? What is the present worth?

Operation.

Principal	$500	Subtracting the discount from
Int. of $1 for 1 y. 3 d.	.0605	the face of the note, the re-
	2500	mainder is the present worth.
	3000	Thus, $500−$30.25=$469.75,
Bank discount	$30.2500	the present worth. Hence,

QUEST.—261. What is bank discount? *Obs.* What is the difference between bank discount and true discount? How is taking interest in advance generally regarded in law? What exception to this rule is allowed? 262. When is it customary to present notes and drafts for collection? What are these 3 days called? Is it customary to charge interest for the days of grace?

* Revised Statutes of New York, Vol. I. p. 741.

262.*a*. To find the *bank* discount on a note or draft.

Cast the interest on the note or draft for three days more than the specified time, and the result will be the discount.

Subtract the discount from the note, and the remainder will be the present worth or proceeds.

Note.—Interest should be reckoned on the *three days grace* in each of the following examples, except the last two.

14. What is the bank discount on a draft of $250, payable in 4 months, at 7 per cent.?

15. What is the bank discount on a draft of $375, payable in 30 days, at 6 per cent.?

16. What is the bank discount on a note of $1000, payable in 60 days, at 5 per cent.?

17. What is the present worth of $1160, payable in 90 days, discounted at a bank, at 6 per cent.?

18. What is the present worth of $750.36, payable in 5 months, at $4\frac{1}{2}$ per cent.?

19. What is the bank discount of $1825.60, payable in 4 months and 15 days, at 6 per cent.?

20. What is the present worth of a draft of $1292, payable in 60 days, at 7 per cent. discount?

21. What is the present worth of a draft of $5000, payable in 15 days, at 6 per cent. discount?

22. What is the present worth of a draft of $15000, payable in 3 days, at 6 per cent. discount?

23. What is the present worth of $1326, payable in 10 months, at $5\frac{1}{2}$ per cent. discount?

24. What is the bank discount, at 7 per cent., on a note of $836.81, payable in 90 days?

25. What is the bank discount, at 8 per cent., on a draft of $1261.38, payable in 60 days?

26. What is the bank discount, at $6\frac{1}{2}$ per cent., on a draft of $10000, payable in 30 days?

27. What is the difference between the true discount and bank discount on $1000, payable in 5 years, at 6 per cent.?

28. What is the difference between the true discount and bank discount on $100000, payable in 1 year, at 7 per cent.?

QUEST.—262.*a*. What is the rule for computing bank discount? How find the present worth or avails of a note?

INSURANCE.

263. INSURANCE is *security* given to *reimburse* losses of property by fire, storms at sea, and other casualties.

OBS. This security is usually effected by contract with Insurance Companies, who, for a stipulated sum, agree to restore to the owners the amount insured on their houses, ships, and other property, if destroyed or injured during the specified time of insurance.

The *written instrument* or *contract* containing the *terms* of insurance, is called the *Policy*.

The *sum* paid for insurance, is called the *Premium*.

The premium is usually reckoned at a *certain per cent.* on the amount of property insured for 1 year, or during a voyage, or other specified time of risk.

Note—1. Insurance on ships and other property, is sometimes effected by contract with individuals. It is then called *out-door insurance*.

2 The insurers, whether an incorporated company or individuals, are called *Underwriters*.

CASE I.—*To find the* PREMIUM, *the sum insured, the rate per cent., and the time being given.*

Ex. 1. How much premium must a mechanic pay annually for the insurance of his shop and tools worth \$350, at $1\frac{1}{2}$ per cent.?

Operation.

$$\begin{array}{r} \$350 \\ .015 \\ \hline 1750 \\ 350 \\ \hline \$5.250 \end{array}$$

Suggestion.—This example is similar to the ordinary examples in interest, and is solved in the same manner. That is, we multiply the sum insured by the rate per cent., and the result \$5.25, is the premium. Hence,

264. To find the *premium*, when the sum insured, the rate per cent., and the time are given.

Multiply the sum insured by the given rate per cent. expressed in decimals, and the product will be the premium.

Note.—This Case is the same as finding the interest on a given sum, at any given rate, when the time is 1 year.

QUEST.—263. What is Insurance? What is a policy? The premium? *Note*, When insurance is effected with individuals, what is it called? What are the insurers sometimes called? 264. How find the premium, when the sum insured, the rate, and time are given?

2. What premium must be paid annually for insuring a house worth \$875, at $\frac{3}{4}$ per cent.?

3. Shipped a box of books valued at \$1000, from New York to New Orleans, and paid $1\frac{1}{4}$ per cent. insurance: what was the premium?

4. A powder mill worth \$925, was insured at $15\frac{1}{2}$ per cent.: what was the premium?

5. A merchant shipped a lot of goods worth \$1560, from Boston to Natchez, and paid $1\frac{3}{4}$ per cent. insurance: what premium did he pay?

6. A gentleman obtained a policy of insurance on his house and furniture to the amount of \$2500, at $3\frac{1}{4}$ per cent. per annum: what premium did he pay a year?

7. A man owning a sixteenth of a whale ship, which cost him \$2750, got it insured, at $7\frac{1}{2}$ per cent. for the voyage: how much did he pay?

8. A man owning a schooner worth \$3800, obtained insurance upon it, at $5\frac{1}{3}$ per cent. for the season: what premium did he pay?

9. A crockery merchant having a stock of goods valued at \$7500, paid 2 per cent. for insurance: how much premium did he pay a year? *Ans.* \$150.

10. A merchant shipped \$3765 worth of flour from Cincinnati to New York, and paid $1\frac{1}{3}$ per cent. insurance: how much premium did he pay?

11. What is the annual premium for insuring a store worth \$7350, at $\frac{4}{5}$ per cent.?

12. An importer effected insurance on a cargo of tea worth \$65000, from Canton to Philadelphia, at 3 per cent.: how much did his insurance cost him?

13. A manufacturer obtained insurance to the amount of \$76500 on his stock and buildings, at $\frac{3}{4}$ per cent.: how much premium did he pay annually?

14. A policy was obtained on a cargo of goods valued at \$95600, shipped from Liverpool to New York, at $2\frac{1}{2}$ per cent.: what was the premium?

15. The owners of the whale ship George Washington obtained a policy of \$58000 on the ship and cargo, at $7\frac{1}{4}$ per cent. for the voyage: what was the premium?

CASE II.—*To find the* SUM INSURED, *the premium and rate per cent. being given.*

16. A gentleman paid $60 annually for insurance on his house and furniture, which was 2 per cent. on its value: what amount of property was covered by the policy?

Suggestion.—Since the rate is 2 per cent., 2 cents is the premium on $1; consequently, $60 must be the premium on as many dollars as 2 cents are contained times in $60. We therefore divide $60 by .02, and the result is the sum insured. Hence,

Operation.
.02)$60.00
Ans. $3000

264. *a.* To find the *sum insured*, when the premium and the rate per cent. are given.

Divide the premium by the rate per cent., expressed in decimals, and the quotient will be the sum insured.

Note.—This Case is similar in principle to Problem III. in Interest.

17. If I pay $250 premium on silks, from Havre to New York, at $1\frac{1}{2}$ per cent., what amount does my policy cover?

18. A merchant paid $1200 premium, at $2\frac{1}{5}$ per cent., on a ship and cargo from London to Baltimore, which was lost on the voyage: what amount of insurance should he recover?

CASE III.—*To find the* RATE PER CENT., *the sum insured, and the annual premium being given.*

19. If a man pays $60 premium annually for the insurance of his house, which is worth $3000, what rate per cent. does he pay?

Analysis.—If $3000 cost $60 premium, $1 will cost 1 *three thousandth* part of $60. We therefore divide $60 by $3000, and the quotient is .02 or 2 per cent. Hence,

Operation.
$3000)$60.00(.02
6000
Ans. 2 per cent.

265. To find the *rate per cent.* when the sum insured and the annual premium are given.

Divide the given premium by the sum insured, and the quotient will be the rate per cent. required.

Note.—This Case is similar in principle to Problem II. in Interest.

QUEST.—264.*a.* How find the sum insured, when the premium and the rate per cent. are given?

20. A merchant paid $40 premium for insuring $5000 on his stock: what rate per cent. did he pay?

21. If a man pays $75 for insuring $15000, what rate per cent. does he pay?

22. If the owner pays $2800 for insuring a ship worth $40000, what rate per cent. does he pay?

CASE IV.—*To find what sum must be insured on a given amount of property, so that if the property is destroyed, its value and the premium may both be recovered.*

23. A blacksmith owns a shop worth $720: what amount must he get insured annually, at 10 per cent., so that in case of loss, both the value of the shop and the premium may be repaid?

Analysis.—Since the rate of insurance is 10 per cent., on a policy of $100, the owner would actually receive but $90; for he pays $10 for insurance. Now, if the recovery of $.90 requires $1 to be insured, the recovery of $720 will require as many dollars to be insured as 90 cents is contained times in $720. We therefore divide $720 by $.90, and the quotient is the answer required. Hence,

Operation.
$.90)$720.00
Ans. $800

265.*a*. To find what sum must be insured on a given amount of property, so that if destroyed, both the *value* of the property and the *premium* may be recovered.

Subtract the rate per cent. from $1, *then divide the value of the property insured by the remainder, and the quotient will be the sum to be insured.*

24. If I send an adventure to China worth $6250, what amount of insurance, at 8 per cent. must I obtain, that in case of a total wreck I may sustain no loss by the operation?

25. What amount of insurance must be effected on $11250, at 5 per cent., in order to cover both the premium and property insured?

QUEST.—265. How find the rate per cent., when the sum insured and the annual premium are given? 265.*a*. How find what sum must be insured on a given amount of property, so that if destroyed, its value and the premium may be recovered?

PROFIT AND LOSS.

266. *Profit* and *Loss* in commerce, signify the *sum gained* or *lost* in ordinary business transactions.

They are reckoned at a certain per cent. on the *purchase price*, or *sum paid* for the articles under consideration.

CASE I.—*To find the* PROFIT *or* LOSS, *the purchase price and rate per cent. being given.*

Ex. 1. A merchant bought a quantity of grain for $375, and sold it for 8 per cent. profit: how much did he gain?

Suggestion.—8 per cent. is 8 cents on 100 cents or $1. Now, if $1 gains 8 cents, $375 will gain 375 times as much. We therefore multiply the cost and the per cent. profit together, and the result $30, is the amount gained. Hence,

Operation.

$$\begin{array}{r} \$375 \\ .08 \\ \hline \textit{Ans. } \$30.00 \end{array}$$

267. To find the *profit* or *loss*, when the purchase price and rate per cent. are given.

Multiply the purchase price by the given per cent. expressed in decimals, and the product will be the profit or loss.

OBS. If the rate per cent. is an *aliquot* part of 100, the *profit* or *loss* may be found by taking a *like part* of the *cost*. Thus, for 25 per cent. take $\frac{1}{4}$; for 20, take $\frac{1}{5}$; for $33\frac{1}{3}$, take $\frac{1}{3}$, &c.

2. A man bought a sleigh for $60, and afterwards sold it for 10 per cent. less than cost: how much did he lose?

3. A grocer bought a cask of oil for $96.50, and retailed it at a profit of 6 per cent.: how much did he make on his oil?

4. A pedlar bought a lot of goods for $215, and retailed them at 20 per cent. advance: how much was his profit?

5. A merchant bought a cargo of coal for $450, which he sold for $12\frac{1}{2}$ per cent. less than cost: what was his loss?

6. A manufacturer bought $1000 worth of wool, and after making it up, sold the cloth for 25 per cent. more than the cost of the materials: what did he receive for his labor?

QUEST.—266. What is meant by profit and loss? How are they reckoned? 267. How do you find the profit or loss, when the cost and rate per cent. are given?

CASE II.—*To find how an article must be sold to* GAIN *or* LOSE *a given per cent., the cost being given.*

7. A man bought a span of horses for $350, and wished to dispose of them for 12 per cent. profit: how much must he sell them for?

Suggestion.—It is manifest he must sell them for the *purchase price*, together with 12 per cent. of that price. We therefore find 12 per cent. of $350, which is $42, and adding it to the cost, the sum $392, is the price for which he must sell the horses.

Operation.

	$350
	.12
	$42.00
Ans.	$392

8. A stage proprietor bought a coach for $480; finding it damaged, he was willing to sell it, at 5 per cent. loss: at what price would he sell it?

Having found the sum lost, (Art. 225,) subtract it from the cost, and the remainder is obviously the selling price. Hence,

Operation.

	$480	purchase price.
	.05	per cent. loss.
	$24.00	sum lost.
Ans.	$456	selling price.

268. To find how an article must be sold, in order to *gain* or *lose* a given rate per cent., when the cost is given.

First find the amount of profit or loss at the given per cent., as in the preceding Case; then the amount thus found added to, or subtracted from the purchase price as the case may be, will give the selling price required.

9. A merchant bought a firkin of butter for $22.75: for how much must he sell it to gain 15 per cent. by his bargain?

10. Bought a chest of tea for $37.50: for how much must I sell it, in order to make 18 per cent. by the operation?

11. Bought a quantity of produce for $89.33, which I sold at 20 per cent. loss: how much did I receive for it?

12. A drover bought a flock of sheep for $275, and taking them to market, sold them at 25 per cent. advance: how much did he sell them for?

QUEST.—268. What is the method of finding how an article must be sold, in order to gain or lose a given per cent., when the cost is given?

13. A merchant had a quantity of groceries on hand, which cost him \$367.13; for the sake of closing up his business he sold them at 15 per cent. less than cost: how much did he get for them?

14. A man bought a farm for \$875, and was offered 33 per cent. advance for his bargain: how much was he offered?

15. A merchant bought a cargo of cotton for \$30000; the price declining, he sold it at $2\frac{1}{2}$ per cent. less than cost: for how much did he sell it?

CASE III.—*To find the* PER CENT. *profit or loss, the cost and selling price being given.*

16. A man bought a cow for \$25, which he afterwards sold for \$29: what per cent. profit did he make?

Analysis.—Subtracting the cost from the selling price, shows that he gained \$4. Now 4 dollars are $\frac{4}{25}$ of 25 dollars; hence, he gained $\frac{4}{25}$ of his *outlay*, or the purchase price of the cow. And $\frac{4}{25}$ reduced to a decimal, is 16 *hundredths*, which is the same as 16 per cent. (Arts. 197, 223. Obs. 3.)

Or, we may reason thus: If 25 dollars (outlay) gain 4 dollars, 1 dollar (outlay) will gain $\frac{1}{25}$ of 4 dollars. We therefore annex ciphers to the gain (\$4), and divide it by the cost \$25; the quotient .16 is the per cent. Hence,

Operation.

```
25)$4.00(.16
    25
    ---
    150
    150
    ---
```

269. To find the per cent. profit or loss, when the *cost* and *selling prices* are given.

First find the gain or loss, as the case may be, by subtraction, then annex ciphers to it, and divide it by the cost; the quotient will be the per cent. required.

Or, make the gain or loss the numerator, and the purchase price the denominator of a common fraction; reduce it to a decimal, and the result will be the per cent. (Art. 197.)

OBS. 1. As *per cent.* signifies *hundredths*, we have seen that the *first two* decimal figures which occupy the place of hundredths, are properly the per cent.; the

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QUEST.—269. How is the per cent. of profit or loss found, when the cost and selling price are given? *Obs.* What figures properly signify the per cent.? *Why?* What do the other decimal figures on the right of hundredths denote? *On* what is the per cent. gained or lost calculated?

other decimals are *parts* of 1 per cent. After obtaining two decimal figures, there is sometimes an advantage in placing the remainder over the divisor, and annexing it to the decimals thus obtained. (Art. 223. Obs. 3.)

2. It should be remembered that the percentage which is *gained* or *lost*, is always calculated on the *purchase* price, or the *sum paid* for the article, and not on the *selling* price, or *sum received*, as it is often supposed.

17. A merchant bought a piece of cloth for $2.75 per yard, and sold it for $3.25: what per cent. did he gain?

18. A boy purchased a book for 20 cents, and sold it for 30 cents: what per cent. did he make?

19. Bought a box of sugar at 6 cents a pound, and sold it for $7\frac{1}{2}$ cents a pound: what per cent. was the profit?

20. A grocer bought eggs at 9 cents, and sold them for 12 cents per dozen: what per cent. was his profit?

21. A man bought a hat for $4.50, and sold it for $6: what per cent. did he gain?

22. A jockey bought a horse for $73, and sold him for $68: what per cent. did he lose?

23. A merchant bought a quantity of goods for $155.63, and sold them for $148.28: what per cent. did he lose?

24. A gentleman bought a house for $3500, and sold it for $150 more than he gave: what per cent. was his profit?

25. A speculator laid out $7500 in land, and afterwards sold it for $10000: what per cent. did he make?

26. A drover bought a herd of cattle for $1175, and sold them for $1365: what per cent. did he gain, and how much did he make by the operation?

27. A merchant bought $10000 worth of wool, and sold it for $12362: what per cent., and how much was his profit?

CASE IV.—*To find the* COST, *the selling price and the per cent. gained or lost being given.*

28. A jockey sold a horse for $250, which was 25 per cent. more than it cost him: how much did he pay for the horse?

Analysis.—It will be observed that the selling price ($250) is equal to the *cost* and the *gain* added together. Now considering the cost a unit or 1, the gain, which is a certain per cent. of the cost, is $\frac{25}{100}$; consequently $1+\frac{25}{100}=\frac{125}{100}$, will denote the sum of the cost and the gain. (Art. 127.) The question therefore resolves itself into this: 250 is $\frac{125}{100}$ of what number? If 250 is $\frac{125}{100}$, $\frac{1}{100}$ is 2; and $\frac{100}{100}$ is 100 times 2, or $200.

Since there is a profit, we divide the price by \$1, increased by the per cent. gain, and the quotient is the cost.

Operation.
\$1.25)\$250.00(\$200
250

PROOF.—\$200 × .25=\$50; and \$200+\$50=\$250, the price.

29. A merchant sold a quantity of goods for \$180, which was 10 per cent. less than cost: how much did the goods cost?

Analysis.—It will be observed that the selling price (\$180) is equal to the cost diminished by the sum lost. Reasoning as in the last example, $1-\frac{10}{100}=\frac{90}{100}$ will denote the cost diminished by the loss. The question then is this: 180 is $\frac{90}{100}$ of what number? If 180 is $\frac{90}{100}$, $\frac{1}{100}$ is 2, and $\frac{100}{100}$ is \$200.

Since there is a loss, we divide the selling price by \$1, diminished by the per cent. loss, and the quotient is the cost.

Operation.
.90)\$180.00
Ans. \$200

PROOF.—\$200 × .10=\$20, and \$200—\$20=\$180, the price.

270. Hence, to find the *cost*, when the *selling price* and the per cent. *gained* or *lost* are given.

Divide the selling price by \$1, *increased or diminished by the per cent. gained or lost, as the case may be, and the quotient will be the cost required.*

Or, make the given per cent. added to or subtracted from 100, *as the case may be, the numerator, and* 100 *the denominator of a common fraction; then divide the selling price by this fraction, and the quotient will be the cost required.*

OBS. 1. It is not unfrequently supposed that if the percentage on the selling price at the given rate is found, and this percentage is added to or subtracted from the selling price, as the case may be, the sum or remainder will be the cost. This is a mistake, and leads to serious errors in the result. It will easily be avoided by remembering, that the basis on which *profit* and *loss* are calculated, is always the *purchase price*, or *sum paid* for the articles under consideration. (Art. 269. Obs. 2.)

30. A grocer sold a hogshead of molasses for \$24, and gained 20 per cent. on the cost: what was the cost of the molasses?

31. A merchant sold a piece of broadcloth for \$85, which was 10 per cent. less than the cost: what was the cost of it?

QUEST.—270. How is the cost found, when the selling price and the per cent. gained or lost, are given? *Obs.* What mistake is sometimes made in finding the cost? How may it be avoided?

32. A butcher sold a yoke of oxen for $125, and thereby made 15 per cent.: how much did they cost him?

33. A bookseller sold a lot of books for $200, which was 12 per cent. more than the cost: what was the cost?

34. A wholesale druggist sold a quantity of medicines for $560, and made 50 per cent. profit on them: what was the cost of them?

35. A merchant sold a cargo of rice for $1500, which was $12\frac{1}{2}$ per cent. less than cost: what was the cost?

EXAMPLES FOR PRACTICE.

1. A merchant bought 25 boxes of raisins for $45: at what price per box must he retail them to gain 10 per cent. by his bargain?

2. A shopkeeper bought a piece of cotton containing 40 yards, at 6 cents a yard, and sold it for 7 cents a yard: what per cent. profit did he gain, and how much did he make by the bargain?

3. A merchant bought 60 yards of domestic flannel, at 25 cents per yard, and sold it at 30 cents per yard: what per cent. was his profit, and how much did he clear by the operation?

4. A bookseller bought 100 Arithmetics, at $31\frac{1}{2}$ cents apiece, and retailed them at $37\frac{1}{2}$ cents apiece: what per cent., and how much did he make by the operation?

5. A drover bought 175 sheep for $350, and sold them so as to gain 15 per cent.: how much did he sell them for per head?

6. A baker paid $2500 for 480 barrels of flour, and finding it damaged, sold it at a loss of 8 per cent.: how much did he sell it for per barrel?

7. A merchant bought 10 pieces of broadcloth, each piece containing 30 yards, for $1400, and retailed the whole at a profit of 20 per cent.: at what price did he sell it per yard?

8. A grocer bought 500 lbs. of butter for $75, and sold it at a loss of 7 per cent.: how much did he get per pound?

9. A merchant bought 12 hogsheads of molasses, at 25 cents per gallon: how must he sell it by the gallon in order to gain 20 per cent.; and what will be his profit?

10. A farmer raises 750 bushels of wheat, at an expense of $675: how must he sell it per bushel, in order to make 18 per cent.?

11. A provision merchant bought 1500 barrels of pork, at $10.25 per barrel, and sold it at a loss of 9 per cent.: how much did he lose, and what did he get per barrel?

12. An inn-keeper bought 150 bushels of oats, at 25 cents a bushel, and retailed them at the rate of $12\frac{1}{2}$ cents a peck: what per cent., and how much did he make on the oats?

13. A miller bought 500 bushels of wheat, at 75 cents per bushel: how much must he sell the whole for in order to gain 20 per cent.?

14. A grocer bought 1630 pounds of tea, at $62\frac{1}{2}$ cents per pound, and sold it at 10 per cent. loss: how much did he sell it at per pound?

15. A merchant bought a bale of calico prints containing 750 yards and paid $75: how must he retail it per yard, in order to gain 20 per cent., and how much would he make on a yard?

16. A bookseller purchased 1000 geographies, at 84 cents apiece: how must he retail them to gain 20 per cent.?

17. A milliner bought 1200 yards of ribbon, at 30 cents per yard: how must she sell it per yard to gain 50 per cent.?

18. A grocer bought 5000 lbs. of sugar for $350, and retailed it at 6 cents per pound: what per cent. loss did he sustain?

19. A man purchased goods amounting to $1635: what per cent. profit must he gain, in order to make $350?

20. A speculator bought 10000 acres of land for $12500, and afterwards sold it at 25 per cent. loss: for how much per acre did he sell it, and how much did he lose by the operation?

21. A grocer sold 116 bbls. of flour, at $\$6\frac{1}{2}$ per barrel, and gained $12\frac{1}{2}$ per cent.: how must he have sold it to lose $12\frac{1}{2}$ per cent.?

22. A merchant sold a lot of cloth at $\$5.37\frac{1}{2}$ a yard, and gained 15 per cent.: what per cent. would he have gained or lost, if he had sold it at $5.25 per yard?

23. Bought 516 yds. of linen, at 75 cents per yard, which shrunk $1\frac{1}{2}$ per cent. in bleaching; after keeping it 6 months it was sold on 6 months' credit, at 20 per cent. advance on a yard: what was it sold for per yard, and how much was made by the operation, allowing 7 per cent. interest on the money invested?

DUTIES.

271. DUTIES, in commerce, are *sums of money* required by Government to be paid on *imported* or *exported goods.*

Duties are of two kinds, *specific* and *ad valorem.*

A *specific duty* is a specified sum imposed on a yard, gallon, pound, &c.

An *ad valorem* duty is a certain *percentage* on the *value* or *cost* of goods.

OBS. 1. The term *ad valorem*, signifies *according to value.*

2. Duties, both specific and ad valorem, are regulated by Government, and have varied at different times and in different countries.

According to the present revenue laws, all duties in this country are levied on the *ad valorem* principle.

272. Before duties are levied, in certain cases *deductions* are made for *Tare*, *Tret* or *Draft*, *Leakage*, and *Breakage.*

Tare is a deduction made from the *weight* of goods on account of the boxes, casks, or bags which contain them.

The *remainder*, after deducting the tare, is sometimes called *suttle weight.*

Tare is of *three kinds*, *actual*, *customary*, and *average.*

Actual tare is the exact weight of the boxes, casks, &c., and is ascertained by weighing them when empty.

Customary tare is an allowance for the supposed weight of boxes, casks, &c., and is established by custom.

Average tare is the *medium* weight of boxes, casks, &c., and is determined by weighing a few, and making the *mean weight* of these the *standard* for the whole.

Draft or *tret* is a deduction made from the weight of goods for waste or refuse matter.

Leakage is an allowance for the waste of liquors contained in casks.

Breakage is an allowance for the waste of liquors contained in bottles.

OBS. In buying and selling groceries in large quantities, allowances were formerly made for tare, draft, leakage, &c., similar to those in reckoning duties; but this practice has very generally fallen into disuse.

QUEST.—271. What are duties in commerce? Of how many kinds are they? What are specific duties? Ad valorem duties? *Note.* What is the meaning of the term ad valorem? 272. What deductions are sometimes made before duties are imposed? What is tare? Draft or tret? Leakage? Breakage?

272.a. *Method of finding the amount of Tare, Tret, &c.*

1. The *tare* allowed in assessing duties in this country and England, at the present time, is *actual* tare, and the *amount* is determined by weighing the goods when landed.

Note.—The amount of *customary* and *average* tare, is found by multiplying the number of pounds per box by the number of boxes.

2. *Draft* or *tret* is not so extensively allowed in levying duties as tare. The present revenue laws of this country do not recognize it. The allowance formerly made on a box weighing 1 cwt. gross (112 lbs.), was 1 lb.

From	1 cwt. to 2 cwt. (224 lbs.),	2 lbs.
"	2 cwt. to 3 cwt. (336 lbs.),	3 lbs.
"	3 cwt. to 10 cwt. (1120 lbs.),	4 lbs.
"	10 cwt. to 18 cwt. (2016 lbs.),	7 lbs.
"	18 cwt. and upwards,	9 lbs.

Note.—In England, the allowance for draft or tret is 4 lbs. on 104 lbs.

3. The *amount* of *leakage* is found by gauging the casks, and the *actual deficiencies* are deducted from the invoice before the duty is assessed.

The *amount* of *breakage* is determined by examination, and the *actual loss* is deducted.

Note.—Formerly 2 per cent. was allowed for leakage on liquors contained in casks; 10 per cent. for breakage on porter, ale, and beer contained in bottles, and 5 per cent. on all other liquors.

4. The *net weight* of merchandise is found by *subtracting* the amount of tare and draft or tret from the *gross weight.*

5. The *net measure* of fluids is found by *deducting* the *leakage* and *breakage* from the *whole quantity.*

OBS. In estimating allowances, when a fraction occurs, equal to a *half pound, gallon,* &c., or more, it is considered a *unit;* if *less* than a half, it is *disregarded.*

Ex. 1. What is the net weight of 4 casks of sugar, the 1st, weighing 2 cwt. 3 qrs. 21 lbs.; the 2d, weighing 3 cwt. 2 qrs. 17 lbs.; the 3d, 4 cwt. 3 qrs. 18 lbs.; and the 4th, 3 cwt. 1 qr. 15 lbs., allowing the former draft and 15 lbs. to a cask for tare?

Operation.

No. 1.	draft	3 lbs.,	gross weight	2 cwt. 3 qrs. 21 lbs.
" 2.	"	4 lbs.,	" "	3 cwt. 2 qrs. 17 lbs.
" 3.	"	4 lbs.,	" "	4 cwt. 3 qrs. 18 lbs.
" 4.	"	4 lbs.,	" "	3 cwt. 1 qr. 15 lbs.
Amt. of draft = 15 lbs.			Amt.	14 cwt. 3 qrs. 15 lbs.
Tare, 15 lbs. × 4 = 60 lbs.		= 75 lbs., or,		2 qrs. 19 lbs.
			The net weight is	14 cwt. 0 qrs. 24 lbs.

2. What is the net weight of 4 bales of cotton, one weighing 561 lbs. gross, another 473 lbs., another 878 lbs., and the other 493 lbs. gross, the tare being 16 lbs. to a bale?

3. What is the net weight of 12 hogsheads of tobacco, each weighing 5 cwt., tare being 27 lbs. to a hogshead?

4. What is the net weight of 21 bbls. of potash, each weighing 341 lbs. gross, allowing 17 lbs. tare per barrel?

5. If the draft and tare are $5\frac{1}{2}$ lbs. per hundred, how much will it be on 15 tons, 12 cwt. 2 qrs. 20 lbs. gross?

6. What is the amount of draft and tare, at $7\frac{1}{4}$ lbs. per hundred, on 27 tons, 16 cwt. 3 qrs. 15 lbs.?

7. What is the leakage on 41 hhds. and 45 gals., at 2 per cent.? What is the net measure?

8. At $3\frac{1}{2}$ per cent., what is the leakage and net measure of 64 casks of ale, which contain 85 gallons, 3 qts. apiece?

9. What is the breakage, at 10 per cent., on 423 bottles of porter? What the net measure?

10. What is the breakage, at 5 per cent., on 65 baskets of wine containing 16 bottles apiece? What is the net measure?

CASE I.—*Specific Duties.*

Ex. 1. What is the specific duty on 10 pipes of wine, at 15 cents per gallon, reckoning the leakage at 2 per cent.?

Operation.

The number of gallons, is 126 × 10, or . .	1260 gals.
Leakage at 2 per cent., 1260 gals. × .02, or .	25.2 gals.
Net gallons on which the duty is to be paid,	1234.8 gals.
Per cent. duty, (specific,)	.15
	61740
	12348
The amount of duty to be paid, is . .	$185.220

273. Hence, to compute *specific* duties on merchandise.

First deduct the legal tare, draft, leakage, or breakage, from the given quantity of goods; then multiply the remainder by the given duty per gallon, pound, yard, &c., and the product will be the duty required.

QUEST.—273. What is the rule for computing specific duties?

2. What is the specific duty, at 2 cents per pound, on 12 boxes of sugar, weighing 900 lbs. apiece, allowing 20 pounds per box for tare?

Ans. { The tare is 240 pounds.
And 2 per ct. on 10560 lbs. is $211.20.

3. At 3 cents a pound, what is the duty on 25 casks of soda, each weighing 125 lbs., allowing 8 pounds on a cask for tare?

4. At 5 cents a pound, what is the specific duty on 75 boxes of raisins, weighing 60 lbs. apiece, allowing 6 pounds a box for draft?

5. At 4 cents per pound, what is the specific duty on 110 chests of cinnamon, each weighing 230 lbs., allowing 16 lbs. per chest for draft?

6. At 15 cents a pound, what is the specific duty on 300 bags of indigo, each weighing 200 lbs., allowing 4 per cent. for tare?

CASE II.—*Ad valorem Duties.*

7. What is the ad valorem duty, at 15 per cent., on an invoice of calico prints, which cost $150 in Liverpool.

Suggestion.—Ad valorem duties, we have seen, are levied on the cost of goods; consequently, there are no deductions to be made in this case. We therefore multiply the invoice price by the given per cent., and the product $22.50 is the duty required. Hence,

Operation.

$150
.15
750
150
$22.50

274. To compute *ad valorem* duties on merchandise.

Multiply the cost by the specified or legal per cent., and the product will be the duty required. (Art. 225.)

OBS. 1. An *invoice* is a written statement of merchandise, with the price of the articles annexed.

2. The law requires that the invoice shall be verified by the owner, or one of the owners of the goods, wares, or merchandise, certifying that the invoice annexed contains a *true and faithful account of the actual costs* thereof, and of all charges thereon, and no other different discount, bounty, or drawback, but such as has been actually allowed on the same; which oath shall be administered by a consul or commercial agent of the United States, or by some public officer duly

QUEST.—274. What is the rule for computing ad valorem duties? *Obs.* What is an invoice? What does the law require respecting the invoice of imported goods?

authorized to administer oaths in the country where the goods were purchased, and the same shall be duly certified by the said consul, &c. Fraud on the part of the owners, or those who administer the oath, is visited with a heavy penalty.—*Laws of the United States.*

8. What is the ad valorem duty, at 30 per cent., on a box of books invoiced at $250?

9. What is the ad valorem duty, at 20 per cent., on a quantity of Java coffee, which cost $356.12?

10. What is the amount of ad valorem duty, at 25 per cent., on a quantity of Turkey carpeting, which cost $526.61?

11. What is the duty on a quantity of bombazines, invoiced at $300.10, at 30 per cent.?

12. What is the duty on a quantity of beeswax, the invoice of which is $460.25, at 15 per cent.?

13. At 25 per cent., what is the duty on an invoice of bleached linens, amounting to $745.85?

14. At 20 per cent., what is the duty on an invoice of jewelry, amounting to $4250?

15. What is the duty on a bale of goods, invoiced at $2500, at 40 per cent.?

16. What is the duty on an invoice of silks, amounting to $5650, at 30 per cent.?

17. What is the duty on a quantity of cutlery, invoiced at $4560, at 33 per cent.?

18. What is the duty on an invoice of broadcloths, which amounts to $8280, at 35 per cent.?

19. What is the duty on an invoice of wines, amounting to $10265, at 35 per cent.?

20. What is the duty on a quantity of cotton fabrics, invoiced at $13637.50, at 33 per cent.?

21. What is the duty on a quantity of ready-made clothing, amounting to $5638.25, at 50 per cent.?

22. Imported 25264 lbs. of sugar costing $6\frac{1}{4}$ cts., per lb., and paid $12\frac{1}{2}$ per cent. duty ad valorem: what was the amount of duty?

23. What is the duty on an invoice of silks amounting to £3256 sterling, at 27 per cent., allowing $4.84 to a pound?

24. What is the duty on an invoice of 650 yards of broadcloths, which cost 16s. 6d. per yard, at 34 per cent. ad valorem, the value of a £ being as above?

ASSESSMENT OF TAXES.

275. A TAX is a *sum* of money assessed on the person or property of a citizen by the government, for public purposes.

Taxes are of two kinds, *direct* and *indirect*. A *direct* tax is one laid upon the property or income of a citizen; an *indirect* tax is one laid on articles of consumption, for which this property is expended, as in duties, excise, &c.

When a tax is assessed on *property*, it is apportioned at a *certain per cent.* on the amount of *real estate* and *personal property* of each taxable individual.

When assessed on the *person*, it is called a *poll-tax*, and is apportioned equally among the citizens, without regard to property.

OBS. 1. Property is of two kinds, *real estate* and *personal* property.

Real estate denotes possessions that are *fixed* or *immovable*; as houses and lands. *All other* property is called *personal*; as money, stocks, mortgages, ships, furniture, carriages, cattle, tools, &c.

2. A poll-tax is sometimes called a *capitation-tax*. The term *poll* signifies the *head* of a person, or the person himself.

276. When a tax is to be assessed, the first step is to make an inventory of all the *taxable* property, both personal and real, in the State, County, Corporation, or District, by which the tax is to be paid; with the number of polls, and the property of every citizen who is to be taxed.

OBS. By the *number of polls* is meant the number of *taxable individuals*, which usually includes every *native* or *naturalized freeman* over the age of 21, and under 70 years. In some States it also includes the young men over the age of eighteen years, who are subject to military duty.

Ex. 1. A certain town is taxed $325. The town contains 200 polls, which are assessed 25 cents apiece; and the whole amount of property, both real and personal, is valued at $13750. What per cent. is the tax, and how much is a man's tax who pays for 1 poll, and whose property is valued at $850?

QUEST.—275. What is a tax? How many kinds of taxes are there? What is a direct tax? An indirect tax? When assessed upon property, how is a tax apportioned? When assessed upon the person, how apportioned, and what called? *Obs.* Of how many kinds is property? What does real estate denote? What is personal property? 276. When a tax is to be assessed, what is the first step? *Obs.* What is meant by the number of polls?

Operation.

Tax on 200 polls, at 25 cents, equals, ...	\$50.00
Tax on polls deducted from whole tax, \$325—\$50=	275.00
Tax on \$1 is equal to \$276÷\$13750=.02 or 2 per cent.	.02
Tax on the man's property \$850 × .02 equals . .	17.00
His poll-tax \$.25 to be added to tax on property, .	.25
Therefore the man's tax is, ...	\$17.25

277. Hence, to assess a State, County, or other tax.

I. *First find the amount of tax on all the polls, if any, at the given rate, and subtract this sum from the whole tax to be assessed; then divide the remainder by the whole amount of taxable property in the State, County, &c., and the quotient will be the per cent. or tax on 1 dollar.*

II. *Multiply the amount of each man's property by the per cent. or tax on 1 dollar, and the product will be the tax on his property.*

Finally, add each man's poll tax to the tax on his property, and the amount will be his whole tax.

278. PROOF.—*Add together the taxes of all the individuals in the State, County, or District, upon which the tax is levied, and if the amount is equal to the whole tax assessed, the work is right.*

2. A certain parish is taxed \$237.50; the whole property of the parish is valued at \$8000; and there are 75 polls, which are assessed 50 cents apiece. What per cent. is the tax; and how much is a man's tax who pays for 3 polls, and whose property is valued at \$500?

Ans. $\begin{cases} 2\frac{1}{4} \text{ per cent.} \\ \$14, \text{ whole tax.} \end{cases}$

3. What amount of tax does a man living in the same parish pay, whose property is valued at \$450, and pays for 2 polls?

4. A tax of \$750 is assessed on a district to build a new school-house; the property of the district is valued at \$15000. What is the tax on a dollar; and what is a man's tax whose property is \$1150?

QUEST.—277. What is the rule for assessing taxes? 278. When a tax bill is made out, how is its correctness proved?

5. What is B's tax for erecting the same school-house, whose property is $1530?

6. A tax of $14752.50 is levied on a certain County, whose property is valued at $562875, and which has a list of 5825 polls, which are assessed at 60 cents apiece. What per cent. is the tax; and what is the amount of C's tax, who pays for 4 polls, and has property valued at $5000?

7. What is D's tax, who, living in the same County, pays for 2 polls, and is worth $3500?

8. What is G's tax, who pays for 5 polls, and is worth $15300?

279. In making out a tax bill, having found the tax on $1, the operation will be greatly facilitated by making a table, showing the amount of tax on any number of dollars from 1 to $10; then on 10, 20, 30, &c., to $100; then on 100, 200, &c., to $1000.

9. A tax of $3506.25 was levied on a corporation composed of 12 individuals, whose property was valued at $175000, and who were assessed for 25 polls, at 25 cents apiece. What was the tax on a dollar. *Ans.* 2 cents on a dollar?

Note.—Having found the tax on $1, we will make a table to aid us in making out the tax bill of the corporation. Since the tax on $1 is $.02, it is obvious that multiplying $.02 by 2 will be the tax on $2; multiplying it by 3, will be the tax on $3, &c.

TABLE.

$1	pays	$.02	$10	pay	$.20	$100	pay	$2.00
2	"	.04	20	"	.40	200	"	4.00
3	"	.06	30	"	.60	300	"	6.00
4	"	.08	40	"	.80	400	"	8.00
5	"	.10	50	"	1.00	500	"	10.00
6	"	.12	60	"	1.20	600	"	12.00
7	"	.14	70	"	1.40	700	"	14.00
8	"	.16	80	"	1.60	800	"	16.00
9	"	.18	90	"	1.80	900	"	18.00
10	"	.20	100	"	2.00	1000	"	20.00

10. In the above assessment, what was A's tax, whose property was valued at $1256, and who pays for 3 polls?

QUEST.—279. When a tax bill is to be made out for a whole town, district, &c., what course do assessors usually take?

Suggestion.—$1256 is composed of 1000+200+50+6. Now if we add the taxes paid on each of these sums together, the amount will be the tax paid on $1256.

Operation.

$1000	pay	$20.00
200	"	4.00
50	"	1.00
6	"	.12
3 polls	"	.75
	Amount,	$25.87

A's tax therefore was $25.87.

11. What was B's tax, who paid for 4 polls, and had property to the amount of $1461?

12. C paid for 1 poll, and the valuation of his property was $5863. What was the amount of his tax?

13. D paid for 1 poll, and the valuation of his property was $7961. What was his tax?

14. E paid for 2 polls, and his property was valued at $14236. What was his tax?

15. F paid for 2 polls, and his real estate was valued at $21000; his personal property at $4500. What was his tax?

16. G's property was valued at $20250, and he paid for 1 poll. What was his tax?

17. H paid for 2 polls, and the valuation of his estate was $15360. What was his tax?

18. J's property was valued at $33000, and he paid for 4 polls. What was his tax?

19. K paid for 1 poll, and his property was valued at $15013. What was his tax?

20. L paid for 3 polls, and his property was valued at $4500. What was his tax?

21. M paid for 1 poll, and the valuation of his property was $30600. What was his tax?

22. The Legislature levied a tax of $5312.50 upon a certain town, having an inventory of $450000, and 1550 polls, which were assessed at $1¼ apiece: what was the tax on a dollar; and what was A's tax, who had $1149 real estate, $1376 personal property, and paid for 3 polls?

23. What was B's tax, who had $2175 real estate, $960 personal property, and paid for 1 poll?

24. C's personal property was $1318, his real estate $1263, and he paid for 2 polls: what was his tax?

25. D paid for 2 polls, his real estate was $1538, and his personal property $1681: what was his tax?

SECTION X.

PROPERTIES OF NUMBERS.*

280. By the term *properties* of numbers, is meant those properties or elements which are inherent and inseparable from them. The following are some of the more prominent:

PROP. 1. The sum of any *two* or *more even* numbers, is an even number.

2. The difference of any *two even* numbers, is an even number.

3. The sum or difference of *two odd* numbers, is *even;* but the sum of *three odd* numbers, is *odd.*

4. The sum of any *even* number of odd numbers, is even; but the sum of any *odd* number of odd numbers, is odd.

5. The sum, or difference, of an *even* and an *odd* number, is an odd number.

6. The product of an *even* and an *odd* number; or of *two even* numbers, is even.

7. If an even number be divisible by an odd number, the *quotient* is an even number.

8. The product of any number of factors, is *even*, if any one of them be even.

9. An odd number cannot be divided by an even number without a remainder.

10. The product of any *two* or *more odd* numbers, is an odd number.

11. If an odd number divides an even number, it will also divide the *half* of it.

12. If an even number is divisible by an odd number, it will also be divisible by *double* that number.

13. The product of any two numbers is the *same*, whichever of the two numbers is the multiplier. (Art. 47.)

14. The *least* divisor of every number, is a *prime* number.

QUEST.—280. What is meant by properties of numbers? What is the least divisor of every number?

* Barlow on the Theory of Numbers; also, Bonnycastle's Arithmetic.

15. Any number expressed by the decimal notation, divided by 9, will leave the *same remainder* as the sum of its *figures* or *digits* divided by 9. The same property belongs to the number 3, and to no other number. Thus, if 236 is divided by 9, the remainder is 2; so, if the sum of its digits, $2+3+6=11$, is divided by 9, the remainder is also 2.

Note.—Upon this property of the number 9, is based a convenient method of proving multiplication and division.

281. To cast the 9s out of a number, begin at the left hand, add the digits together, and as soon as the sum is 9 or over, drop the 9, and add the remainder to the next digit, and so on. For example, to cast the 9s out of 8626557, we proceed thus: 8 and 6 are 14; drop the 9, and add the 5 to the next figure. 5 and 2 are 7 and 6 are 13; drop the 9, and add the 4 to the next figure. 4 and 5 are 9; drop the 9 as above. 5 and 7 are 12; dropping the 9, the remainder is 3.

OBS. When the sum is over 9, we may simply add its digits together, and proceed to the next figure. Thus, 8 and 6 are 14; now adding its digits, 1 and 4 are 5 and 2 are 7 and 6 are 13. Adding the digits in this sum, 1 and 3 are 4, proceed to the next figure, &c.

PROOF OF MULTIPLICATION BY CASTING OUT THE NINES.

282. *First, cast the 9s out of the multiplicand and multiplier; multiply their remainders together, and cast the 9s out of their product, and set down the excess; then cast the 9s out of the answer obtained, and if this excess be the same as that obtained from the multiplier and multiplicand, the work may be considered right.*

What is the product of 565 multiplied by 356?

Operation.	*Proof.*
565	The excess of 9s in the multiplicand is 7.
356	" " 9s " multiplier is 5.
3390	$7\times5=35$; and the excess of 9s is 8.
2825	
1695	
Prod. 201140	The excess of 9s in the *Ans.* is also 8.

QUEST.—282. How is multiplication proved by casting out the 9s?

PROOF OF DIVISION BY CASTING OUT THE NINES.

283. *First, cast the 9s out of the divisor and quotient, and multiply the remainders together; to the product add the remainder, if any, after division; cast the 9s out of this sum, and set down the excess; finally, cast the 9s out of the dividend, and if the excess is the same as that obtained from the divisor and quotient, the work may be considered right.*

AXIOMS.

284. In mathematics, there are certain propositions whose *truth* is so *evident* at sight, that no process of reasoning can make it plainer. These propositions are called *axioms*. Hence,

An *axiom* is a *self-evident* proposition.

1. Quantities which are equal to the *same* quantity, are equal to each other.

2. If the same or equal quantities are *added* to equal quantities, the *sums* will be equal.

3. If the same or equal quantities are *subtracted* from equals, the *remainders* will be equal.

4. If the same or equal quantities are *added* to *unequals*, the *sums* will be unequal.

5. If the same or equal quantities are *subtracted* from *unequals*, the *remainders* will be unequal.

6. If equal quantities are *multiplied* by the same or equal quantities, the *products* will be equal.

7. If equal quantities are *divided* by the same or equal quantities, the *quotients* will be equal.

8. If the same quantity is both *added* to and *subtracted* from another, the value of the latter will not be altered.

9. If a quantity is both *multiplied* and *divided* by the same or an equal quantity, its value will not be altered.

10. The *whole* of a quantity is *greater* than a *part*.

11. The *whole* of a quantity is equal to the sum of *all its parts*.

OBS. The term *quantity*, signifies anything which can be *multiplied*, *divided*, or *measured*. Thus, *numbers*, *yards*, *bushels*, *weight*, *time*, &c., are called quantities.

QUEST.—283. How is division proved by casting out the 9s? 284. What is an axiom? What is the first axiom? The Second? Third? Fourth? Fifth? Sixth? Seventh? Eighth? Ninth? Tenth? Eleventh? *Obs.* What is meant by quantity?

GENERAL PRINCIPLES AND PROBLEMS.

286. When the *sum* of two numbers and *one* of the numbers are given, to find the *other* number.

From the sum subtract the given number, and the remainder will be the other number.

Ex. 1. The sum of two numbers is 25, and one of them is 10: what is the other number?

Solution. 25—10=15, the other number. (Art. 40.)

PROOF. 15+10=25, the given sum. (Art. 284. Ax. 11.)

2. A and B together own 36 cows, 9 of which belong to A: how many does B own?

3. Two farmers bought 300 acres of land together, and one of them took 115 acres: how many acres had the other?

287. When the *difference* and the *greater* of two numbers are given, to find the *less.*

Subtract the difference from the greater, and the remainder will be the less number.

4. The greater of two numbers is 37, and the difference between them is 10: what is the less number?

Solution. 37—10=27, the less number. (Art. 40.)

PROOF. 27+10=37, the greater number. (Art. 40. Obs.)

5. A had 48 dollars in his pocket, which was 12 dollars more than B had: how many dollars had B?

6. D had 450 sheep, which was 63 more than E had: how many had E?

289. When the *difference* and the *less* of two numbers are given, to find the *greater.*

Add the difference to the less number, and the sum will be the greater. (Art. 40. Obs.)

QUEST.—286. When the sum of two numbers and one of them are given, how is the other found? 287. When the difference and the greater of two numbers are given, how is the less found? 289. When the difference and the less of two numbers are given, how is the greater found?

7. The difference between two numbers is 5, and the less number is 15: what is the greater number. *Ans.* 20.

8. A is 16 years old, and B is 8 years older than A: how old is B?

9. The number of male inhabitants in a certain town, is 935; and the number of females exceeds the number of males by 115: how many females does the town contain?

290. When the *sum* and *difference* of two numbers are given, to find the *numbers*.

From the sum subtract the difference, and half the remainder will be the smaller number.

To the smaller number add the given difference, and the sum will be the larger number.

10. The sum of two numbers is 35, and their difference is 11: what are the numbers? *Ans.* 12 and 23.

11. The sum of the ages of 2 boys is 25 years, and the difference between them is 5 years: what are their ages?

12. A man bought a chest of tea and a hogshead of molasses for $63; the tea cost $9 more than the molasses: what was the price of each?

291. When the *product* of two numbers and *one* of the numbers are given, to find the *other* number.

Divide the product by the given number, and the quotient will be the number required. (Art. 77.*a.*)

13. The product of two numbers is 84, and one of the numbers is 7: what is the other number? *Ans.* 12.

14. The product of A and B's ages is 120 years, and A's age is 12 years: how old is B?

15. A certain field contains 160 square rods, and the length of the field is 20 rods: what is its breadth?

Note.—The area of a field is found by multiplying its *length* and *breadth* together. (Art. 153. Obs. 3.) Hence the given *area* may be considered as a *product.*

QUEST.—290. When the sum and difference of two numbers are given, how are the numbers found? 291. When the product of two numbers and one of them are given, how is the other found?

292. When the *divisor* and the *quotient* are given, to find the *dividend.*

Multiply the given divisor and the quotient together, and the product will be the dividend. (Art. 72.)

16. If a certain divisor is 9, and the quotient is 12, what is the dividend? *Ans.* 108.

17. A man having 11 children, gave them $75 apiece: how many dollars did he give them all?

18. A farmer divided a quantity of apples among 90 boys, giving each boy 15 apples: how many did he give them all?

293. When the *dividend* and *quotient* are given, to find the *divisor.*

Divide the dividend by the given quotient, and the quotient thus obtained will be the divisor required. (Art. 72. Obs. 2.)

19. A certain dividend is 130, and the quotient is 10: what is the divisor? *Ans.* 13.

20. A gentleman divided $120 equally among a company of sailors, giving them $10 apiece: how many sailors were there in the company?

21. A farmer having 600 sheep, divided them into flocks of 75 each: how many flocks had he?

294. When the *product* of three numbers and *two* of the numbers are given, to find the *other* number.

Divide the given product by the product of the two given numbers, and the quotient will be the other number.

22. There are three numbers whose product is 60; one of them is 3, and another 5: it is required to find the other number. *Ans.* 4.

23. The product of A, B, and C's ages, is 210 years; the age of A is 5 years, and that of B is 6 years: what is the age of C?

24. The product of three boys' marbles, is 1728; two of them have a dozen apiece: how many has the other?

QUEST.—292. When the divisor and quotient are given, how is the dividend found? 293. When the dividend and quotient are given, how is the divisor found? 294. When the product of three numbers and two of them are given, how is the other found?

SECTION XI.

ANALYSIS.

ART. **295.** Business men have a method of solving practical questions, which is frequently *shorter* and *more expeditious* than that of arithmeticians fresh from the schools. If asked by what rule they perform them, their reply is, "they do them in *their head*," or by the "*no rule method*."

Their method consists in *Analysis*, and may, with propriety, be called the COMMON SENSE RULE.

295.*a.* The term *analysis*, in its general sense, signifies the *resolving* of a compound body into its *elements*.

The *analysis* of a *number* is the resolving of it into its *factors*.

The *analysis* of a *question* or *problem*, is the resolving of it into its *several conditions* or *parts*.

An *analytic solution* is the process of finding the *answer* to a question or problem, by tracing the relation of its parts to each other; or by reasoning from a *given number* or *part* to *one*, then from *one* to the *number* or *part* required, according to the conditions of the question.

OBS. In the preceding sections, the student has become acquainted with the method of *analyzing particular examples* and *combinations* of numbers, and thence deducing general *principles* and *rules*. But analysis may be applied with advantage not only to the development of mathematical *truths*, but also to the *solution* of large classes of problems both in arithmetic and practical life.

MENTAL EXERCISES.

Ex. 1. If 8 barrels of flour cost $40, how much will 5 barrels cost?

Analysis.—1 is 1 eighth of 8: therefore 1 barrel will cost 1 eighth as much as 8 barrels; and 1 eighth of $40 is $5. Now if 1 barrel costs $5, 5 barrels will cost 5 times as much; and

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QUEST.—295. What is said of the method by which business men solve practical questions? In what does their method consist? What may it with propriety be called? 295.*a.* What is the meaning of the term analysis? What is meant by the analysis of a number? The analysis of a question or problem? What is an analytic solution? *Obs.* To what may analysis be advantageously adapted?

5 times \$5 are \$25. Therefore, if 8 barrels of flour cost \$40, 5 barrels will cost \$25.

Or, we may reason thus: 5 barrels are $\frac{5}{8}$ of 8 barrels; therefore 5 barrels will cost $\frac{5}{8}$ of \$40. Now 1 eighth of \$40 is \$5, and 5 eighths is 5 times \$5, which is \$25.

2. If 7 lbs. of tea cost 42 shillings, what will 10 lbs. cost?

3. If 9 sheep are worth \$27, how much are 15 sheep worth?

4. If 10 barrels of flour cost \$60, what will 12 barrels cost?

5. Suppose 30 gallons of molasses cost \$15, how many dollars will 7 gallons cost?

6. If a man earns 54 shillings in 6 days, how much can he earn in 15 days?

7. If 12 men can build 48 rods of wall in a day, how many rods can 20 men build in the same time?

8. A gentleman divided 90 shillings equally among 15 beggars: how many shillings did 7 of them receive?

9. Suppose 75 lbs. of butter last a family of boarders 25 days, how many pounds will supply them for 12 days?

10. If 7 yards of cloth cost \$30, how much will 9 yds. cost?

11. If 10 bbls. of beef cost \$72, how much will 8 bbls. cost?

12. If 7 acres of land cost \$50, what will 12 acres cost?

13. A farmer bought an ox-cart, and paid \$15 down, which was $\frac{3}{10}$ of the price of it: what was the price of the cart; and how much did he owe for it?

Analysis.—The question is simply this: 15 is $\frac{3}{10}$ of what number? If 15 is $\frac{3}{10}$, $\frac{1}{10}$ is $\frac{1}{3}$ of 15, which is 5. Now if 5 is 1 tenth, 10 tenths is 10 times 5, which is 50. Therefore the price of the cart was \$50; and the sum he owed was \$50—\$15, which is \$35.

Note.—In solving examples of this kind, the learner is often perplexed in finding the value of $\frac{1}{10}$, &c. This difficulty arises from supposing that if $\frac{3}{10}$ of a certain number is 15, $\frac{1}{10}$ of it must be $\frac{1}{10}$ of 15. This mistake will be easily avoided by substituting in his mind the word *parts* for the given *denominator*.

Thus, if 3 parts cost \$15, 1 part will cost $\frac{1}{3}$ of \$15, which is \$5. But this part is a *tenth*. Now if 1 tenth cost \$5, then 10 tenths will cost 10 times as much.

14. A man bought a yoke of oxen, and paid \$56 cash down, which was $\frac{7}{8}$ of the price of them: what did they cost?

15. A merchant bought a quantity of wood and paid \$45 in goods, which was $\frac{5}{8}$ of the whole cost: how much did he pay for the wood?

16. The crew of a whale ship having been out 24 months, found they had consumed $\frac{4}{7}$ of their provisions: how many months' provisions had they when they embarked; and how much longer would their provisions last? 14 72

17. How many times 7 in $\frac{4}{5}$ of 35?

Analysis.—$\frac{1}{5}$ of 35 is 7, and $\frac{4}{5}$ is 4 times 7, which is 28. Now 7 is contained in 28, 4 times. Therefore 7 is contained in $\frac{4}{5}$ of 35, *four* times.

18. How many times 6 in $\frac{7}{9}$ of 45?
19. How many times 10 in $\frac{7}{6}$ of 60?
20. How many times 12 in $\frac{6}{7}$ of 84?
21. $\frac{5}{7}$ of 42 are how many times 6?
22. $\frac{7}{8}$ of 40 are how many times 5?
23. $\frac{6}{10}$ of 80 are how many times 12?
24. $\frac{5}{6}$ of 48 are how many times 4?
25. $\frac{7}{8}$ of 64 are how many times 7?
26. $\frac{9}{10}$ of 100 are how many times 12?
27. $\frac{7}{11}$ of 110 are how many times 8?
28. $\frac{8}{9}$ of 180 are how many times 10?
29. $\frac{5}{12}$ of 84 are how many times 9? 3 $\frac{8}{9}$

30. How many yards of cloth, at $7 per yard, can be bought for $\frac{7}{9}$ of $54? 6

31. How many barrels of flour, at $5 per barrel, can be bought for $\frac{2}{3}$ of $60? 8

32. A man had $64 in his pocket, and paid $\frac{5}{8}$ of it for 10 barrels of flour: how much was that per barrel? 4

33. 40 is $\frac{5}{9}$ of how many times 6?

Analysis.—Since 40 is $\frac{5}{9}$, $\frac{1}{9}$ is $\frac{1}{5}$ of 40, which is 8; and $\frac{9}{9}$ is 9 times 8, or 72. Now 6 is contained in 72, 12 times. Therefore 40 is $\frac{5}{9}$ of 12 times 6.

34. 56 is $\frac{8}{9}$ of how many times 7?
35. 81 is $\frac{9}{10}$ of how many times 30?
36. 72 is $\frac{8}{11}$ of how many times 9?
37. 96 is $\frac{8}{9}$ of how many times 12?
38. 64 is $\frac{8}{10}$ of how many times 20?
39. 54 is $\frac{6}{8}$ of how many times 24?
40. 108 is $\frac{9}{11}$ of how many times 12?

41. Frank sold 10 peaches, which was $\frac{2}{9}$ of all he had: he then divided the remainder equally among 5 companions: how many did they receive apiece? 7

42. Lincoln spent 60 cents for a book, which was $\frac{10}{12}$ of his money; the remainder he laid out for oranges, at 4 cents apiece: how many oranges did he buy?

43. A man paid away \$35, which was $\frac{5}{7}$ of all he had; he then laid out the rest in cloth at \$2 per yard: how many yards did he obtain?

44. A farmer bought a quantity of goods, and paid \$20 down, which was $\frac{2}{3}$ of the bill: how many cords of wood, at \$3 per cord, will it take to pay the balance?

45. 60 is $\frac{5}{7}$ of how many times the product of 2×3?

46. $\frac{3}{9}$ of 27 is $\frac{3}{4}$ of what number?

Analysis.—$\frac{3}{9}$ of 27 is 9. And if 9 is $\frac{3}{4}$ of a certain number, $\frac{1}{4}$ of that number is 3; and $\frac{4}{4}$ is 4 times 3, which is 12. Therefore $\frac{3}{9}$ of 27 is $\frac{3}{4}$ of 12.

47. $\frac{4}{10}$ of 30 is $\frac{2}{7}$ of what number?

48. $\frac{3}{8}$ of 40 is $\frac{5}{7}$ of what number?

49. $\frac{4}{7}$ of 35 is $\frac{2}{10}$ of what number?

50. $\frac{7}{9}$ of 54 is $\frac{6}{15}$ of what number?

51. $\frac{4}{5}$ of 30 is $\frac{12}{10}$ of how many times 8?

Analysis.—$\frac{1}{5}$ of 30 is 6, and $\frac{4}{5}$ is 4 times 6, or 24. Now, if 24 is $\frac{12}{10}$ of a certain number, $\frac{1}{10}$ of that number is $\frac{1}{12}$ of 24, which is 2, and $\frac{10}{10}$ or the whole number is 10 times 2, or 20. But 8 is contained in 20, $2\frac{1}{2}$ times. Therefore, $\frac{4}{5}$ of 30 is $\frac{12}{10}$ of $2\frac{1}{2}$ times 8.

52. $\frac{5}{8}$ of 48 is $\frac{10}{11}$ of how many times 7?

53. $\frac{6}{9}$ of 36 is $\frac{8}{12}$ of how many times 10?

54. $\frac{3}{9}$ of 45 is $\frac{5}{10}$ of how many times 9?

55. $\frac{4}{7}$ of 56 is $\frac{3}{20}$ of how many times 12?

56. $\frac{4}{6}$ of 30 is $\frac{5}{7}$ of how many eighths of 40?

Analysis.—$\frac{1}{6}$ of 30 is 5, and $\frac{4}{6}$ is 4 times 5, or 20. Now, as 20 is $\frac{5}{7}$ of a number, $\frac{1}{7}$ of that number is $\frac{1}{5}$ of 20, which is 4, and $\frac{7}{7}$ or the whole is 7 times 4, or 28. Again, $\frac{1}{8}$ of 40 is 5, and 5 is in 28, $5\frac{3}{5}$ times. Therefore, $\frac{4}{6}$ of 30 is $\frac{5}{7}$ of $5\frac{3}{5}$ *eighths* of 40.

57. $\frac{2}{3}$ of 24 is $\frac{4}{5}$ of how many sevenths of 28?

58. $\frac{3}{4}$ of 36 is $\frac{9}{10}$ of how many sixths of 42?

59. $\frac{5}{6}$ of 48 is $\frac{8}{9}$ of how many tenths of 60?

60. $\frac{7}{9}$ of 54 is $\frac{6}{10}$ of how many ninths of 81?

EXERCISES FOR THE SLATE.

296. No specific directions can be given for solving examples by *analysis.* None in fact, are requisite. The *judgment,* from the conditions of the question, will *suggest the process.*

It may however be remarked that, in general, we reason from the *given number* to 1, then from 1 to the *number required.*

OBS. In reciting the following examples, the pupil should analyze each of them, and give the reason for every step, as in the preceding mental exercises.

Ex. 1. If 40 barrels of beef cost \$320, how much will 52 barrels cost?

Analytic Solution.—Since 40 bbls. cost \$320, 1 bbl. will cost $\frac{1}{40}$ of \$320, or \$8. Now if 1 bbl. cost \$8, 52 bbls. will cost 52 times as much, or \$416. Therefore, if 40 bbls. of beef cost \$320, 52 bbls. will cost \$416.

First Method.

$$\begin{array}{rll} \$40)\underline{\$320}, & \text{cost of 40 bbls.} \\ \$8 & \text{" 1 bbl.} \\ \underline{52} & \\ \$416 & \text{" 52 bbls.} \end{array}$$

Or, thus: 52 bbls. are $\frac{52}{40}$ of 40 bbls.; therefore 52 bbls. will cost $\frac{52}{40}$ of \$320; and $\frac{52}{40}$ of \$320=\$416. (Arts. 132, 133.)

Second Method.

$\$320 \times \frac{52}{40} = \$416.$

OBS. 1. Other solutions of this example might be given; but our present object is to show how this and similar examples may be solved by analysis. The former method is the simplest, though not so short as the latter. It contains two steps:

First, we divide the price of 40 bbls. (\$320) into 40 equal parts, to find the value of one part, or the cost of 1 bbl., which is \$8.

Second, we multiply the price of 1 bbl. (\$8) by 52, the number of barrels, whose cost is required, and the product is the answer sought.

2. This, and similar questions, are usually placed under Simple Proportion, or the "Rule of Three;" but business men almost invariably solve them by *analysis.*

2. If 30 cows cost \$360.90, how much will 47 cows cost, at the same rate?

3. If 25 barrels of apples cost \$15, how much will 37 barrels cost?

4. If 15 hogsheads of molasses cost \$450, how much will 21 hogsheads cost?

QUEST.—296. Can any particular rules be given for solving questions by analysis? How then will you know how to proceed?

5. If 31 yards of cloth cost $127, how much will 89 yards cost?

6. If 55 tons of hay cost $660, what will 17 tons come to?

7. An agent paid $159 for 530 pounds of wool: how much was that per 100?

8. A man bought 30 cords of wood for $76.80: how much must he pay for 65 cords?

9. A gentleman bought 85 yards of carpeting for $106.25: how much would 38 yards cost?

10. A drover bought 350 sheep for $525: how much would 65 cost, at the same rate?

11. If $12\frac{1}{2}$ pounds of coffee cost $1.25, how much will 45 pounds cost?

12. If $16\frac{1}{2}$ bushels of corn are worth $8, how much are 25 bushels worth?

13. Paid $20 for 60 pounds of tea: how much would $12\frac{1}{2}$ pounds cost, at the same rate?

14. Bought 41 yards of flannel for $16.40: how much would $8\frac{3}{5}$ yards cost?

15. Bought 18 pounds of ginger for $4.50: how much will $10\frac{3}{5}$ pounds cost?

16. If a stage goes 84 miles in 12 hours, how far will it go in $15\frac{1}{2}$ hours?

17. If 16 horses eat 72 bushels of oats in a week, how many bushels will 25 horses eat in the same time?

18. If a railroad car runs 120 miles in 5 hours, how far will it run in $12\frac{3}{4}$ hours?

19. If a steamboat goes 180 miles in 12 hours, how far will it go in $5\frac{3}{8}$ hours?

20. If 4 men can do a job of work in 48 days, how long will it take 24 men to do it?

Analysis.—Since the job requires 4 men 48 days, it will evidently require 1 man 4 times 48 days, or 192 days. Again, if it requires 1 man 192 days, it will require 24 men $\frac{1}{24}$ part of 192 days, which is 8 days. Therefore, if 4 men can do the job in 48 days, 24 men can do it in 8 days.

Operation.

$$\begin{array}{r} 48 \text{ days.} \\ \underline{4} \\ 24)\overline{192} \text{ days.} \\ \textit{Ans. } 8 \text{ days.} \end{array}$$

21. If 18 men eat a barrel of flour in 72 days, how long will it last 27 men?

22. If a given quantity of corn lasts 90 horses 96 days, how long will the same quantity last 45 horses?

23. If 36 men can build a house in 180 days, how long will it take 120 men to build it?

24. If 100 barrels of pork last a crew of 20 men 45 months, how long will it last a crew of 28 men?

25. If 4 stacks of hay will keep 60 cattle 120 days, how long will they keep 25 cattle?

26. If $\frac{3}{8}$ of a bushel of wheat cost 30 cents, what will $\frac{7}{8}$ of a bushel cost?

27. If $\frac{5}{8}$ of a ton of hay cost \$7, what will $\frac{3}{9}$ of a ton cost?

28. If $\frac{3}{5}$ of a pound of imperial tea cost 27 cents, how much will $\frac{4}{5}$ of a pound cost?

29. If $\frac{3}{7}$ of a ton of coal cost \$2.61, how much will $\frac{6}{7}$ of a ton cost?

30. If $\frac{2}{3}$ of a yard of silk cost 11 shillings, how much will $\frac{7}{8}$ of a yard cost?

Analysis.—Since $\frac{2}{3}$ of a yard cost 11s. $\frac{1}{3}$ yard will cost $\frac{1}{2}$ of 11s. or $5\frac{1}{2}$s., and $\frac{3}{3}$ will cost 3 times as much, which is $16\frac{1}{2}$s. Again, if 1 yard costs $16\frac{1}{2}$s., $\frac{1}{8}$ yard will cost $\frac{1}{8}$ of $16\frac{1}{2}$s. or $2\frac{1}{16}$s., and $\frac{7}{8}$ will cost 7 times as much, which is $14\frac{7}{16}$s. Therefore, if $\frac{2}{3}$ of a yard of silk cost 11s., $\frac{7}{8}$ of a yard will cost $14\frac{7}{16}$s.

Operation.

	2)11s.	cost	$\frac{2}{3}$ y.
	$5\frac{1}{2}$s.	"	$\frac{1}{3}$ y.
	3		
	8)$16\frac{1}{2}$s.	"	1 y.
	$2\frac{1}{16}$s.	"	$\frac{1}{8}$ y.
	7		
Ans.	$14\frac{7}{16}$s.	"	$\frac{7}{8}$ y.

31. If $\frac{4}{5}$ of a cord of wood cost \$1.80, how much will $\frac{3}{7}$ of a cord cost?

32. If $\frac{2}{9}$ of a yard of broadcloth cost 14 shillings, how much will $\frac{3}{4}$ of a yard cost?

33. A man bought $\frac{7}{8}$ of an acre of land for \$56, and afterwards sold $\frac{2}{5}$ of an acre at cost: how much did he receive for it?

34. A grocer bought 7 barrels of vinegar for \$28, and sold $\frac{3}{4}$ of a barrel at cost: how much did it come to?

35. A grocer bought a firkin of butter containing 56 lbs. for \$11.20, and sold $\frac{3}{4}$ of it for $\$8\frac{2}{5}$: how much did he get a pound?

36. If $6\frac{1}{4}$ bushels of peas are worth \$5.50, how much are $20\frac{1}{2}$ bushels worth?

37. If a man pays \$47 for building $23\frac{1}{2}$ rods of ornamental fence, how much would it cost him to build $42\frac{3}{4}$ rods?

38. A farmer paid \$45.42 for making $36\frac{2}{5}$ rods of stone wall: how much will it cost him to make $60\frac{7}{10}$ rods?

39. A man paid $\frac{20}{100}$ of a dollar for 4 pounds of veal: how much would a quarter of veal cost, which weighs 20 pounds?

40. If 5 pounds of butter cost $4\frac{2}{7}$ shillings, how much will 42 pounds cost?

Suggestion.—$4\frac{2}{7}$s. $=\frac{30}{7}$s. (Art. 122.) Therefore 1 pound will cost $\frac{6}{7}$s.; and 42 lbs. will cost 42 times as much, or 36s. *Ans.*

41. If 20 lbs. of cheese cost $\$3\frac{6}{8}$, how much will 168 pounds cost?

42. If 30 yards of cotton cost $\$4\frac{1}{2}$, how much will a piece containing 19 yards cost?

43. If $\frac{5}{12}$ of a cord of wood costs $\frac{5}{8}$ of a dollar, how much will $\frac{3}{4}$ of a cord cost?

Analysis.—Since $\frac{5}{12}$ of a cord cost $\$\frac{5}{8}$, $\frac{1}{12}$ will cost $\$\frac{1}{8}$; and $\frac{12}{12}$ or 1 cord will cost $\$\frac{12}{8}$. Again, if 1 cord cost $\$\frac{12}{8}$, $\frac{1}{4}$ of a cord will cost $\$\frac{3}{8}$; and $\frac{3}{4}$ will cost $\frac{9}{8}$, or $\$1\frac{1}{8}$. Therefore, if $\frac{5}{12}$ cord of wood cost $\$\frac{5}{8}$, $\frac{3}{4}$ of a cord, at the same rate, will cost $\$1\frac{1}{8}$.

$\$\frac{5}{8}\div 5 = \$\frac{1}{8}$, cost $\frac{1}{12}$ c.
$\$\frac{1}{8}\times 12 = \frac{12}{8}$, " 1 c.
$\$\frac{12}{8}\div 4 = \$\frac{3}{8}$, " $\frac{1}{4}$ c.
$\$\frac{3}{8}\times 3 = \$\frac{9}{8}$, " $\frac{3}{4}$ c.
$\$\frac{9}{8}=\$1\frac{1}{8}$. *Ans.*
Or, $\$\frac{5}{8}\div\frac{5}{12}=\$\frac{60}{40}$, cost 1 c.
$\$\frac{60}{40}\times\frac{3}{4}=\$\frac{180}{160}$, or $\$1\frac{1}{8}$.

44. If $\frac{3}{4}$ of a yard of cloth cost £$\frac{6}{7}$, how much will $\frac{7}{8}$ of a yard cost?

45. If $\frac{3}{16}$ of a ship cost \$16000, how much is $\frac{5}{8}$ of her worth?

46. A man bought a quantity of land, and sold $\frac{9}{10}$ of an acre for \$63, which was only $\frac{3}{4}$ of the cost: how much did he give per acre?

47. If $7\frac{1}{2}$ yards of satinet cost $\$9\frac{3}{8}$, how much will $18\frac{1}{2}$ yards cost?

48. A ship's company of 30 men have 4500 pounds of flour: how long will it last them, allowing each man $2\frac{1}{2}$ lbs. per day?

49. How long will 56700 pounds of meat last a garrison of 756 soldiers, allowing each man $\frac{3}{4}$ lb. per day?

50. How long will the same quantity of meat last the same garrison, allowing $1\frac{1}{2}$ lb. apiece per day?

51. A merchant sold 22 yards of silk, at 7 shillings per yard, and took his pay in wheat, at 11 shillings per bushel: how many bushels did it take?

Suggestion.—We first find the cost of the silk, which is 154s. The next step is to find how many bushels of wheat it will take to pay this 154s. Now as the wheat is 11s. a bushel, it will evidently take as many bushels as 11s. are contained times in 154s., which is 14. Therefore it will take 14 bu. of wheat, at 11s. per bu., to pay for 22 yds. of silk, at 7s. per yard.

Operation.

$$\begin{array}{r} 7\text{s.} \\ 22 \\ 11\overline{)154}\text{s.} \\ \textit{Ans.}\ \overline{14}\ \text{bu.} \end{array}$$

297. The last and similar examples are sometimes placed under a rule called *Barter*.

Barter signifies an exchange of articles of commerce, at prices agreed upon by the parties.

OBS. Such examples are so easily solved by *Analysis*, that a *specific rule* for them is *unnecessary*.

52. A shoemaker sold 64 pair of boots at 32s. 6d. a pair, and took his pay in corn at 3s. 4d. per bushel: how many bushels did he receive?

53. A man bought 50 pounds of sugar at $12\frac{1}{2}$ cents a pound, and was to pay for it in wood at \$3.12$\frac{1}{2}$ per cord: how many cords did it take?

54. How many pair of hose, at 3s. 9d. a pair, will it take to pay for 135 pounds of tea at 6s. 4d. a pound?

55. How many pounds of butter at $17\frac{1}{2}$ cents a pound, must be given in exchange for 186 yards of calico at $18\frac{3}{4}$ cents per yard?

56. How many pounds of tobacco at $16\frac{1}{4}$ cents a pound, must be given in exchange for 256 pounds of sugar at $6\frac{1}{4}$ cents a pound?

57. A farmer bought 325 sheep at \$2$\frac{1}{4}$ apiece, and paid for them in hay at \$10$\frac{1}{2}$ per ton: how many tons did it take?

58. A man bought a hogshead of molasses worth $37\frac{1}{2}$ cents per gallon, and gave $331\frac{1}{2}$ pounds of cheese in exchange: how much was the cheese a pound?

QUEST.—297. What is meant by Barter? *Obs.* Is a specific rule necessary for such operations?

59. Bought 74 bushels of salt at $42\frac{1}{2}$ cents per bushel, and paid in oats at $\frac{1}{4}$ of a dollar per bushel: how many oats did it require?

60. A bookseller exchanges 400 dictionaries worth $87\frac{1}{2}$ cents apiece, for 700 grammars: how much did the grammars cost apiece?

61. How many yards of silk worth $\$1\frac{3}{4}$ per yard, will pay for $249\frac{1}{2}$ yards of cloth worth $\$5\frac{1}{4}$ per yard?

62. Bought 19 cwt. 2 qrs. 15 lbs. of sugar at $\$9\frac{1}{2}$ per hundred, and paid for it in butter at $7\frac{1}{2}$ cents a pound: how much butter did it take?

63. Bought 263 yds. 3 qrs. of satinet at $\$1\frac{3}{4}$ per yard, and paid for it in cheese at $\$9\frac{1}{2}$ per hundred: how much cheese did it take?

64. Bought 125 hhds. 22 gals. 3 qts. of molasses at $37\frac{1}{2}$ cents per gallon, and paid for it in wool at $62\frac{1}{2}$ cents a pound: how much wool did it take?

65. Bought .778125 ton of indigo at $\$4\frac{1}{2}$ a pound: how much cloth at $\$5\frac{7}{8}$ per yard will pay for it?

66. Bought .45683 acre of land at $\$\frac{3}{4}$ per square foot: how many cords of wood did it require at $\$3\frac{3}{5}$ per cord to pay for it?

67. How many barrels of flour at $\$6\frac{1}{4}$ per barrel, must be given in exchange for 45 tons, 15 cwt. 20 lbs. of coal, at $\$7\frac{1}{2}$ per ton?

68. A goldsmith sold a tankard for £12, 8s., which was 5s. 4d. per ounce, and agreed to take as many yards of silk as there were ounces in the tankard: how many yards did he receive?

69. Bought 432 sheep at $\$2\frac{1}{4}$ apiece, for which I paid 144 barrels of flour: what was the flour per barrel?

70. If 15 yards of domestic flannel are worth 25 yards of muslin, how many yards of flannel are worth 15 yards of muslin?

71. A market-woman bought 10 dozen oranges at the rate of 3 for 4 cents, and then exchanged them for eggs at the rate of 4 for 5 cents: how many eggs did she receive?

72. If 15 lbs. of pepper are worth 25 lbs. of ginger, how many pounds of ginger must be given for 195 lbs. of pepper?

73. If 11 boys can earn as much as 5 men, how many boys can earn as much as 145 men?

74. How much cotton at \$7.50 per hundred, must be given for 175 tons, 10 cwt. 15 lbs. of flour, at \$2.25 per hundred?

75. How much soap at $10\frac{1}{2}$ cents a pound, must be given for 17 cwt. $10\frac{1}{2}$ lbs. of potash at $6\frac{1}{4}$ cents a pound?

76. Three men, A, B, and C join in an adventure; A puts in \$200; B, \$300; and C, \$400; and they gain \$72: how much is each man's share of the gain?

Analysis.—The whole sum invested is \$200+\$300+\$400=\$900. Now, since \$900 gain \$72, \$1 will gain $\frac{1}{900}$ of \$72; and \$72÷900=\$.08. Again,

If \$1 gains 8c.,	\$200 will gain	\$200×.08=	\$16,	A's share.
" 1 "	300 "	300×.08=	24,	B's "
" 1 "	400 "	400×.08=	32,	C's "

Or, we may reason thus: since the sum invested is \$900,

A's part of the investment is $\frac{200}{900}$, which is equal to $\frac{2}{9}$;
B's " " is $\frac{300}{900}$, " " $\frac{3}{9}$;
C's " " is $\frac{400}{900}$, " " $\frac{4}{9}$;

A must therefore receive $\frac{2}{9}$ of \$72 (the gain)=\$16
B " " $\frac{3}{9}$ of 72 " = 24
C " " $\frac{4}{9}$ of 72 " = 32

PROOF.—The whole gain is . . . \$72. (Ax. 11.)

298. When two or more individuals associate themselves together for the purpose of carrying on a joint business, the union is called a *partnership* or *copartnership*.

OBS. The process by which examples like the last one are commonly solved, is called *Partnership*, or *Fellowship*.

77. A and B entered into partnership; A furnished \$400, and B \$500; they gained \$300: how much was each man's share of the gain?

78. A, B, and C hired a farm together, for which they paid \$175 rent; A advanced \$75; B, \$60; and C, \$40. They raised 250 bushels of wheat: what was each man's share?

79. A, B, and C together spent \$1000 in lottery tickets. A put in \$400; B, \$250; and C, \$350; they drew a prize of \$1500: how much was each man's share?

80. A, B, C, and D fitted out a whale ship; A advanced \$10000; B, \$12000; C, \$15000; and D, \$8000; the ship brought home 3000 bbls. of oil: what was each man's share?

81. A, B, and C formed a partnership; A furnished $900; B, $1500; and C, $1200; they lost $1260: what was each man's share of the loss?

82. X, Y, and Z entered into a joint speculation, on a capital of $20000, of which X furnished $5000; Y, $7000; and Z the balance; their net profits were $5000 per annum: what was the share of each?

83. A bankrupt owes one of his creditors $300; another $400; and a third $500; his property amounts to $800: how much can he pay on a dollar; and how much will each of his creditors receive?

Note.—The solution of this example is the same in principle as that of example seventy-sixth.

299. A *bankrupt* is a person who is insolvent, or unable to pay his just debts.

OBS. Examples like the preceding one are sometimes arranged under a rule called *Bankruptcy.*

84. A bankrupt owes $2000, and his property is appraised at $1600: how much can he pay on a dollar?

85. A man failing in business, owes A $156.45; B $256.46; and C $360.40; and his effects are valued at $317: how much will each man receive?

86. The whole effects of a man failing in business amounted to $3560, he owed $35600: how much can he pay on a dollar; and how much will B receive, who has a claim on him of $5000?

87. A man died insolvent, owing $55645; and his property was sold at auction for $2350: how much will his estate pay on a dollar?

88. How much can a bankrupt, who has $6540 real estate, and owes $56000, pay on a dollar?

300. It often happens in storms and other casualties at sea, that masters of vessels are obliged to throw portions of their cargo overboard, or sacrifice their ship and crew. In such cases, the law requires that the loss shall be divided among the owners of the vessel and cargo, in proportion to the amount of each one's property at stake.

The process of finding each man's loss, in such instances, is called *General Average.*

OBS. The operation is the same as that in solving questions in bankruptcy and partnership.

89. A, B, and C freighted a sloop with flour from New York to Boston; A had on board 600 barrels; B, 400; and C, 200. On her passage 200 barrels were thrown overboard in a gale, and the loss was shared among the owners according to the quantity of flour each had on board: what was the loss of each?

90. A Liverpool packet being in distress, the master threw goods overboard to the amount of $10000. The whole cargo was valued at $72000, and the ship at $28000: what per cent. loss was the general average; and how much was A's loss, who had goods aboard to the amount of $15000?

91. A coasting vessel being overtaken in a gale, the master was obliged to throw overboard part of his cargo, valued at $15500. The whole cargo was worth $85265, and the vessel $17000: what per cent. was the general average; and what was the loss of the master, who owned $\frac{1}{4}$ of the vessel?

92. A farmer mixed 15 bushels of oats worth 2 shillings per bushel, with 5 bushels of corn worth 4 shillings per bushel: what is the mixture worth per bushel?

Solution.—15 bu. at 2s.=30s., value of oats.
 5 bu. at 4s.=20s., value of corn.
 20 bu. mixed 50s., value of whole mixture.

Now, if 20 bu. mixture are worth 50s., 1 bu. is worth $\frac{1}{20}$ of 50s., which is $2\frac{1}{2}$s., the answer required.

PROOF.—20 bu. $\times$ $2\frac{1}{2}$s.=50s. the value of the whole mixture.

93. A miller has a quantity of rye worth 6s. per bushel, and wheat worth 9s. per bushel; he wishes to make a mixture of them which shall be worth 8s. per bushel: what part of each must the mixture contain?

Analysis.—The difference in their prices per bushel is 3s.; hence, the difference in the price of 1 third of a bushel of each is 1s. Now, if 1 third of a bushel is taken from a bushel of rye, the remaining 2 thirds will be worth 4s.; and if 1 third of a bushel of wheat, which is worth 3s., be added to the rye, the mixture will be worth 7s. Again, if $\frac{2}{3}$ of a bushel is taken from a bushel of rye, the remaining third will be worth 2s.,

and if $\frac{2}{3}$ of a bushel of wheat, which is worth 6s., be added to the rye, the mixture will be worth 8s.; therefore $\frac{1}{3}$ of a bushel of rye added to $\frac{2}{3}$ of wheat, will make a mixture of 1 bushel, which is worth 8 shillings; consequently the mixture must be $\frac{1}{3}$ rye and $\frac{2}{3}$ wheat; or 1 part rye to 2 parts wheat.

PROOF.—Since 1 bushel of rye is worth 6s., $\frac{1}{3}$ bu. is worth $\frac{1}{3}$ of 6s., or 2s.; and as 1 bu. of wheat is worth 9s., $\frac{2}{3}$ bu. is worth $\frac{2}{3}$ of 9s., or 6s.; and 6s.+2s.=8s.

OBS. If we make the difference between the less price and the price of the mixture, the numerator, and the difference between the prices of the commodities to be mixed, the denominator, the fraction will express the part to be taken of the higher priced article; and if we place the difference between the higher price and the price of the mixture over the same denominator, the fraction will express the part to be taken of the lower priced article.

94. A goldsmith has a quantity of gold 16 carats fine, and another quantity 22 carats fine; he wishes to make a mixture 20 carats fine: what part of each will the mixture contain?

Ans. $\frac{2}{6}$ of 16 carats fine, and $\frac{4}{6}$ of 22 carats fine.

301. Examples requiring a mixture of commodities of different values, like the last three, are commonly arranged under the rule of *Alligation.*

OBS. Alligation is usually divided into *medial* and *alternate.* The 92d example is an instance of Medial Alligation; the 93d and 94th are instances of Alternate Alligation. Questions in the latter very seldom occur in practical life.

95. A grocer mixes 50 pounds of tea worth 4 shillings a pound, with 100 lbs. worth 7s. a pound, what is a pound of the mixture worth?

96. A milk-man mixed 30 quarts of water with 120 quarts of milk, worth 5 cents per quart: what is a quart of the mixture worth?

97. A farmer made a mixture of provender containing 30 bushels of oats, worth 25 cents per bushel; 10 bushels of peas, worth 75 cents per bushel, and 15 bushels of corn, worth 50 cents per bushel: what is the value of the whole mixture; and what is it worth per bushel?

98. An oil-dealer mixed 60 gallons of whale oil, worth $31\frac{1}{4}$ cents per gallon, with 85 gallons of sperm oil, worth 90 cents per gallon: what is the mixture worth per gallon?

99. A grocer had three kinds of sugar, worth 6, 8, and 12

cents per pound; he mixed 112 lbs. of the first, 150 lbs. of the second, and 175 of the third together: what was the mixture worth per pound?

100. A goldsmith melted 10 oz. of gold 20 carats fine, with 8 oz. 22 carats fine, and 4 oz. of alloy: how many carats fine was the mixture?

101. If 4 men reap 12 acres in 2 days, how long will it take 9 men to reap 36 acres?

Analysis.—If 4 men can reap 12 acres in 2 days, 1 man can reap $\frac{1}{4}$ of 12 acres in the same time; and $\frac{1}{4}$ of 12 acres is 3 acres. But if 1 man can reap 3 acres in 2 days, in 1 day he can reap $\frac{1}{2}$ of 3 acres, and $\frac{1}{2}$ of 3 is $1\frac{1}{2}$ acre. Again, if $1\frac{1}{2}$ acre requires a man 1 day, 36 acres will require him as many days as $1\frac{1}{2}$ is contained times in 36; and $36 \div 1\frac{1}{2} = 24$ days. Now if 1 man can reap the given field in 24 days, 9 men will reap it in $\frac{1}{9}$ of the time; and $24 \div 9 = 2\frac{2}{3}$.

Ans. 9 men can reap 36 acres in $2\frac{2}{3}$ days.

OBS. This and similar examples are usually placed under Compound Proportion, or "Double Rule of Three."

102. If 7 men can reap 42 acres in 6 days, how many men will it take to reap 100 acres in 5 days?

103. If 14 men can build 84 rods of wall in 3 days, how long will it take 20 men to build 300 rods?

104. If 1000 barrels of provisions will support a garrison of 75 men for 3 months, how long will 3000 barrels support a garrison of 300?

105. If a man travels 320 miles in 10 days, traveling 8 hours per day, how far can he go in 15 days, traveling 12 hours per day?

106. If 24 horses eat 126 bushels of oats in 36 days, how many bushels will 32 horses eat in 48 days?

107. A lad returning from market being asked how many peaches he had in his basket, replied that $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of them made 52: how many peaches had he?

Analysis.—The sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4} = \frac{13}{12}$. (Art. 127.) The question then resolves itself into this: 52 is $\frac{13}{12}$ of what number? Now if 52 is $\frac{13}{12}$, $\frac{1}{12}$ is $\frac{1}{13}$ of 52, which is 4; and $\frac{12}{12}$ is $4 \times 12 = 48$. *Ans.* 48 peaches.

PROOF.—$\frac{1}{2}$ of 48 is 24; $\frac{1}{3}$ is 16; and $\frac{1}{4}$ is 12. Now, $24 + 16 + 12 = 52$.

302. This and similar examples are commonly placed under the Rule of *Position*.

OBS. The shortest and easiest method of solving them is by *Analysis*.

108. A farmer lost $\frac{1}{2}$ of his sheep by sickness; $\frac{1}{3}$ were destroyed by wolves; and he had 72 sheep left: how many had he at first?

109. A person having spent $\frac{1}{2}$ and $\frac{1}{3}$ of his money, finds he has \$48 left: what had he at first?

110. After a battle a General found that $\frac{1}{6}$ of his army had been taken prisoners, $\frac{1}{8}$ were killed, $\frac{1}{12}$ had deserted, and he had 900 left: how many had he at the commencement of the action?

111. What number is that $\frac{1}{3}$ and $\frac{1}{4}$ of which is 84?

112. What number is that $\frac{1}{2}$ and $\frac{1}{3}$ of which being added to itself, the sum will be 110?

113. A certain post stands $\frac{1}{3}$ in the mud, $\frac{1}{4}$ in the water, and 10 feet above the water: how long was the post?

114. Suppose I pay \$85 for $\frac{5}{8}$ of an acre of land: what is that per acre?

115. A man paid \$2700 for $\frac{3}{16}$ of a vessel: what is the whole vessel worth?

116. A gentleman spent $\frac{1}{3}$ of his life in Boston, $\frac{1}{4}$ of it in New York, and the rest of it, which was 30 years, in Philadelphia: how old was he?

117. What number is that $\frac{7}{8}$ of which exceeds $\frac{2}{3}$ of it by 10?

118. In a certain school $\frac{1}{3}$ of the scholars were studying arithmetic, $\frac{1}{4}$ algebra, $\frac{1}{6}$ geometry, and the remainder, which was 18, were studying grammar: how many scholars were there in the school?

119. A owns $\frac{1}{8}$ and B $\frac{1}{12}$ of a ship; A's part is worth \$650 more than B's: what is the value of the ship?

120. In a certain orchard $\frac{1}{3}$ are apple trees, $\frac{1}{4}$ peach trees, $\frac{1}{6}$ plumb trees, and the remaining 15 were cherry trees: how many trees did the orchard contain?

121. What will 567 yds. of sarcenet cost, at $33\frac{1}{3}$ cts. per yard?

Analysis.—The price $33\frac{1}{3}$ cents is an *aliquot* part of \$1, viz: $\frac{1}{3}$ of a dollar. Now if the price were \$1 per yard, it is plain that the cost of the whole would be as many *dollars* as there are *yards*.

Operation.

$$\begin{array}{r} 3)\underline{567} \\ \$189 \text{ } Ans. \end{array}$$

But the price per yard is $\frac{1}{3}$ of a dollar; consequently the cost

of the whole must be 1 *third* as many dollars as there are yards. And $\frac{1}{3}$ of 567 is 189. Therefore 567 yards of sarcenet, at $33\frac{1}{3}$ cents per yard, will cost $189.

122. What cost 680 tons of chalk, at 10 shillings sterling per ton?

Suggestion.—The price 10s. is an *aliquot* part of £1, (20s.) and is equal to £$\frac{1}{2}$ sterling. Therefore the cost will be 1 *half* as many pounds sterling as there are tons of chalk.

303. The *method* of solving questions by *aliquot parts*, is often called *Practice*, from the circumstance that business men practice it.

The term *practice*, however, conveys no idea of the *nature* of the operation, nor of the *principles* upon which it is based, and is falling into disuse.

OBS. If the price itself is not an aliquot part of $1, or £1, &c., it may be divided into aliquot parts of $1, or £1, or into such parts as are aliquot parts of each other. Thus, $87\frac{1}{2}$ cts. is not an aliquot part of $1, but $87\frac{1}{2}$ cts.= $50+25+12\frac{1}{2}$ cts. Now 50 cts.=\$$\frac{1}{2}$; 25 cts.=\$$\frac{1}{4}$; and $12\frac{1}{2}$ cts.=\$$\frac{1}{8}$. Or thus: 50 cts.=\$$\frac{1}{2}$, 25 cts.=$\frac{1}{2}$ of 50 cts., and $12\frac{1}{2}$ cts.=$\frac{1}{2}$ of 25 cts.

Note.—For Tables of aliquot parts of $1, £1, 1s., &c., see p. 149.

Aliquot parts of Federal Money.

123. What will 968 bushels of corn cost, at $62\frac{1}{2}$ cents per bushel?

Analysis.—$62\frac{1}{2}$ cents$=50+12\frac{1}{2}$ cents; but 50 cts.=\$$\frac{1}{2}$, and $12\frac{1}{2}$ cents=\$$\frac{1}{8}$. Now if the price were $1 per bushel, the cost would be $968. Hence,

At 50 cts., the cost would be $\frac{1}{2}$ of $968, which is $484
At $12\frac{1}{2}$ cts., " " $\frac{1}{8}$ of $968, " 121
Therefore, at $50+12\frac{1}{2}$ cts., the cost must be $605

124. What will 1268 baskets of peaches cost, at 25 cents a basket?

125. What is the cost of 480 yds. of ribbon, at $6\frac{1}{4}$ cts. per yard?

126. What cost 750 bushels of potatoes, at $33\frac{1}{3}$ cents per bushel?

127. What cost 360 barrels of cider, at $66\frac{2}{3}$ cents per barrel?

128. What cost 1564 Grammars, at $37\frac{1}{2}$ cents apiece?

129. What cost 1875 Histories, at 65 cents apiece?

130. What cost 2160 Geographies, at 84 cents apiece?

Aliquot parts of Sterling Money.

131. What cost 720 bushels of corn, at 2 shillings and 6 pence per bushel?

Suggestion.—2s. 6d.=£$\frac{1}{8}$; and 720×$\frac{1}{8}$=£90. *Ans.*

132. At 10s. 6d. per barrel, what will 350 barrels of mackerel come to?

133. At 17s. 6d. apiece, what will 540 hats come to?

134. What cost 33750 sheep, at 6s. 8d. apiece?

Aliquot parts of New York currency.

303.*a*. Notwithstanding the law requires accounts to be kept in *Federal Money*, a large amount of retail business is still done in the *denominations* of the old State currencies.

135. What will 766 Arithmetics cost, at 4s. apiece?

OBS. In N. Y. currency 8s. make \$1; therefore 4s.=\$$\frac{1}{2}$. *Ans.* \$383.

The answers to the following examples, in which the prices are given in N. Y. and N. E. currencies, are required in Federal Money.

136. What cost 1360 knives, at 2s. 8d. apiece?

137. What cost 1760 brooms, at 1s. apiece?

138. At 2s. apiece, what will 968 melons cost?

139. At 3s. a pair, what will 848 pair of gloves come to?

140. At 5s. 4d. a yard, what cost 1875 yards of balzorine?

141. At 6s. a pair, what will 2163 pair of slippers cost?

142. At 4s. 6d. a bushel, what cost 1942 bu. of wheat?

143. At 1s. 4d. apiece, what will 1673 inkstands cost?

144. At 5s. 8d. apiece, what will 1386 brushes cost?

145. What cost 1068 caps, at 6s. 6d. apiece?

146. What cost 2960 lbs. of flax, at 2s. 8d. per pound?

Aliquot parts of New England Currency.

147. What cost 861 pails, at 2s. apiece?

OBS. In N. E. cur. 6s. make \$1, therefore 2s.=\$$\frac{1}{3}$. *Ans.* 287.

148. What cost 840 chairs, at 3s. apiece?

149. What cost 1360 melons, at 1s. 6d. apiece?

150. At 4s. a bushel, what will 1124 bu. of apples cost?

151. At 4s. 6d. apiece, what will 972 thermometers cost?

152. At 5s. 3d. a pair, what cost 1372 pair of shoes?

304. In the preceding examples, the *quantity* is a *simple* number, and the *price* is an *aliquot part*, or is easily separated into aliquot parts. But questions in which the *quantity*, and those in which the *price* and *quantity* both are *compound* numbers, may also be solved by aliquot parts.

153. Cost 8 cwt. 2 qrs. 15 lbs. venison, at £2, 5s. 6d. per cwt.?

2 qrs.	$\frac{1}{2}$	£2, 5s. 6d.,	price of 1 cwt.
		8	
		18, 4, 0	" " 8 cwt.
10 lbs.	$\frac{1}{5}$	1, 2, 9	" " 2 qrs.
5 lbs.	$\frac{1}{2}$	0, 4, $6\frac{3}{5}$	" " 10 lbs.$=\frac{1}{5}$ of 2 qrs.
		2, $3\frac{3}{10}$	" " 5 lbs.$=\frac{1}{2}$ of 10 lbs.
	Ans.	£19, 13s. $6\frac{9}{10}$d.	

154. Cost 3 cwt. 2 qrs. $12\frac{1}{2}$ lbs. raisins, at $12.40 per cwt.?
155. Cost 9 cwt. 1 qr. 10 lbs. cheese, at $10.60 per cwt.?
156. Cost 12 cwt. 3 qrs. 5 lbs. sugar, at $9.356 per cwt.?
157. Cost 21 cwt. 1 qr. 10 lbs. tobacco, at $17.20 per cwt.?
158. Cost 35 cwt. 2 qrs. 20 lbs. honey, at $21.84 per cwt.?
159. Cost 43 tons, 4 cwt. 1 qr. coal, at $6.25 per ton?
160. Cost 52 tons, 5 cwt. 2 qrs. hay, at $17.30 per ton?
161. Cost 260 tons, 2 cwt. 1 qr. iron, at 45.60 per ton?
162. Cost 45 yds. 2 qrs. 1 na. satin, at $1.34 per yard?
163. Cost 84 yds. 1 qr. 2 na. cloth, at 7.90 per yard?
164. Cost 45 acres, 2 R. 20 rods land, at $24.20 per acre?
165. Cost 63 acres, 1 R. 10 rods land, at $43.64 per acre?
166. Amount of wages for 5 yrs. 6 m. 10 d., at $384 a yr.?
167. Amount of salary for 16 yrs. 4 m. 15 d., at $1872 a yr.?
168. Rent of a house 6 yrs. 3 m. 5 d., at $864 a year?
169. Cost 64 cwt. 1 qr. 10 lbs. of rice, at £2, 6s. 4d. per cwt.?
170. Cost 94 cwt. 2 qrs. 5 lbs. figs, at £3, 17s. 3d. per cwt.?
171. Cost 17 tons, 5 cwt. 2 qrs. wool, at £6, 15s. 7d. per ton?
172. Cost 35 hhds. 9 gals. 2 qts. wine, at £16, 8s. 3d. per hhd.?
173. Cost 87 hhds. 7 gals. 1 qt. oil, at £17, 9s. 7d. per hhd.?
174. Cost 139 yds. 2 qrs. 1 na. cloth, at 19s. $5\frac{1}{2}$d. per yard?
175. Cost 295 bu. 2 pks. 4 qts. wheat, at $$1.18\frac{3}{4}$ per bu.?
176. Cost 165 A. 2 R. 10 r. land, at $$76.37\frac{1}{2}$ per acre?
177. Cost 5268 quills, at $6\frac{1}{4}$ cents per dozen?
178. Cost 15263 oranges, at $$3\frac{3}{4}$ per hundred?
179. Cost 25570 cucumbers, at $$12\frac{1}{2}$ per thousand?

SECTION XII.

RATIO AND PROPORTION.

ART. **305.** RATIO *is that relation between two numbers which is expressed by the* QUOTIENT *of the one divided by the other.* Thus, the ratio of 6 to 2 is $6 \div 2$, or 3; for 3 is the quotient of 6 divided by 2.

306. The two given numbers thus compared, when spoken of together, are called a *couplet;* when spoken of separately, they are called the *terms* of the ratio.

The *first* term is the *antecedent;* and the *last,* the *consequent.*

307. Ratio is expressed in two ways:

First, in the form of a fraction, making the *antecedent* the *numerator,* and the *consequent* the *denominator.* Thus, the ratio of 8 to 4 is written $\frac{8}{4}$; the ratio of 12 to 3, $\frac{12}{3}$, &c.

Second, by placing two points or a colon (:) between the numbers compared. Thus, the ratio of 8 to 4, is written 8 : 4; the ratio of 12 to 3, 12 : 3, &c.

OBS. 1. The expressions $\frac{8}{4}$, and 8 : 4 are equivalent to each other, and one may be exchanged for the other at pleasure.

2. The English mathematicians put the antecedent for the numerator and the consequent for the denominator, as above; but the French put the consequent for the numerator and the antecedent for the denominator. The English method appears to be equally simple, and is claimed to be the most in accordance with reason.

3. In order that concrete numbers may have a ratio to each other, they must necessarily express objects so far of the same nature, that one can be properly said to be *equal* to, or *greater,* or *less* than the other. (Art. 89. 2.) Thus a foot has a ratio to a yard; for one is *three times* as long as the other; but a foot has not properly a ratio to an hour, for one cannot be said to be *longer* or *shorter* than the other.

QUEST.—305. What is ratio? 306. What are the two given numbers called when spoken of together? What, when spoken of separately? 307. In how many ways is ratio expressed? What is the first? The second? *Obs.* Which of the terms do English mathematicians put for the numerator? Which do the French? In order that concrete numbers may have a ratio to each other, what kind of objects must they express?

308. A *direct* ratio is that which arises from dividing the antecedent by the consequent, as in Art. 305.

309. An *inverse* or *reciprocal* ratio, is the ratio of the *reciprocals* of two numbers. (Art. 89. Def. 9.) Thus, the direct ratio of 9 to 3, is 9 : 3, or $\frac{9}{3}$; the reciprocal ratio is $\frac{1}{9} : \frac{1}{3}$, or $\frac{1}{9} \div \frac{1}{3} = \frac{3}{9}$; (Art. 139;) that is, the consequent 3, is divided by the antecedent 9.

Note.—The term *inverse*, signifies *inverted*. Hence,

A reciprocal ratio is expressed by inverting the fraction which expresses the direct ratio; or when the notation is by points, by inverting the order of the terms. Thus, 8 is to 4, inversely, as 4 to 8.

309.a. A *simple* ratio is a ratio which has but *one antecedent* and *one consequent*, and may be either direct or inverse; as 9 : 3, or $\frac{1}{9} : \frac{1}{3}$.

310. A *compound* ratio is the ratio of the *products* of the corresponding terms of two or more simple ratios. Thus,

The simple ratio of . . . . . 9 : 3 is 3;
And " " of . . . . . 8 : 4 is 2;
The ratio compounded of these is 72 : 12 = 6.

OBS. 1. A compound ratio is of the *same nature* as any other ratio. The term ratio is used to denote the *origin* of the ratio in particular cases.

2. A compound ratio is equal to the product of the simple ratios.

Ex. 1. What is the ratio of 14 to 7? *Ans.* 2.

2. What is the ratio of 3 to 7? *Ans.* $\frac{3}{7}$.

3. What is the ratio of 10 to 2? 16 to 4? 18 to 9? 24 to 3? 30 to 6? 25 to 5? 27 to 9? 40 to 8? 56 to 7? 84 to 12?

4. What is the ratio of 9 to 13? 10 to 17? 21 to 43?

Required the ratio of the following numbers:

5. 63 to 7	11. 11 to 55	17. 15 lbs. to 3 lbs.
6. 90 to 15	12. 12 to 84	18. 21 lbs. to 7 lbs.
7. 120 to 12	13. 15 to 105 7	19. 35 bu. to 5 bu.
8. 117 to 13	14. 21 to 168 8	20. 84 yds. to 12 yds.
9. 168 to 24 7	15. 33 to 132	21. 96 gals. to 24 gals.
10. 266 to 38	16. 46 to 184 4	22. 75s. to 15s. 5

QUEST.—308. What is a direct ratio? 309. What is an inverse or reciprocal ratio? How is a reciprocal ratio expressed by a fraction? How by points? 309.a. What is a simple ratio? 310. What is a compound ratio? *Obs.* Does it differ in its nature from other ratios?

23. What is the ratio of £1 to 10s.?

Note.—£1 is 20s. The question then is simply this: what is the ratio of 20 to 10s.? *Ans.* 2.

24. What is the ratio of £2 to 5s.? 8 Of £3 to 12s.? 5

311. From the *definition* of ratio and the *mode* of expressing it in the form of a fraction, it is obvious that the *ratio* of two numbers is the same as the *value* of a fraction whose *numerator* and *denominator* are respectively equal to the *antecedent* and *consequent* of the given couplet; for, *each* is the *quotient* of the numerator divided by the denominator. (Arts. 110, 305.)

Obs. From the principles of fractions already established, we may, therefore, deduce the following *general principles* respecting ratios.

312. *To multiply the antecedent of a couplet by any number, multiplies the ratio by that number; and to divide the antedent, divides the ratio:* for, multiplying the numerator, multiplies the value of the fraction by that number, and dividing the numerator, divides the value. (Arts. 111, 112.)

Thus, the ratio of $16 : 4$ is 4;
The ratio of $16 \times 2 : 4$ is 8, which equals 4×2;
And the ratio of $16 \div 2 : 4$ is 2, " " $4 \div 2$.

313. *To multiply the consequent of a couplet by any number, divides the ratio by that number; and to divide the consequent, multiplies the ratio;* for, multiplying the denominator, divides the value of the fraction by that number, and dividing the denominator, multiplies the value. (Arts. 113, 114.)

Thus, the ratio of $16 : 4$ is 4;
The " $16 : 4 \times 2$ is 2, which equals $4 \div 2$;
And " $16 : 4 \div 2$ is 8, " " 4×2.

314. *To multiply or divide both the antecedent and consequent of a couplet by the same number, does not alter the ratio;* for, multiplying or dividing both the numerator and denomi-

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Quest.—311. What is the ratio of two numbers equal to? 312. What is the effect of multiplying the antecedent of a couplet by any number? Of dividing the antecedent? How does this appear? 313. What is the effect of multiplying the consequent by any number? Of dividing the consequent? Why? 314. What is the effect of multiplying and dividing both the antecedent and consequent by the same number? Why?

nator by the same number, does not alter the value of the fraction. (Art. 116.)

Thus, the ratio of $12 : 4$ is 3;
The " $12 \times 2 : 4 \times 2$ is 3;
And " $12 \div 2 : 4 \div 2$ is 3.

315. If the two numbers compared are *equal*, the *ratio* is a *unit* or 1, and is called a *ratio of equality*. Thus, the ratio of $6 \times 2 : 12$ is 1; for the value of $\frac{12}{12}=1$. (Arts. 117, 121.)

316. If the antecedent of a couplet is *greater* than the consequent, the ratio is *greater* than a *unit*, and is called a *ratio of greater inequality*. Thus, the ratio of $12 : 4$ is 3; for the value of $\frac{12}{4}=3$. (Art. 117.)

317. If the antecedent is *less* than the consequent, the ratio is *less* than a *unit*, and is called a *ratio of less inequality*. Thus, the ratio of $3 : 6$ is $\frac{3}{6}$, or $\frac{1}{2}$; for $\frac{3}{6}=\frac{1}{2}$. (Art. 120.)

OBS. 1. The *direct* ratio of two fractions which have a common *numerator*, is the same as the reciprocal ratio of their denominators. Thus, the ratio of $\frac{2}{4} : \frac{2}{8}$ is the same as $\frac{1}{4} : \frac{1}{8}$, or $8 : 4$.

2. The ratio of two *fractions* which have a common denominator, is the same as the ratio of their *numerators*. Thus, the ratio of $\frac{8}{2} : \frac{4}{2}$ is the same as that of $8 : 4$, viz: 2. Hence,

317.a. The ratio of any two fractions may be expressed in whole numbers, by reducing them to a common denominator, and then using the numerators for the terms of the ratio. (Art. 314.) Thus, the ratio of $\frac{1}{2}$ to $\frac{1}{6}$ is the same as $\frac{6}{12} : \frac{2}{12}$, or $6 : 2$.

25. What is the direct ratio of $3 : 9$, expressed in the lowest terms? What the inverse ratio?

Ans. $\frac{1}{3}$; and $\frac{1}{3} \div \frac{1}{9}=3$. (Arts. 308, 309.)

26. What is the inverse ratio of 4 to 12? Of 6 to 18? Of 9 to 24? Of 21 to 25? Of 40 to 56?

27. What is the direct ratio of 15s. to £2? Of 13s. 6d. to £1? Of £2, 10s. to £3, 5s.?

28. What is the direct ratio of 6 inches to 3 feet?

29. What is the direct ratio of 15 oz. to 1 cwt.?

QUEST.—315. When the two numbers compared are equal, what is the ratio? What is it called? 316. When the antecedent is greater than the consequent, what is the ratio? What is it called? 317. If the antecedent is less than the consequent, what is the ratio? What is it called?

PROPORTION.

318. PROPORTION *is an equality of ratios.* Thus, the two ratios 6 : 3 and 4 : 2 form a proportion; for $\frac{6}{3}=\frac{4}{2}$.

OBS. The terms of the two couplets, or the numbers of which the proportion is composed, are called *proportionals.*

319. *Proportion* may be expressed in two ways.

First, by the sign of equality (=) placed between the two ratios.

Second, by four points or a double colon (: :) placed between the two ratios. Thus, each of the expressions, 12 : 6 = 4 : 2, and 12 : 6 : : 4 : 2, is a proportion, one being equivalent to the other.

OBS. The latter expression is read, "the ratio of 12 to 6 equals the ratio of 4 to 2," or simply, "12 is to 6 as 4 is to 2."

320. The number of *terms* in a proportion must at least be *four,* for the equality is between the ratios of *two couplets,* and each couplet must have an antecedent and a consequent.

There may, however, be a proportion formed from *three numbers,* for one of the numbers may be repeated so as to form *two terms.* Thus, the numbers 8, 4, and 2, are proportional; for the ratio of 8 : 4 = 4 : 2. It will be seen that 4 is the consequent in the first couplet, and the antecedent in the last. It is therefore a *mean proportional* between 8 and 2.

OBS. 1. In this case, the number repeated is called the *middle term,* or *mean proportional* between the other two numbers.

The *last* term is called a *third* proportional to the other two numbers. Thus 2 is a third proportional to 8 and 4.

2. Care must be taken not to confound *proportion* with *ratio.* In a simple ratio there are but *two* terms, an antecedent and a consequent; whereas in a proportion there must at least be *four* terms or *two couplets.* (Arts. 305, 318.)

Again, one *ratio* may be *greater* or *less* than another; the ratio of 9 to 3 is greater than the ratio of 8 to 4, and less than 18 to 2. One *proportion,* on the other hand, cannot be *greater* or *less* than another; for *equality* does not admit of degrees.

QUEST.—318. What is proportion? *Obs.* What are the numbers of which a proportion is composed, called? 319. In how many ways is proportion expressed? What is the first? The second? 320. How many terms must there be in a proportion? Why? Can a proportion be formed of three numbers? How? Will there be four terms in it? *Obs.* What is the number repeated called? What is the last term called in such a case? What is the difference between proportion and ratio?

321. The *first* and *last* terms of a proportion are called the *extremes;* the other two, the *means.*

OBS. *Homologous* terms are either the two antecedents, or the two consequents. *Analogous* terms are the antecedent and consequent of the same couplet.

322. *Direct* proportion is an equality between two *direct* ratios. Thus, 12 : 4 : : 9 : 3 is a direct proportion.

OBS. In a direct proportion, the first term has the same ratio to the second, as the third has to the fourth.

323. *Inverse* or *reciprocal* proportion is an equality between a *direct* and a *reciprocal* ratio. Thus, $8:4::\frac{1}{3}:\frac{1}{6}$; or 8 is to 4, reciprocally, as 3 is to 6.

OBS. In a reciprocal or inverse proportion, the first term has the same ratio to the second, as the fourth has to the third.

324. *If four numbers are proportional, the product of the extremes is equal to the product of the means.* Thus, 8 : 4 : : 6 : 3 is a proportion: for $\frac{8}{4}=\frac{6}{3}$. (Art. 318.)

Now $8\times3=4\times6$.

Again, $12:6::\frac{1}{3}:\frac{1}{6}$ is a proportion. (Art. 323.)

And $12\times\frac{1}{6}=6\times\frac{1}{3}$.

OBS. 1. The truth of this proposition may also be illustrated thus:

The numbers 2 : 3 : : 6 : 9 are obviously proportional. (Art. 318.)

For $\frac{2}{3}=\frac{6}{9}$. (Art. 120.) Now,

Multiplying each ratio by 27, (the product of the denominators,)

The proportion becomes $\frac{2\times27}{3}=\frac{6\times27}{9}$ (Art 284. Ax. 6.)

Dividing both the numerator and the denominator of the first couplet by 3; (Art. 116;) or canceling the denominator 3, and the same factor in 27; (Art. 136;) also canceling the 9, and the same factor in 27, we have $2\times9=6\times3$. But 2 and 9 are the extremes of the given proportion, and 3 and 6 are the means; hence, the product of the extremes $2\times9=6\times3$, the product of the means.

2. Conversely, if the product of the extremes is equal to the product of the means, the four numbers are proportional; and if the products are not equal, the numbers are not proportional.

325. *Proportion* is divided into *Simple* and *Compound.*

QUEST.—321. Which terms are the extremes? Which the means? *Obs.* What are homologous terms? Analogous terms? 322. What is direct proportion? *Obs.* In direct proportion what ratio has the first term to the second? 323. What is inverse proportion? *Obs.* What ratio has the first term to the second in this case? 324. If four numbers are proportional, what is the product of the extremes equal to? *Obs.* If the product of the extremes is equal to the product of the means, what is true of the four numbers? If the products are not equal, what is true of the numbers? 325. Into what is proportion divided?

SIMPLE PROPORTION.

326. SIMPLE PROPORTION *is an equality between two simple ratios.* It may be either *direct* or *inverse.* (Art. 309.a.)

If four numbers are in proportion, we have seen that the *product* of the *extremes* is equal to the *product* of the *means.*

Hence, if the *product* of the means is divided by *one* of the extremes, the *quotient* will be the *other extreme;* and if the *product* of the extremes is divided by *one* of the means, the *quotient* will be the *other mean.* For, if the product of two factors is divided by one of them, the quotient will be the other factor. (Art. 291.)

Take the proportion	$8:4::6:3$.
Now the product	$8\times3\div4=6$, one of the means;
So the product	$8\times3\div6=4$, the other mean;
Again, the product	$4\times6\div8=3$, one of the extremes;
And the product	$4\times6\div3=8$, the other extreme. Hence,

326.a. *If any three terms of a proportion are given, the fourth may be found by dividing the product of two of them by the other term.*

OBS. Simple Proportion is often called the *Rule of Three*, from the circumstance that three terms are given to find a fourth. In the older arithmetics, it is also called the *Golden Rule.* But the fact that these names convey no idea of the nature or object of the rule, seems to be a strong objection to their use, not to say a sufficient reason for discarding them.

Ex. 1. If the first three terms of a proportion are 4, 6, 8, what is the fourth term?

Suggestion.—Since 6 and 8 are the two means, we divide their product by 4, which is one of the extremes, and the quotient is the other extreme or 4th term.

Operation.

$$\begin{array}{l} 4:6::8:\text{to 4th term.} \\ \quad\;\underline{8} \\ 4)\overline{48} \\ \quad\;\overline{12}\ \textit{Ans.} \end{array}$$

PROOF.—$4\times12=6\times8$. (Art. 324. Obs. 2.)

QUEST.—326. What is simple proportion? If the product of the means is divided by one of the extremes, what will the quotient be? If the product of the extremes is divided by one of the means, what will the quotient be? 326.a. When three terms of a proportion are given, how is the fourth found? *Obs.* What is simple proportion often called?

2. If 12 bbls. of flour cost $72, what will 4 bbls. cost, at the same rate?

Suggestion.—It is evident 12 bbls. have the same ratio to 4 bbls., as the cost of 12 bbls. ($72) has to the cost of 4 bbls., which is required. That is,

$$12 \text{ bbls.} : 4 \text{ bbls.} :: \$72 : \text{to cost of } 4 \text{ bbls.}$$

$$\begin{array}{r} 4 \\ 12\overline{)288} \\ \hline \$24 \textit{ Ans.} \end{array}$$

OBS. 1. It will be noticed that we placed the given *number* of *dollars* for the *third term*. This we did because the answer required is dollars.

2. We placed the *smaller* of the other two numbers for the *second* term, and the *larger* for the *first*, because 12 bbls. will cost more than 4 bbls.; and therefore the *answer* must be *smaller* than the *third* term.

327. From the preceding illustrations and principles, we deduce the following general

RULE FOR SIMPLE PROPORTION.

I. *Place that number for the third term, which is of the same kind as the answer required.*

II. *Then, if by the nature of the question the answer must be greater than the third term, place the greater of the other two numbers for the second term; but if it is to be less, place the less of the other two numbers for the second term, and the other for the first.*

III. *Finally, multiplying the second and third terms together, divide the product by the first, and the quotient will be the answer in the same denomination as the third term.*

PROOF.—*Multiply the first term and the answer together, and if the product is equal to the product of the second and third terms, the work is right.* (Art. 324.)

OBS. 1. If the first and second terms are *compound* numbers, they must be reduced to the *lowest denomination* mentioned in either.

QUEST.—327. In arranging the terms in simple proportion, which number do you place for the third term? How arrange the other two numbers? Having stated the question, how is the answer found? Of what denomination is the answer? How is simple proportion proved? *Obs.* If the first and second terms contain different denominations, how proceed?

When the third term contains *different* denominations, it must also be reduced to the *lowest* denomination mentioned in it.

2. The process of arranging the terms of a question for solution, or putting it into the *form* of a *proportion*, is called *stating the question.*

3. After solving the following examples by proportion, it will be an excellent exercise for the pupil to solve them by *analysis*. (Art. 296.)

Demonstration.—1. The *reason* for placing that number, which is the same kind as the answer, for the *third* term, instead of the second, is *twofold:*

First, this *number*, in many cases, has no *ratio* to the *first* term; consequently, it is absurd to place it for the *second* term. (Art. 307. Obs. 3.)

Second, this arrangement of the terms of a proportion, avoids the necessity of what is called the *Rule of Three Inverse.*

2. The *reason* for placing the *greater* of the other two numbers for the *second* term, when the answer is greater than the *third* term, arises from the fact, that the *first* term of a proportion has the same ratio to the *second*, which the *third* has to the *fourth* or *answer;* consequently, if the *answer* is *greater* than the *third* term, the *second* term must be *greater* than the *first;* and if the answer is *less* than the third term, the second must be *less* than the first.

3. The *reason* that dividing the *product* of the second and third terms by the first, gives the *answer*, is because the product of the *means* is equal to the product of the *extremes;* and if the *product* of two numbers is divided by *one* of the numbers, the *quotient* will be the *other* number. (Arts. 291, 324.)

3. If 6 men dig a cellar in 12 days, how many men will it take to dig it in 4 days?

Suggestion.—Since it will require more men to dig the cellar in 4 days than it will to dig it in 12 days, we put the larger number of days for the second term, and the smaller for the first term.

Statement.

4 d. : 12 d. : : 6 m. : Ans.

$$\begin{array}{r} 6 \\ \hline 4)72 \\ \hline \end{array}$$

Ans. 18 men.

4. If 6 yards of broadcloth cost 30 dollars, how much will 20 yards cost?

5. If 8 bbls. of flour cost $40, what will 15 bbls. cost?

6. If 16 lbs. of tea cost $12, what will 41 lbs. cost?

7. If 12 acres of land produce 240 bushels of wheat, how much will 57 acres produce?

8. If a man can travel 400 miles in 15 days, how far can he travel in 9 days?

QUEST.—When the third term contains different denominations, what is to be done? What is meant by stating the question? *Dem.* Why place that number for the third term which is the same kind as the answer? Why place the greater of the other two numbers for the second term, when the answer is greater than the third term? How does it appear that dividing the product of the second and third terms by the first will give the answer?

SIMPLE PROPORTION BY CANCELLATION.

9. If 63 barrels of beef cost $504, what will 7 barrels cost?

Suggestion.—Having stated the question, we cancel the factor 7, which is common to the *first* and *second* terms, then proceed as before.

bbls. bbls. dolls.

$\cancel{63} : \cancel{7} : : 504 : \text{Ans.}$

$9 : 1$

$504 \div 9 = \$56.$ *Ans.*

PROOF.—$\cancel{63} : \cancel{7} : : \cancel{504} : \cancel{56}.$ Hence,

328. When the first term has factors common to either of the other two terms.

CANCEL *the factors which are common, then proceed according to the rule above.* (Arts. 92. 136.)

PROOF.—*Place the answer for the fourth term, then cancel all the factors common both to the means and extremes, and if the work is right, none will be left.*

OBS. 1. The question should be *stated*, before *canceling* the common factors.

2. When the terms are of different denominations, the reduction of them may sometimes be shortened by cancellation.

10. If 12 yds. of lace cost £1, what will 1 qr. of a yard cost?

Suggestion.—Multiply the *first* term by 4 to reduce it to quarters, and the *third* term by 20 and 12, to reduce it to pence.

yds. qr. £.

$12 \times 4 : 1 : : 1 \times 20 \times 12 : \text{Ans.}$

Then $\dfrac{\overset{5,}{\cancel{20}} \times \cancel{12}}{\cancel{12} \times \cancel{4}} = 5$ pence. *Ans.*

PROOF.—$\cancel{12} \times \cancel{4} : 1 : : 1 \times \cancel{20} \times \cancel{12} : \cancel{5}.$

11. If 6 men can build a wall in 36 days, how long will it take 18 men to build it?

12. If 10 quintals of fish cost $35, how much will 17 quintals cost?

13. If a ship has water sufficient to last a crew of 25 men for 8 months, how long will it last 15 men?

14. If 12 lbs. sugar cost $1, how much will 84 lbs. cost?

QUEST.—328. When the first term has factors common to either of the other two terms, how may the operation be shortened? How prove simple proportion by cancellation?

15. If 15 lbs. lard cost \$1.15, how much will 80 lbs. cost?

16. If $\frac{5}{8}$ of an acre of land cost £$\frac{3}{7}$, how much will $\frac{7}{8}$ of an acre cost?

Operation.

Acre.	Acre.	£.	

$$\frac{5}{8} : \frac{7}{8} :: \frac{3}{7} : \text{Ans.}$$

$$\frac{\cancel{8}}{5}\times\frac{\cancel{7}}{\cancel{8}}\times\frac{3}{\cancel{7}}=£\frac{3}{5}\ \textit{Ans.}$$

Suggestion.—State the question as in whole numbers, then inverting the first term, which is a divisor, cancel the factors common to the numerators and denominators, and the result £$\frac{3}{5}$, is the answer required.

Sometimes it may be more convenient to arrange the terms of the fractions on each side of a perpendicular line, as in division of fractions.

Or thus,

$$\begin{array}{r|l} 5 & \cancel{8} \\ \cancel{8} & \cancel{7} \\ \cancel{7} & 3 \\ \hline \textit{Ans.} & £\frac{3}{5}. \end{array}$$

17. If $\frac{4}{7}$ of a hogshead of molasses cost \$28, how much will 16 hogsheads cost?

18. If $2\frac{1}{4}$ yds. of broadcloth cost \$18, what will 27 yds. cost?

19. If 6 acres and 40 rods of land cost \$125, how much will 25 acres and 120 rods cost?

20. If 15 yds. of silk cost £4, 10s., what will 75 yds. cost?

21. If a railroad car goes 35 m. in 1 hr. 45 min., how far will it go in 3 days?

22. If $4\frac{1}{2}$ lbs. of chocolate cost 9s., what will $22\frac{1}{2}$ lbs. cost?

23. If $35\frac{2}{3}$ lbs. of butter cost \$4, what will $15\frac{1}{8}$ lbs. cost?

24. If 84 lbs. of cheese cost $\$5\frac{2}{8}$, what will 60 lbs. cost?

25. If $\frac{3}{8}$ of a ship is worth \$6000, how much is $\frac{5}{16}$ of her worth?

26. If $4\frac{1}{2}$ bu. of wheat make 1 barrel of flour, how many barrels will 84 bu. make?

27. If the interest of \$1500 for 12 mo. is \$90, what will be the interest of the same sum for 8 mo.?

28. If a tree 20 ft. high, casts a shadow 30 ft. long, how long will be the shadow of a tree 50 ft. high?

29. How long will it take a steamship to sail round the globe, allowing it to be 25000 miles in circumference, if she sails at the rate of 3000 miles in 12 days?

30. How many acres of land can a man buy for \$840, if he pays at the rate of \$56 for every 7 acres?

31. How much will 85 cwt. of iron cost, at the rate of \$91 for 13 cwt.?

32. At the rate of \$45 for 6 cwt. of beef, how much can be bought for \$980?

33. If 9 ounces of silver will make 4 tea spoons, how many spoons will 25 pounds of silver make?

34. If 15 tons of wool are worth \$90000, how much is 5 cwt. worth?

35. If $5\frac{1}{2}$ yds. of cloth are worth \$$27\frac{1}{2}$, how much are $50\frac{1}{4}$ yards worth?

36. If 60 men can build a house in $90\frac{1}{2}$ days, how long will it take 15 men to build it?

37. A bankrupt owes \$25000, and his property is worth \$20000: how much can he pay on a dollar?

38. At 7s. 6d. per week, how long can a man board for £24, 10s.?

39. What cost 94 tons of coal, if 141 tons cost £85?

40. What cost 291 yds. of cambric, if 13 yds. cost £8, 6s. $3\frac{1}{2}$d.?

41. What cost 3 lbs. of raisins, at £6, 7s. 6d. per 100 lbs.?

42. If 20 sheep cost £37, $12\frac{1}{2}$s., what will 311 cost?

43. At 7s. 6d. per ounce, what is the value of a silver pitcher weighing 9 oz. 13 pwt. 8 grs.?

44. If 405 yards of linen cost £69, 7s. 6d., what will 243 yards cost?

45. If A can saw a cord of wood in 6 hours, and B in 9 hours, how long will it take both together to saw a cord?

46. A cistern has 3 cocks, the first of which will empty it in 10 min.; the second, in 15 min.; and the third, in 30 min.: how long will it take all of them together to empty it?

47. A man and a boy together can mow an acre of grass in 4 hours; the man can mow it alone in 6 hours: how long will it take the boy to mow it?

48. If 265.6 yards of cloth cost £673, 15s. $6\frac{1}{2}$d., how much will 123.4 yards cost?

49. What cost $6\frac{5}{8}$ ounces of silver, at 12s. 8d. per ounce?

50. If $\frac{3}{16}$ of a ship cost £$273\frac{1}{8}$, what is $\frac{5}{32}$ worth?

51. What will $49\frac{3}{11}$ yards of velvet cost, if $7\frac{2}{8}$ yards cost £7, 18s. 4d.?

52. If £100 of bank stock is worth £$98\frac{7}{8}$, what is £362, 8s. $7\frac{1}{2}$d. of stock worth?

53. If you pay £37, 10s. per ton for iron, at what rate must you sell it to gain the price of 1 ton on 15 tons?

54. What will be the rent of 35 acres, 2 roods, 10 r. of land, if 46 acres, 3 roods, 14 r. are worth £50?

55. If a landlord deducts $\frac{2}{3}$ on a shilling to his tenant, what will be the deduction on £76, 3s. $4\frac{1}{3}$d.?

56. If $\frac{1}{3}$ and $\frac{1}{10}$ of a pasture cost £4, 10s., what will the whole pasture cost?

57. Bought 840 apples, at the rate of 10 for a penny, and 240 more, at 8 for a penny: if I sell them at 36 for 4d., shall I gain or lose by the operation?

58. If 27 tons, 3 qrs. 15 lbs. of coal cost $217.83, what will 119 tons, 1 qr. 10 lbs. come to?

59. If a horse can travel 18 m. 3. fur. 25 rods in 3 hours, 40 min., how far can he travel in $48\frac{1}{2}$ hours?

60. If $\frac{11}{15}$ of a melon cost $\$\frac{8}{15}$, what will $\frac{11}{20}$ cost?

61. If $\frac{19}{20}$ of a cord of wood cost £$\frac{13}{20}$, what will $\frac{29}{35}$ cost?

62. A jockey bought a horse for $125, and sold him for $$162\frac{1}{2}$: what per cent. did he make?

63. A man bought a house for $7265: for how much must he sell it to gain $15\frac{5}{8}$ per cent.?

64. A man bought 175 bbls. of beef, at $$9.62\frac{1}{2}$ per barrel, and sold it at a loss of $7\frac{1}{2}$ per cent.: how much did he lose?

65. If the interest of $675.25 is $55.625 for 1 year, how much will be the interest of $2368.85?

66. A man pays $1565.50 interest annually, which is 7 per cent. on his indebtedness: how much does he owe?

67. A merchant paid $45265 ad valorem duties, which was $23\frac{1}{2}$ per cent. on the goods he imported: what was the value of the goods?

68. A speculator sold a house 15 per cent. less than cost, and thereby lost $500; if he had kept it 1 day longer he could have sold it so as to make 15 per cent. How much did the house cost, and what could he have got by keeping it?

69. What must be the length of a board which is $9\frac{3}{4}$ in. wide, to make a square foot?

70. If $87\frac{1}{2}$ yds. carpeting $1\frac{1}{4}$ yard wide will cover a floor, how many yards $\frac{3}{4}$ yd. wide will it take to cover it?

COMPOUND PROPORTION.

329. COMPOUND PROPORTION *is an equality between a compound ratio and a simple one.* (Arts. 309.*a.*, 310.)

Thus, $\left.\begin{array}{l}8:4\\ \text{Into}\ \ 6:3\end{array}\right\}::12:3$, is a compound proportion.

That is, $8\times6:4\times3::12:3$; for, $8\times6\times3=4\times3\times12$.

OBS. Compound proportion is sometimes called *Double Rule of Three*, and is chiefly applied to the solution of examples which would require *two or more statements* in simple proportion.

Ex. 1. If 4 men can earn $24 in 6 days, how much can 8 men earn in 10 days?

Operation.

$$\left.\begin{array}{l}4\text{m.}:\ 8\text{m.}\\ 6\text{d.}:10\text{d.}\end{array}\right\}::\$24:\text{Ans.}$$

$24\times8\times10=1920$,

and $4\times\ 6=24$.

Now $1920\div24=80$.

Ans. 80 dollars.

Suggestion.—We place the $24, which is the same kind as the answer, for the third term. We then take the other numbers in pairs, two of a kind, and arrange each pair according as the answer would be greater or less than the third term, if it depended on this pair alone. Thus, since 8 men will earn more than 4 men, we place the larger for the second term and the smaller for the first, as in simple proportion. Again, since the given men can earn more in 10 days than in 6 days, we place the 10 for the second term and 6 for the first. Finally, we divide the product of all the numbers standing in the 2d and 3d places of the proportion, by the product of those standing in the first place, and the quotient is the answer.

Note.—1. The learner will observe, that it is not the ratio of 4 to 8 alone, nor that of 6 to 10, which is equal to the ratio of 24 to the answer, as it is sometimes stated; but it is the ratio *compounded* of 4 to 8 and 6 to 10, which is equal to the ratio of 24 to the answer. Thus, $4\times6:8\times10::24:80$, the answer.

2. A compound proportion, when stated as above, is read, "the ratio of 4 into 6 is to 8 into 10 as 24 to the answer."

QUEST.—329. What is compound proportion? *Obs.* To what is it chiefly applied? What is it sometimes called?

330. From the preceding illustrations and principles, we derive the following general

RULE FOR COMPOUND PROPORTION.

I. *Place that number which is of the same kind as the answer required for the third term.*

II. *Then take the other numbers in pairs, or two of a kind, and arrange them as in simple proportion.* (Art. 327.)

III. *Finally, multiply together all the numbers in the second and third terms, divide the result by the product of those in the first term, and the quotient will be the fourth term or answer required.*

PROOF.—*Multiply the answer into all the first terms or antecedents of the several couplets, and if the product is equal to the product of the second and third terms, the work is right.* (Art. 324.)

OBS. 1. Among the given numbers there is but *one* which is of the same kind as the answer. This is sometimes called the *odd* term, and must always be placed for the *third* term.

2. Questions in compound proportion may be solved by *Analysis;* also by *Simple Proportion*, by making *two* or *more* separate statements.

It will be an excellent exercise for the pupil to solve the following examples by each of these methods.

1. If 5 men can mow 20 acres of grass in 4 days, working 10 hours per day, how much can 8 men mow in 5 days, working 12 hours per day?

Operation.

$$\left.\begin{array}{r} 5\text{ m.} : 8\text{ m.} \\ 4\text{ d.} : 5\text{ d.} \\ 10\text{ hr.} : 12\text{ hr.} \end{array}\right\} :: 20 : \text{Ans.} \qquad \begin{array}{r} 8\times5\times12\times20=9600. \\ 5\times4\times10=200. \\ 9600\div200=48 \text{ acres, } \textit{Ans.} \end{array}$$

2. A man having agreed to build a wall 27 rods long, found that 12 men had built only 9 rods of it in 6 days: how many men must be employed to build the remainder in 4 days?

QUEST.—330. In stating a question in compound proportion, which number do you put for the third term? How arrange the other numbers? Having stated the question, how is the answer found? How is compound proportion proved? *Obs.* Among the given numbers, how many are of the same kind as the answer? How may questions in compound proportion be solved by simple proportion?

COMPOUND PROPORTION BY CANCELLATION.

3. If 8 men can clear 30 acres of land in 63 days, working 10 hours a day, how many acres can 10 men clear in 72 days, working 12 hours a day?

Having stated the question, cancel all the factors in the *first* terms which are common to the *second* or *third* terms.

Statement.

$$\left.\begin{array}{l} 8\text{ m.} : 10\text{ m.} \\ 63\text{ d.} : 72\text{ d.} \\ 10\text{ hr.} : 12\text{ hr.} \end{array}\right\} :: 30 : \text{Ans.} \quad \text{Acres.}$$

$$\frac{10\times72\times12\times30}{8\times63\times10}=\frac{\cancel{10}\times\overset{9}{\cancel{72}}\times12\times30}{\cancel{8}\times\underset{7}{\cancel{63}}\times\cancel{10}}=\frac{360}{7}=51\tfrac{3}{7}\text{ a. } Ans.$$

331. Hence, when the *first terms* have factors *common* to the *second* or *third* terms.

Cancel the factors which are common, then proceed according to the rule above. (Art. 330.)

PROOF.—*Place the answer for the fourth term, cancel all the factors in the first and fourth terms, which are common to the second and third terms; if the work is right, none will remain.*

OBS. Instead of placing points between the first and second terms, it is sometimes more convenient to put a perpendicular line between them as in division of fractions. (Art. 140.) This will bring all the terms, whose product is to be the dividend on the right of the line, and those whose product is to form the divisor, on the left. In this case the third term should be placed below the second terms, with the sign of proportion (::) before it, to show its *origin*, and its *relation* to the answer.

4. If a man can walk 192 miles in 4 days, traveling 12 hours a day, how far can he go in 24 days, traveling 8 hours a day?

The product of the antecedents, 4×12, has the same ratio to the product of the consequents, 24×8, as 192 has to the Ans.

Operation.

$$\begin{array}{r|l} \cancel{4}\text{ d.} & \cancel{24}\text{ d. } 2 \\ \cancel{12}\text{ hr.} & \cancel{8}\text{ hr. } 2 \\ & ::192\text{ miles.} \\ \hline \textit{Ans.} & 192\times2\times2=768\text{ m.} \end{array}$$

5. If 8 men can make 9 rods of wall in 12 days, how many men will it require to make 36 rods in 4 days? *Ans.* 96 m.

QUEST.—331. When the first terms have factors common to the second and third terms, how proceed? How prove compound proportion by cancellation?

6. If 5 men make 240 pair of shoes in 24 days, how many men will it require to make 300 pair in 15 days?

7. If 60 lbs. of meat will supply 8 men 15 days, how long will 72 lbs. last 24 men?

8. If 12 men can reap 80 acres of wheat in 6 days, how long will it take 25 men to reap 200 acres?

9. If 18 horses eat 128 bushels of oats in 32 days, how many bushels will 12 horses eat in 64 days?

10. If 8 men can build a wall 20 ft. long, 6 ft. high, and 4 ft. thick, in 12 days, how long will it take 24 men to build one 200 ft. long, 8 ft. high, and 6 ft. thick?

11. If 8 men reap 36 acres in 9 days, working 9 hours per day, how many men will it take to reap 48 acres in 12 days, working 12 hours per day?

12. If $100 gain $6 in 12 months, how long will it take $400 to gain $18? *Ans.* 9 mos.

13. If $200 gain $12 in 12 m., what will $400 gain in 9 m.?

14. If 8 men spend £32 in 13 weeks, how much will 24 men spend in 52 weeks?

15. If 6 men can dig a drain 20 rods long, 6 feet deep, and 4 feet wide, in 16 days, working 9 hours each day, how many days will it take 24 men to dig a drain 200 rods long, 8 feet deep, and 6 feet wide, working 8 hours per day?

16. If 3 lbs. of yarn will make 10 yards of cloth $1\frac{1}{2}$ yard wide, how many pounds will be required to make a piece 100 yards long, and $1\frac{1}{4}$ yd. wide?

17. A general wished to remove 80000 lbs. of provision from a fortress in 9 days, and it was found that in 6 days 18 horses had carried away but 15 tons: how many horses would be required to carry the remainder in 3 days?

18. If a man travels 130 miles in 3 days, when the days are 14 hours long, how long will it take him to travel 390 miles when the days are 7 hours long?

19. If the price of 10 oz. of bread is 5d., when corn is 4s. 2d. per bushel, what must be paid for 3 lbs. 10 oz. when corn is 5s. 6d. per bushel?

20. If 6 journeymen make 132 pair of boots in $4\frac{1}{2}$ weeks, working $5\frac{1}{2}$ days a week, and $12\frac{3}{4}$ hours per day, how many pair will 18 men make in $13\frac{1}{3}$ weeks, working $4\frac{1}{4}$ days per week, and 11 hours per day?

SECTION XIII.

DUODECIMALS.

ART. **332.** DUODECIMALS are a species of *compound numbers*, the *denominations* of which increase and decrease uniformly in a *twelvefold ratio.* Its denominations are *feet, inches* or *primes*, *seconds, thirds, fourths, fifths, &c.*

Note.—The term *duodecimal* is derived from the Latin numeral *duodecim*, which signifies *twelve*.

TABLE.

12 fourths	('''')	make	1 third,	marked	'''
12 thirds		"	1 second,	"	''
12 seconds		"	1 inch or prime,	"	*in.* or '
12 inches or primes		"	1 foot,	"	*ft.*

Hence, $1' = \frac{1}{12}$ of 1 foot.

$1'' = \frac{1}{12}$ of 1 in., or $\frac{1}{12}$ of $\frac{1}{12}$ of 1 ft. $= \frac{1}{144}$ of 1 ft.

$1''' = \frac{1}{12}$ of $1''$, or $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of 1 ft. $= \frac{1}{1728}$ of 1 ft.

OBS. The accents used to distinguish the different denominations below feet, are called *Indices*.

333. *Duodecimals* are *added* and *subtracted* in the same manner as other compound numbers. (Arts. 168, 169.)

1. Add together 15 ft. 6′ 9″; 20 ft. 7′ 8″; 11 ft. 8′ 5″ and 41 ft. 7′ 3″. *Ans.* 89 ft. 6′ 1″.
2. Add together 21 ft. 3′ 7″; 43 ft. 4′ 8″; 13 ft. 8′ 9″.
3. Add together 45 ft. 2′ 1″; 68 ft. 5′ 3″; 79 ft. 5′ 10″.
4. Add together 93 ft. 5′ 4″; 69 ft. 4′ 6″; 84 ft. 9′ 4″.
5. Add together 68 ft. 3′ 9″; 89 ft. 8′ 7′; 94 ft. 8′ 3″.
6. Add together 173 ft. 8′ 9″; 241 ft. 6′ 5″; 476 ft. 9′ 10″.
7. From 46 ft. 5′ 7″ subtract 19 ft. 8′ 10″. *Ans.* 26 ft. 8′ 9″.
8. From 78 ft. 4′ 5″ subtract 36 ft. 6′ 8″.

QUEST.—332. What are duodecimals? What are its denominations? *Note.* What is the meaning of the term duodecimal? Repeat the Table. *Obs.* What are the accents called which are used to distinguish the different denominations? 333. How are duodecimals added and subtracted?

9. If from a board measuring 19 ft. you take 7 ft. 6′ 6″, how much will be left?

10. What is the sum and difference of 28 ft. 5′ and 43 ft.?

11. What is the sum and difference of 68 ft. 4′ 6″ and 51 ft.?

12 What is the sum and difference of 125 ft. 8′ 5″ and 108 ft. 9′ 4″?

MULTIPLICATION OF DUODECIMALS.

334. Duodecimals are principally applied to the measurement of *surfaces* and *solids.* (Arts. 153, 154.)

Ex. 1. How many square feet are there in a board 8 ft. 9 in. long, and 2 ft. 6 in. wide?

Operation.

8 ft.	9′	length.
2 ft.	6′	width.
17 ft.	6′	
4 ft.	4′ 6″	
21 ft.	10′ 6″.	*Ans.*

Suggestion.—We first multiply each denomination of the multiplicand by the number of feet in the multiplier, beginning at the right hand. Thus, 2 times 9′ are 18′, equal to 1 ft. and 6′. Set the 6′ under inches, and carry the 1 ft. to the next product. 2 times 8 ft. are 16 ft., and 1 to carry makes 17 ft. Again, since $6'=\frac{6}{12}$ of a ft., and $9'=\frac{9}{12}$ of a ft., 6′ into 9′ is $\frac{54}{144}$ of a ft.=54″, or 4′ and 6″. Write the 6″ one place to the right of inches, and carry the 4′ to the next product. Then 6′ or $\frac{6}{12}$ of a foot multiplied into 8 ft.$=\frac{48}{12}$ of a ft., or 48′ and 4′ to carry make 52′; but 52′=4 ft. and 4′. Now adding the partial products, the sum is 21 ft. 10′ 6″.

OBS. It will be seen from this operation, that feet multiplied into feet, produce feet; feet into inches, produce inches; inches into inches, produce seconds; and in each case the product of any two factors has as many accents as the factors have. Hence,

335. To find the *denomination* of the product of any two factors in duodecimals.

Add the indices of the two factors together, and the sum will be the index of their product.

Thus, feet into feet, produce feet; feet into inches, produce

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QUEST.—334. To what are duodecimals chiefly applied? 335. How find the denomination of the product in duodecimals? What do feet into feet produce? Feet into inches? Feet into seconds? What do inches into inches produce? Inches into thirds? Inches into fourths? Seconds into seconds? Seconds into thirds? Seconds into eighths? Thirds into thirds? Thirds into sixths?

inches; feet into seconds, produce seconds; feet into thirds, produce thirds, &c.

Inches into inches, produce seconds; inches into seconds, produce thirds; inches into fourths, produce fifths, &c.

Seconds into seconds, produce fourths; seconds into thirds, produce fifths; seconds into sixths, produce eighths, &c.

Thirds into thirds, produce sixths; thirds into fifths, produce eighths; thirds into sevenths, produce tenths, &c.

Fourths into fourths, produce eighths; fourths into eighths, produce twelfths, &c.

Note.—The foot is considered the *unit*, and has no *index*.

For the explanation of the apparent contradiction involved in multiplying *feet* by *feet*, &c., see Higher Arithmetic. (Art. 516. Obs. 3.)

336. From these illustrations we have the following

RULE FOR MULTIPLICATION OF DUODECIMALS.

I. *Place the several terms of the multiplier under the corresponding terms of the multiplicand.*

II. *Multiply each term of the multiplicand by each term of the multiplier separately, beginning with the lowest denomination in the multiplicand, and the highest in the multiplier, and write the first figure of each partial product one or more places to the right, under its corresponding denomination.* (Art. 335.)

III. *Finally, add the several partial products together, carrying* 1 *for every* 12 *both in multiplying and adding, and the sum will be the answer required.*

OBS. 1. It is sometimes asked whether the inches in duodecimals are *linear*, *square*, or *cubic*. The answer is, they are *neither*. An *inch* is 1 *twelfth* of a foot. Hence, in measuring surfaces an inch is $\frac{1}{12}$ of a *square* foot; that is, a surface 1 foot *long* and 1 inch *wide*. In measuring solids, an inch denotes $\frac{1}{12}$ of a *cubic* foot. In common language, these inches are called *lumber inches*, or *carpenters' inches*.

2. Mechanics, and surveyors of wood and lumber, in taking dimensions of their work, lumber, &c., often call the inches a *fractional* part of a foot, and then find the contents in feet and a *fraction* of a foot. Sometimes inches are regarded as *decimals* of a foot.

QUEST.—336. What is the rule for multiplication of duodecimals? *Obs.* What kind of inches are those spoken of in measuring surfaces by duodecimals? In measuring solids? In common language what are they called?

2. How many square feet are there in a board 18 feet 9 inches long, and 2 feet 6 inches wide?

3. How many square feet are there in a board 14 feet 10 inches long, and 11 inches wide?

4. How many square feet in a gate 12 feet 5 inches wide, and 6 feet 8 inches high?

5. How many square feet in a floor 16 feet 6 inches long, and 12 feet 9 inches wide?

6. How many square feet in a ceiling 53 feet 6 inches long, and 25 feet 6 inches wide?

7. How many square feet are there in a stock of 6 boards 17 feet 7 inches long, and 1 foot 5 inches wide?

8. How many feet in a stock of 10 boards 12 feet 8 inches long, and 1 foot 1 inch wide?

9. How many cubic feet in a stick of timber 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick?

10. How many cubic feet in a block of marble 8 feet 4 inches long, 2 feet 6 inches wide, and 1 foot 10 inches thick?

11. How many cubic feet in a load of wood 6 feet 7 inches long, 3 feet 5 inches high, and 3 feet 8 inches wide?

12. How many feet in a load of wood 7 feet 2 inches long, 4 feet high, and 3 feet wide?

13. How many feet in a load of wood 9 feet long, 4 feet 3 inches wide, and 5 feet 6 inches high?

14. How many feet in a pile of wood 100 feet long, $5\frac{1}{2}$ feet high, and 4 feet wide?

15. How many feet in a pile of wood 150 feet long, $8\frac{1}{2}$ feet high, and 5 feet wide?

16. How many cubic feet in a wall 40 feet 6 inches long, 5 feet 10 inches high, and 2 feet thick?

17. How many solid feet in a vat 10 feet 8 inches long, 7 feet 2 inches wide, and 6 feet 4 inches deep?

18. How many bricks 8 inches long, 4 inches wide, and 2 inches thick, are there in a wall 20 feet long, 10 feet high, and $1\frac{1}{2}$ feet thick?

19. How much will the flooring of a room which is 20 feet long, and 18 feet wide come to, at $6\frac{1}{4}$ cents per square foot?

20. How much will the plastering of a wall 16 feet square come to, at $12\frac{1}{2}$ cents per square yard?

SECTION XIV.

INVOLUTION.

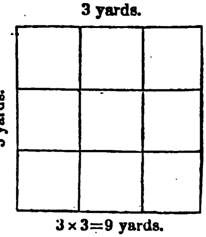
ART. **337.** INVOLUTION *is the process of finding any power of a number by multiplying it into itself.*

338. A POWER *is the product arising from multiplying a number into itself.* Thus, $3 \times 3 = 9$. Here 9 is the *second power* of 3. Again, $3 \times 3 \times 3 = 27$; and 27 is the *third* power of 3, &c.

339. *Powers are divided into different orders; as the first, second, third, fourth, fifth power,* &c. They take their name from the *number of times* the given number is used as a *factor*, in producing the *given power.*

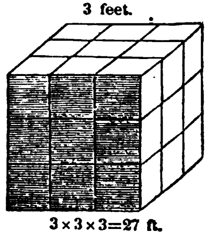
OBS. 1. The *original* number is called the *first* power. Strictly speaking, it is not a *power*, but a root. (Art. 344.)

2. The *second* power of a number is called the *square;* because the *area* of a *square* is obtained by multiplying one side *into* itself, or using it *twice* as a factor. Thus, if the side of a square is 3 yards, then $3 \times 3 = 9$ yards, its *area;* and $3 \times 3 = 9$, the *second power* of 3. (Art. 153. Obs. 3.)



3. The *third* power of a number is also called the *cube*, (Art. 154.) because the solidity of a cubical body is obtained by multiplying one side into itself *twice*, or using it *three times* as a factor. Thus, if the side of a cube is 3 feet, then $3 \times 3 \times 3 = 27$ ft. its *solidity;* and $3 \times 3 \times 3 = 27$, the *third power* of 3. (Art. 154. Obs. 3.)

4. The *fourth* power of a number is called the *biquadrate.*



QUEST.—337. What is involution? 338. What is a power? 339. How are powers divided? From what do they take their name? *Obs.* What is said to be the first power? What is the second power called? The third? The fourth? What is the square of 2? Of 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? What is the cube of 2? 3? 4? 5? What is the fourth power of 2? 3? The fifth power of 2?

340. Powers are denoted by a *small figure* placed above the given number, at the right hand.

This figure is called the *index* or *exponent*, and shows how *many times* the given number is employed as a *factor* to produce the *required power*. Thus,

The index of the *first* power is 1;
The index of the *second* power is 2;
The index of the *third* power is 3;
The index of the *fourth* power is 4; &c. That is,

$2^1 = 2$, the first power of 2;
$2^2 = 2 \times 2$, the square, or 2d power of 2;
$2^3 = 2 \times 2 \times 2$, the cube, or 3d power of 2;
$2^4 = 2 \times 2 \times 2 \times 2$, the biquadrate, or 4th power of 2;
$2^5 = 2 \times 2 \times 2 \times 2 \times 2$, the fifth power of 2.

OBS. The *index* of the *first* power is commonly omitted; for $2^1 = 2$.

1–10. Express the third power of 6. The fourth power of 12. The square of 16. The cube of 20. The fourth power of 25. The fifth power of 72. The sixth power of 100. The tenth power of 500. The 15th power of 786.

341. To *involve* a number to any required power.

Multiply the given number into itself, till it is taken as a factor as many times as there are units in the index of the power to which the number is to be raised. (Art. 339.)

OBS. 1. The *number* of *multiplications* in raising a number to any given power, is *one* less than the index of the required power. Thus, the square of 3 is written 3^2, and $3 \times 3 = 9$, the 3 is taken twice as a factor, but there is but one multiplication.

2. A *Fraction* is raised to a power by *involving* both the *numerator* and *denominator*, or multiplying the fraction into itself. Thus, the square of $\frac{2}{3}$ is $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$.

Mixed numbers may be reduced to improper fractions, or the common fraction be reduced to a decimal, then raised to the required power.

3. All powers of 1 are 1; for $1 \times 1 \times 1 \times 1$, &c. $= 1$.

4. If two or more *powers* are multiplied together, their product is that power whose *index* is the *sum* of the indices of the factors. Thus, $2 \times 2 = 4$, the 2d power of 2; and $4 \times 4 = 16$, the 4th power of 2; and $16 \times 16 = 256$, the 8th power 2.

QUEST.—340. How are powers denoted? What is this figure called? What does it show? What is the index of the first power? Of the second? Third? Fourth? Fifth? Sixth? 341. How is a number involved to any required power? *Obs.* How many multiplications are there in raising a number to a given power? How is a fraction raised to a power? A mixed number? What are all powers of 1? If two or more powers are multiplied together, what power is the product?

Again, the *second* power into the third produces the *fifth* power; the *fourth* into the *fifth*, the *ninth* power, &c.

11. What is the square of 24?

Common Operation.	*Analytic Operation.*
24	24=2 tens or 20+4 units.
24	24=2 tens or 20+4 units.
96	80+16
48	400+80
576 *Ans.*	And 400+160+16=576.

It will be seen from the analytic operation that the square of $20+4$ contains the square of the first part, viz: $20\times20=400$, added to twice the product of the two parts, viz: $20\times4+20\times4=160$, added to the square of the last part, viz: $4\times4=16$. Hence,

342. *The square of any number which consists of two figures, is equal to the square of the tens, added to twice the product of the tens into the units, added to the square of the units.*

OBS. 1. The product of any two factors cannot have more figures than both factors, nor but one less than both. For example, take 9, the greatest number which can be expressed by one figure. (Art. 7.) And $(9)^2$, or $9\times9=81$, has two figures, the same number which both factors have. 99 is the greatest number which can be expressed by two figures; and $(99)^2$, or $99\times99=9801$, has four figures, the same as both factors have?

Again, 1 is the smallest number expressed by one figure, and $(1)^2$, or $1\times1=1$, has but one figure less than both factors. 10 is the smallest number which can be expressed by two figures; and $(10)^2$, or $10\times10=100$, has one figure less than both factors. Hence,

2. *Any square number cannot have more figures than double the number of the root or first power, nor but one less.*

3. *A cube cannot have more figures than triple the number of the root or first power, nor but two less.*

12. What is the square of 45? 50? 75? 100? 540?
13. What is the cube of 5? Of 8? 10? 12? 60?
14. What is the fourth power of 3? Of 4? 16? 20?
15. What is the fifth power of 2? Of 3? 4? 5? 6?
16. What is the square of $\frac{1}{2}$? Of $\frac{1}{4}$? $\frac{1}{5}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{8}$?
17. What is the cube of $\frac{2}{3}$? Of $\frac{4}{5}$? Of $\frac{7}{8}$? Of $\frac{10}{20}$?
18. What is the square of $2\frac{1}{2}$? Of $3\frac{1}{4}$? $5\frac{2}{3}$? $10\frac{3}{4}$?

QUEST.—342. What is the square of any number consisting of two figures equal to? Obs. How many figures are there in the product of any two factors? How many figures will the square of a number contain? The cube?

19. What is the square of 1.5? Of 3.25? Of 10.25?
20. What is the cube of .5? .05? .005? .0005? .00005?
21. What is the 4th power of 2.5? 5.01? 20.02? 45.1?
22. What is the 3d power of $\frac{9}{10}$? $\frac{21}{35}$? $\frac{138}{232}$? $\frac{237}{425}$?
23. What is the cube of $115\frac{3}{4}$? $228\frac{11}{12}$? $463\frac{7}{8}$?
24. The 5th power of .045?
25. The 12th power of .007?
26. The 20th power of 5?
27. The 30th power of 4?
28. The 40th power of 3?
29. The 51st power of 2?

EVOLUTION.

343. EVOLUTION *is the process of finding the root of a given number by resolving it into equal factors.*

344. *The* ROOT *of a number is a factor, which being multiplied into itself a certain number of times, will produce that number.* Thus, 2 is a root of 4, because when multiplied into itself, it produces 4. So 3 is a root of 27, because $3 \times 3 \times 3 = 27$.

OBS. The *number of times* the root must be taken as a factor to produce the given number, is denoted by the *name* of the root. Thus, when it is *said* that 2 is the 4th root of 16, the name of the root shows that 2 must be taken 4 times as a factor to produce 16; and $2 \times 2 \times 2 \times 2 = 16$.

345. Evolution is the *opposite* of involution. (Art. 337.) One is finding a *power* of a number by multiplying it into itself; the other is finding a *root* by resolving a number into equal factors.

Roots.	1	2	3	4	5	6	7	8	9	10	11	12
Squares.	1	4	9	16	25	36	49	64	81	100	121	144
Cubes.	1	8	27	64	125	216	343	512	729	1000	1331	1728

OBS. 1. *Powers* and *roots* are therefore *correlative* terms. If one number is a *power* of another, the latter is a *root* of the former. Thus, 27 is the cube of 3; and 3 is the cube root of 27.

2. In *subtraction* a number is resolved into *two parts*;
In *division*, a number is resolved into *two factors*;
In *evolution*, a number is resolved into *equal factors*.

QUEST.—343. What is Evolution? 344. What is a root? *Obs.* How do you know how many times the root must be taken as a factor to produce the given number? What does the square root denote? The cube root? The fourth root? 345. Of what is evolution the opposite? Into what are numbers resolved in subtraction? In division? In evolution?

MENTAL EXERCISES.

1. What is the square root of 25? *Ans.* 5.

3–5. The square of 9? 16? 36? 49? 64? 81? 100? 121? 144?

6. What is the cube or third root of 8?

Solution.—If we resolve 8 into three equal factors, each of these factors is 2: for $2\times2\times2=8$. Therefore, the cube root of 8 is 2.

7–9. What is the cube root of 27? 64? 125?

10. What is the fourth root of 16?

11. What is the square root of $\frac{9}{16}$?

Solution.—The square root of the numerator 9, is 3; and the square root of the denominator 16, is 4. Therefore $\frac{3}{4}$ is the square root of $\frac{9}{16}$.

12. What is the square root of $\frac{1}{9}$? *Ans.* $\frac{1}{3}$.

13. What is the square root of $\frac{16}{25}$? Of $\frac{25}{49}$?

14. What is the square root of $\frac{36}{81}$? Of $\frac{64}{100}$?

15. What is the cube root of $\frac{1}{8}$? *Ans.* $\frac{1}{2}$.

16. What is the cube root of $\frac{1}{27}$? Of $\frac{27}{64}$?

346. Roots are expressed in *two* ways:

First, by the *radical sign* ($\sqrt{}$) placed before a number.

Second, by a *fractional index* placed above the number on the right hand. Thus, $\sqrt{4}$, or $4^{\frac{1}{2}}$ denotes the *square* or 2d root of 4; $\sqrt[3]{27}$, or $27^{\frac{1}{3}}$ denotes the *cube* or 3d root of 27; $\sqrt[4]{16}$, or $16^{\frac{1}{4}}$ denotes the 4th root of 16.

OBS. 1. The figure placed over the radical sign, denotes the *root*, or the number of equal factors into which the given number is to be resolved. The figure for the *square* root is usually omitted, and simply the radical sign $\sqrt{}$ is placed before the given number. Thus, the square root of 25 is written $\sqrt{25}$.

2. When a root is expressed by a *fractional* index, the *denominator*, like the figure over the radical sign, denotes the *root* of the given number. Thus, $(25)^{\frac{1}{2}}$ denotes the square root of 25; $27^{\frac{1}{3}}$ denotes the cube root of 27.

QUEST.—346. In how many ways are roots expressed? What are they? *Obs.* What does the figure over the radical sign denote? What does the denominator of a fractional index denote?

347. A number which can be resolved into *equal* factors, or whose root can be *exactly* found, is called a *perfect power*, and its root is called a *rational number*. Thus, 16, 25, 27, &c., are perfect powers, and their roots 4, 5, 3, are rational numbers.

348. A number which *cannot* be resolved into *equal* factors, or whose root *cannot* be *exactly* found, is called an *imperfect power;* and its root is called a *Surd*, or *irrational number*. Thus, 15, 17, 45, &c., are imperfect powers, and their roots 3.8+; 4.1+; 6.7+, &c., are surds, for their exact roots cannot be extracted.

OBS. A number may be a *perfect* power of one degree and an *imperfect* power of another degree. Thus, 16 is a perfect power of the second degree, but an imperfect power of the third degree; that is, it is a perfect *square* but not a perfect *cube*. Indeed numbers are seldom perfect powers of more than one degree. 16 is a perfect power of the 2d and 4th degrees: 64 is a perfect power of the 2d, 3d and 6th degrees.

349. Every *root*, as well as every power of 1, is 1. Thus, $(1)^2$, $(1)^3$, $(1)^6$, $\sqrt{1}$, $\sqrt[3]{1}$, $\sqrt[6]{1}$, are all equal. (Art. 341. Obs. 3.)

17. What is the square root of .25? *Ans.* .5.

18. What is the square root of .16? .49? .36? .81?

19. What is the square root of .4?

Suggestion.—The numerator .4 is a perfect power, but the denominator 10 is an imperfect power; therefore the root cannot be exactly found.

20. What is the square root of .9?

21. What is the square root of .04? The square root of .09? Of .27? .64?

22. Express the cube root of 45 both ways upon your slate, or on the black board.

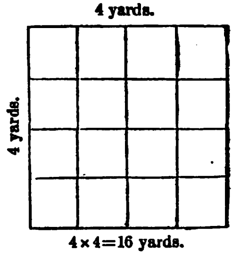
23–30. Express the cube root of 64 both ways; the fourth root of 181; the 5th root of 32; the 6th root of 64; the 7th root of 84; the 8th root of 91; the 9th root of 105; the 10th of 256.

QUEST.—347. What is a perfect power? What is a rational number? 348. What is an imperfect power? What is a surd? *Obs.* Are numbers ever perfect powers of one degree and imperfect powers of another degree? Are they often perfect powers of more than one degree? 349. What are all roots and powers of 1?

EXTRACTION OF THE SQUARE ROOT.

350. *To extract the square root of a number is to find a factor, which, being multiplied into itself, will produce the given number.* (Art. 344. Obs.)

Ex. 1. What is the length of one side of a square room which contains 16 square yards?



Suggestion.—Let the room be represented by the adjoining figure. It is divided into 16 equal squares, which we will call square yards.

Since the room is square, the question is simply this: What is the square root of 16? Now if we resolve 16 into two equal factors, each of those factors will be the square root of 16. But $16=4\times4$. Therefore the side of the room is 4 yards.

2. What is the length of one side of a square room which contains 576 square feet?

Operation.

$$\dot{5}7\dot{6}(24$$
$$4$$
$$44)\overline{176}$$
$$\underline{176}$$

Suggestion.—Since we may not see at once what the root of 576 is, we separate it into periods of two figures each, by putting a point over the 5, and the 6. This shows that the root is to have two figures, and thus enables us to find the root of part of the number at a time. (Art. 342. Obs. 2.) Now the greatest square of 5, the left hand period, is 4, the root of which is 2. We place the 2 on the right of the number for the first part of the root; then subtract its square from 5, the period under consideration, and to the right of the remainder bring down 76, the next period, for a dividend. To find the next figure in the root, we double the 2, the part of the root already found, and placing it on the left of the dividend for a trial divisor, find how many times it is contained in the dividend, omitting the right hand figure.

QUEST.—350. What is it to extract the square root of a number?

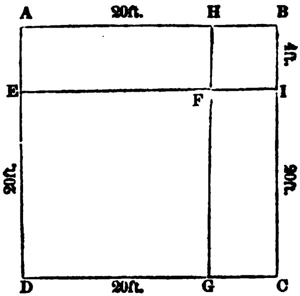
Now 4 is contained in 17, 4 times. Placing the 4 on the right of the root, also on the right of the trial divisor, we multiply 44, the divisor thus completed, by 4, the last figure in the root, and subtracting the product 176 from the dividend, find there is no remainder. The answer therefore is 24.

Obs. **Since the root is to contain two figures, the 2 stands in tens' place; hence, the first part of the root found is properly 20; which being doubled, gives 40 for the divisor. For convenience we omit the cipher on the right; and to compensate for this, we omit the right hand figure of the dividend. This is the same as dividing both the divisor and the dividend by 10, and therefore does not alter the quotient. (Art. 88.)**

PROOF.—24=2 tens, or 20+4 units.

$$\begin{array}{lrcl} & 24 = 2 \text{ "} & & 20+4 \text{ "} \\ \hline & 96 & & 80+16 \\ & 48 & & 400+80 \\ \hline (24)^2 = & 576 & = & 400+160+16. \end{array} \quad \text{(Art. 342.)}$$

ILLUSTRATION BY GEOMETRICAL FIGURE.



Let the large square ABCD, represent the room in the last example; then the square DEFG will be the greatest square of the left hand period, the root of which is 20 ft., and $20 \times 20 = 400$, the number of ft. in the area. (Art. 153. Obs. 3.) But this square 400 ft. taken from 576 ft. leaves a remainder of 176 ft. Now it is plain, if the space remaining is all added to one side of this square, its sides will become unequal; consequently it will cease to be a square. (Art. 153. Obs. 1.) But if it is equally enlarged on two sides, it will obviously continue to be a square. For this reason the root is doubled for a divisor in the operation. The parallelograms AEFH and GFIC will therefore represent the additions made to the two sides, each of which is 4 ft. wide; consequently the area of each is $20 \times 4 = 80$ ft., and the area of both is $40 \times 4 = 160$ ft.

QUEST.—*Obs.* **What place does the first figure of the root occupy in the example above? Why is the right hand figure omitted?**

But, having made these additions to two sides of the square, there is a vacancy at the corner. The square BIFH represents this vacancy, the side of which is 4 ft., the same as the width of the additions; and its area is $4 \times 4 = 16$ ft. For convenience in finding the area of this vacancy, we place the last figure of the root on the right of the divisor, and thus it is multiplied into itself. We now have a perfect square, the length of whose side is $20 + 4 = 24$ ft.

351. From the preceding illustrations and principles, we derive the following general

RULE FOR EXTRACTING THE SQUARE ROOT.

I. *Separate the given number into periods of two figures each, by placing a point over the units' figure, another over the hundreds, and so on over each alternate figure.*

II. *Find the greatest square number in the first or left hand period, and place its root on the right of the number for the first figure in the root. Subtract the square of this figure of the root from the period under consideration; and to the right of the remainder bring down the next period for a dividend.*

III. *Double the root just found, and placing it on the left of the dividend for a trial divisor, find how many times it is contained in the dividend, omitting the right hand figure; place the quotient on the right of the root, also on the right of the trial divisor; multiply the divisor thus completed by the last figure of the root, subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

IV. *Double the root already found for a new divisor, or bring down the last divisor, doubling its right hand figure, and proceed as before; thus continue the operation till the root of all the periods is found.*

PROOF.—*Multiply the root into itself; and if the product is equal to the given number, the work is right.* (Art. 344.)

OBS. 1. The product of the divisor into the figure last placed in the root, cannot *exceed* the *dividend*. Hence, in finding the figure of the root, some allowance must be made for carrying, when the product of this figure into itself *exceeds* 9.

QUEST.—351. What is the first step in extracting the square root? The *second? Third?* Fourth? How is square root proved?

2. If the trial divisor *is not contained* in the dividend, place a *cipher* in the root, also on the right of the divisor, and bring down the next period.

3. If there is a remainder after all the periods are brought down, *periods of ciphers* may be annexed, and the figures of the root thus obtained, will be *decimals.*

351.a. *Demonstration.*—The *reason* for the several steps in the rule, may be inferred from the preceding illustrations. The following is a summary of them:

1. Separating the given number into *periods of two figures* each, shows how many figures the root is to contain, and thus enables us to find *part* of the *root* at a time. (Art. 342. Obs. 2.)

2. The *square* of the first figure of the root, shows the number of feet, yards, &c., disposed of by the first figure of the root; it is subtracted from the period to find how many feet, yards, &c., *remain* to be added.

3. The root is *doubled* for a trial divisor, because the addition must be made on *two sides* of the square already found, or it will cease to be a *square.*

4. In dividing, the *right hand figure* of the dividend is *omitted,* because the cipher on the right of the divisor is omitted; otherwise the quotient would be 10 times too large for the next figure in the root.

5. The last figure of the root is placed on the right of the divisor for convenience of multiplying. The divisor is then multiplied by the last figure of the root to find the area of the several additions thus made.

3. Square root of 625?
4. Square root of 900?
5. Square root of 1225? 35
6. Square root of 1764?
7. Square root of 2916?
8. Square root of 4761?
9. Square root of 8649?
10. Square root of 12321?
11. Square root of 53824?
12. Square root of 531441?

352. *Decimals* must be separated into periods like whole numbers, by placing a point over *units,* then over *hundredths,* and so on, annexing a cipher to the right hand period, if deficient.

OBS. There will always be as many decimal figures in the root, as there are periods of decimals in the given number.

353. To extract the square root of a *common fraction.*

Reduce the fraction to its simplest form, then extract the root of the numerator and denominator.

OBS. When either the numerator or denominator is not a *perfect square,* the fraction should be reduced to a decimal, and the root found as above.

A *mixed number* should be reduced to an improper fraction.

QUEST.—351.*a. Dem.* Why separate the given number into periods of two figures each? Why subtract the square of the first figure in the root from the first period? Why double the root thus found for a divisor? Why omit the right hand figure of the dividend? Why place the last figure of the root on the right of the divisor? Why multiply the divisor by the last figure in the root? 352. When there are decimals in the given number, how are they pointed off? When there is a remainder, how proceed? *Obs.* How do you determine how many decimal figures there should be in the root? How is the square root of a common fraction found? Of a mixed number?

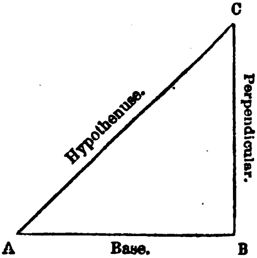
13. What is the square root of 6.25? *Ans.* 2.5.
14. Square root of 1.96?
15. Square root of 29.16?
16. Square root of 284.09?
17. Square root of .1225?
18. Square root .776161?
19. Square root of 2?
20. Square root of 17?
21. Square root of 175?
22. Square root of 116964?
23. Square root of 10316944?
24. Square root of $\frac{36}{49}$?
25. Square root of $\frac{144}{576}$?
26. Square root of $6\frac{1}{4}$?
27. Square root of $52\frac{9}{16}$?
28. Square root 22420225?
29. Square root 5764801?
30. Square root 16777216?
31. Square root 43046721?
32. Square root 3486784401?
33. Square root 1073.741824?
34. Square root of $\frac{256}{1296}$?
35. Square root of $\frac{1024}{7776}$?
36. Square root of $\frac{4096}{46656}$?
37. Square root of $\frac{16384}{65536}$?
38. Square root of $40\frac{24}{25}$?
39. Square root of $113\frac{28}{36}$?
40. Square root .00053361?
41. Square root .00038025?
42. What is the square root of $\frac{4}{9}$ of $\frac{16}{25}$ of $\frac{49}{81}$ of 144?
43. What is the square root of $\frac{9}{64}$ of $\frac{25}{49}$ of $\frac{81}{121}$ of 4096?
44. Required the square root of 3 to 7 decimals.
45. Required the square root of 12 to eight decimals.

APPLICATIONS OF THE SQUARE ROOT.

354. A *triangle* is a figure which has *three sides* and *three angles*. When one of the sides of a triangle is *perpendicular* to another side, the angle between them is called a *right-angle.*

355. A *right-angled triangle* is a triangle which has a *right-angle.*

The side opposite the right-angle is called the *hypothenuse*, and the other two sides, the *base* and *perpendicular.* The triangle ABC is right-angled at B, and the side AC is the hypothenuse.

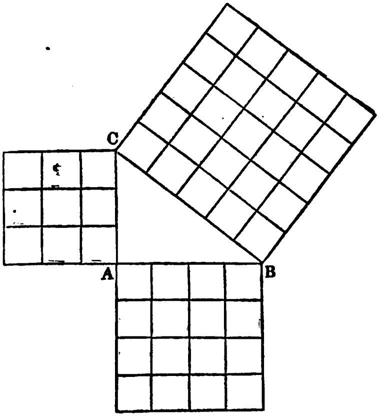


QUEST.—354. What is a triangle? What is a right-angle? 355. What is a right-angled triangle? Draw a right-angled triangle upon the black-board. What is the side opposite the right-angle called? What are the other two sides called?

356. The *square* described on the *hypothenuse* of a right-angled triangle, is equal to the *sum* of the *squares* described on the *other two* sides. (Thomson's Legendre's Geom. B. IV. 11.)

OBS. The truth of this proposition may be seen from the following geometrical illustration.

Let ABC be a right-angled triangle, the base AB =4 ft., and the perpendicular AC=3 ft. The square described on the base contains 16 sq. ft. and the square described on the perpendicular contains 9 sq. ft.; now the sum of these squares 16+9=25 sq. ft. But the square described on the hypothenuse also contains 25 sq. ft.; and is therefore equal to the sum of the other two.



OBS. Since the square of the hypothenuse BC, is 25, it follows that the square root of 25, which is 5, must be the hypothenuse itself. Hence,

357. When the base and perpendicular are given, to find the *hypothenuse.*

Add the square of the base to the square of the perpendicular, and the square root of the sum will be the hypothenuse.

Thus, in the right-angled triangle, ABC, the base is 4, and the perpendicular is 3, then $(4)^2+(3)^2=25$, and $\sqrt{25}=5$, the hypothenuse.

358. When the hypothenuse and base are given, to find the *perpendicular.*

From the square of the hypothenuse subtract the square of the base, and the square root of the remainder will be the perpendicular.

Thus, if the hypothenuse is 5, and the base 4, then $(5)^2-(4)^2=9$, and $\sqrt{9}=3$, the perpendicular.

QUEST.—356. What is the square described on the hypothenuse equal to? Draw a right-angled triangle, and describe a square on each of its sides? 357. When the base and perpendicular are given, how is the hypothenuse found?

359. When the hypothenuse and the perpendicular are given, to find the *base.*

From the square of the hypothenuse subtract the square of the perpendicular, and the square root of the remainder will be the base.

Thus, if the hypothenuse is 5, and the perpendicular 3, then $(5)^2-(3)^2=16$, and $\sqrt{16}=4$, the base.

46. What is the length of a ladder which will just reach to the top of a house 32 feet high, when its foot is placed 24 feet from the house?

Operation.

Perpendicular $(32)^2=32\times 32=1024$
Base $(24)^2=24\times 24=\underline{\ 576}$
The square root of their sum $1600=40$ ft. *Ans.*

47. The side of a certain school-room having square corners, is 8 yards, and its width 6 yards: what is the distance between two of its opposite corners?

48. Two men start from the same place and at the same time; one goes exactly south 40 miles a day, the other goes exactly west 30 miles a day: how far apart will they be at the close of the first day?

49. How far apart will the same travelers be at the end of 4 days?

50. A line 75 feet long fastened to the top of a flag-staff reaches the ground 45 feet from its base: what is the height of the flag-staff?

51. Suppose a house is 40 feet wide, and the length of the rafters is 32 feet: what is the distance from the beam to the ridge-pole?

52. The side of a square field is 30 rods: how far is it between its opposite corners?

53. If a square field contains 10 acres, what is the length of its side, and how far apart are its opposite corners?

54. If a school room is 40 feet long, 30 feet wide, and 14 feet high, what is the length of a diagonal drawn upon the floor; and what is the length of a diagonal drawn from the floor to the ceiling?

QUEST.—358. When the hypothenuse and base are given, how is the perpendicular found? 359. When the hypothenuse and perpendicular are given, how is the *base found?*

360. *The side of a square equal in area to any given surface, is found by extracting the square root of the given surface.*

OBS. To find the *dimensions* of a rectangular field, equal in *area* to a given surface, when its length is *double*, *triple*, or *quadruple*, &c., of its breadth, find the *square root* of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, of the given surface, and this will be the *width*; and the width being *doubled*, *tripled*, or *quadrupled*, as the case may be, will be the *length*.

55. What is the side of a square, whose area is equal to that of a circle which contains 225 sq. yds.? *Ans.* 15 yds.

56. A general has 906304 soldiers: how many must he place in rank and file to form them into a square?

57. A man bought a square tract of land containing 3840 acres: how many rods square is the tract?

58. He afterwards divided his tract into four equal and square farms: what is the length of one of their sides?

59. A man having a garden 465 yards square, wished to extend it so as to make it 9 times as large: how many yards square will it then be?

60. What is the side of a square, whose area is equal to that of a triangle containing 576 sq. ft.?

61. The length of a rectangular field containing 80 acres, is twice its breadth: what are its length and breadth?

62. The breadth of a rectangular farm containing 160 acres, is $\frac{1}{4}$ its length: what are its length and breadth?

63. What is the side of a square equal in area to a rectangular field 32 rods long and 18 rods wide?

361. The areas of *all similar figures* are to each other as the *square* of their *similar sides* or *dimensions*. (Leg. IV. 25, 27. V. 10.)

64. If a pipe 1 inch in diameter will fill a cistern in 60 minutes, how long will it take a pipe 2 inches in diameter to fill the same cistern?

65. If a gate 9 inches in diameter will empty a mill-pond in 16 hours, how large must a gate be to empty it in 4 hours?

QUEST.—360. How do you find the side of a square equal in area to any given surface? *Obs.* How find the dimensions of a rectangular field equal in area to a given surface, when its length is double, triple, &c., of its breadth? 361. In what ratio are the areas of similar figures to each other.

66. If one side of the base of a triangular pyramid measuring 16 square feet, is 20 inches in length, what is the length of a side of a similar pyramid, which measures 36 square feet?

67. A man owns a building lot containing 20 square rods in the shape of a right-angled triangle, the perpendicular of which is 20 yards in length: what is the perpendicular of a a similar lot, which contains 30 square rods?

362. ***A mean proportional between two numbers is found by multiplying the given numbers together, and extracting the square root of the product.*** (Art. 320. Obs. 1.)

68. What is the mean proportional between 9 and 16?

Solution.—$16 \times 9 = 144$; and $\sqrt{144} = 12$. *Ans.*

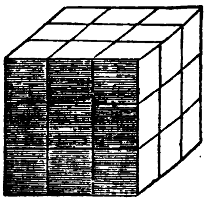
Find the mean proportional between the following numbers:

69. 4 and 16.	74. 28 and 54.	79. $\frac{1}{4}$ and $\frac{1}{9}$.
70. 9 and 25.	75. 45 and 96.	80. $\frac{4}{9}$ and $\frac{16}{25}$.
71. 25 and 36.	76. .04 and .16.	81. $\frac{25}{36}$ and $\frac{49}{81}$.
72. 49 and 64.	77. .64 and 6.25.	82. $\frac{36}{64}$ and $\frac{16}{100}$.
73. 81 and 64.	78. .09 and .36.	83. $\frac{49}{121}$ and $\frac{81}{144}$.

EXTRACTION OF THE CUBE ROOT.

363. ***To extract the cube root is to find a factor which being multiplied into itself twice, will produce the given number.*** (Art. 344.)

1. What is the side of a cubical block containing 27 solid feet?



$3 \times 3 \times 3 = 27$.

Suggestion.—Let the given block be represented by the adjoining cubical figure, each side of which is divided into 9 equal squares, which we will call square feet. Now, since the length of a side is 3 feet, if we multiply 3 into 3 into 3, the product 27, will be the solid contents of the cube. (Art. 154. Obs. 3.) Hence, if we reverse the pro-

QUEST.—362. How find a mean proportional between two numbers? 363. What is it to extract the cube root?

cess, i. e. if we resolve 27 into three equal factors, one of these factors will be the side of the cube. (Art. 344. Obs.) *Ans.* 3 ft.

2. A man wishes to form a cubical mound containing 15625 solid feet of earth: what is the length of its side?

Operation.

	$1\dot{5}62\dot{5}$(25
	8
Divisor. 1200	7625 div.
300	
25	
1525	7625

1. We first separate the given number into periods of three figures each, by placing a point over the units' figure, then over thousands. This shows us that the root must have two figures, (Art. 342. Obs. 3,) and thus enables us to find part of it at a time.

2. Beginning with the left hand period, we find the greatest cube of 15 is 8, the root of which is 2. Placing the 2 on the right of the given number, we subtract its cube from the period, and to the remainder bring down the next period for a dividend. This shows that we have 7625 solid feet to be added to the cubical mound already found.

3. We square the part of the root thus found, which in reality is 20, for since there is to be another figure annexed to it, the 2 is tens; then multiplying its square 400 by 3, we write the product on the left of the dividend for a trial divisor; and finding it is contained in the dividend 5 times, we place the 5 in the root.

4. We next multiply 20, the root already found, by 5, the last figure placed in the root; then multiply this product by 3 and write it under the divisor. We also write the square of 5, the last figure placed in the root, under the divisor, and adding these three results together, multiply their sum 1525 by 5, and subtract the product from the dividend. The answer is 25.

PROOF.—*Multiply the root into itself twice, and if the last product is equal to the given number, the work is right.*

Thus, $25 \times 25 \times 25 = 15625$.

OBS. The simplest method of illustrating the process of extracting the cube root to those unacquainted with algebra and geometry, is by means of *cubical blocks*. A set of these blocks contains 1st, a *cube*, the side of which is usually about 1½ in. square; 2d, *three side* pieces about ½ in. thick, the upper and lower base of which is just the size of a side of the cube; 3d, *three corner* pieces, whose ends are ½ in. square, and whose length is the same as that of the side pieces; 4th, a *small cube*, the side of which is equal to the end of the corner pieces. It is desirable for every teacher and pupil to have a set. If not conveniently procured at the shops, any one can easily make them for himself.

364. From the preceding illustrations and principles, we derive the following general

RULE FOR EXTRACTING THE CUBE ROOT.

I. *Separate the given number into periods of three figures each, placing a point over units, then over every third figure towards the left in whole numbers, and over every third figure towards the right in decimals.*

II. *Find the greatest cube in the first period on the left hand; then placing its root on the right of the number, subtract the cube from the period, and to the remainder bring down the next period for a dividend.*

III. *Square the part of the root thus found with a cipher annexed to it; multiply this square by 3, and place the product on the left of the dividend for a trial divisor; find how many times it is contained in the dividend, and place the result in the root.*

IV. *Multiply the root previously found with a cipher annexed by this last figure placed in it, then multiply this product by 3, and write the result under the divisor; under this result write also the square of the last figure placed in the root.*

V. *Finally, add these results to the trial divisor; multiply the sum by the last figure placed in the root, and subtract the product from the dividend. To the right of the remainder bring down the next period for a new dividend; find a new divisor as before, and thus proceed till the root of all the periods is found.*

OBS. 1. When there is a remainder, *periods of ciphers* may be added, and the figures of the root thus obtained will be decimals.

2. If the right hand period of decimals is deficient, this deficiency must be supplied by ciphers.

3. When there are *decimals* in the given example, find the root as in whole numbers; then point off as many decimal figures in the answer, as there are *periods of decimals* in the given number.

4. If the trial divisor is *not contained* in the dividend, place a *cipher* in the root, also *two ciphers* on the right of the divisor, and bring down the next period.

5. In finding the cube root of a *common fraction*, first reduce the fraction to its lowest terms, then extract the root of its numerator and denominator.

When either the numerator or denominator is not a *perfect cube*, the fraction should be reduced to a decimal, and the root of the decimal be found as above.

A *mixed* number should be reduced to an improper fraction.

QUEST.—364. What is the first step in extracting the cube root? The second? Third? Fourth? Fifth? How is the cube root proved?

DEMONSTRATION BY CUBICAL BLOCKS.

1. The *reason* for dividing the number into *periods* of *three figures*, is *two* fold: *First*, it shows us how many figures the *root* will contain: *Second*, it enables us to find *part* of it at a time. Now, placing the large cube upon a table or stand, let it represent the greatest cube in the left hand period, which in the example above is 8, the root of which is 2. We subtract this cube from the left hand period, and to the remainder bring down the next period, in order to find how many feet remain to be added. In making this addition, it is plain the cube must be equally increased on *three sides*; otherwise its sides will become *unequal*, and it will then cease to be a *cube*. (Art. 154. Obs. 2.)

2. The *object of squaring* the part of the root already found with a cipher annexed, is to find the area of one side of the cube. (Art. 153. Obs. 3.) The cipher is annexed because the part of the root thus found, denotes *tens* of the order next following. We multiply its square by 3, because the additions are to be made to three of its sides; and, dividing the dividend by this product shows the thickness of these additions. Now placing one of the side pieces on the top, and the other two on two adjacent sides of the cube, they will represent these additions.

3. But we perceive there is a vacancy at three corners, each of which is of the same length as the root already found, or the side of the cube, viz: 20 ft., and the breadth and thickness of each is 5 ft., the thickness of the side additions. Placing the corner pieces in these vacancies, they will represent the additions necessary to fill them. The object of multiplying the root already found by the figure last placed in it, is to obtain the area of a side of one of those additions; we then multiply this area by 3, to find the area of a side of each of them.

4. We find also another vacancy at one corner, whose length, breadth, and thickness are each 5 ft., the same as the thickness of the side additions. This vacancy therefore is cubical. It is represented by the small cube, which being placed in it, will render the mound an exact cube again. The object of squaring the 5, or the figure last placed in the root, is to find the area of a side of this cubical vacancy. We now have the area of one side of each of the side additions, the area of one side of each of the corner additions, and the area of one side of the cubical vacancy, the sum of which is 1525. We next multiply the sum of these areas by the figure last placed in the root, in order to find the cubical contents of the several additions. (Art. 154. Obs. 3.) These areas are added together, and their sum multiplied by the last figure placed in the root, for the sake of finding the solidity of all the additions at once. The result would obviously be the same, if we multiply them separately, and then subtracted the sum of their products from the dividend.

3. What is the cube root of 1728?

4. Cube root of 13824?

5. Cube root of 373248?

6. Cube root of 571787?

7. Cube root of 1953125?

8. Cube root of 27?

9. Cube root 2357947691?

QUEST.—*Dem.* Why separate the given number into periods of three figures each? Why subtract the greatest cube from the left hand period? Why square the part of the root already found? Why annex a cipher to it? Why multiply its square by 3? Why divide the dividend by this product? Why multiply the root already found by the last figure placed in it? Why multiply this product by 3? Why square the figure last placed in the root? Why multiply the sum of these areas by the last figure placed in the root?

10. Cube root of 12.167?
11. Cube root of 91.125?
12. Cube root of $\frac{27}{64}$?
13. Cube root of $\frac{125}{729}$?
14. Cube root of $\frac{378}{1750}$?
15. Cube root of $\frac{1520}{5130}$?
16. Cube root of $\frac{729}{4096}$?
17. Cube root of $\frac{15625}{46656}$?
18. Cube root of $18\frac{2}{3}$?
19. Cube root of $37\frac{1}{27}$?
20. Cube root of 1092727?
21. Cube root 27054036008?
22. Cube root 164.566592?
23. Cube root 122615.327232?

APPLICATIONS OF THE CUBE ROOT.

365. To find the side of a cube equal to any given solid.

Extract the cube root of the given quantity, and it will be the length of the side required.

24. What is the length of a side of a cubical box, which contains 389017 solid inches?

25. What is the side of a cubical vat, which contains 48228544 solid feet?

26. What is the side of a cubical mound, which contains 1259712 solid yards?

27. What is the side of a cube equal to a stick of timber 2 feet square, and 128 feet long?

28. What is the side of a cubical bin, which contains 500 bushels, allowing 2150.4 cu. in. to a bushel?

29. What is the side of a cubical cistern, which holds 100 wine hogsheads?

30. What is the side of a cube equal to a pile of wood 2421 ft. long, 12 ft. wide, and 7 feet high?

366. *The contents of similar solids are to each other as the cubes of their similar sides or dimensions.* (Leg. VII. 20. VIII. 11. Cor.)

OBS. Conversely, the *cubes* of the similar sides of similar solids are to each other as their *contents*.

31. If a globe 4 inches in diameter weighs 32 lbs., what is the weight of a globe whose diameter is 5 inches?

Note.—The *diameters* of globes or balls are *similar dimensions*. Therefore,

$4^3 : 5^3 :: 32$ lbs. : Ans.; or, $64 : 125 : 32$ lbs. : Ans.

125×32 lbs. $= 4000$ lbs., and 4000 lbs. $\div 64 = 62.5$ lbs. *Ans.*

QUEST.—365. How find the side of a cube equal to any given solid? 366. What is the ratio of similar solids to each other?

32. If a ball 3 inches in diameter weighs 4 lbs., what is the diameter of a ball which weighs 32 lbs?

Solution.—4 lbs. : 32 lbs. : : 3^3 : cube of diameter required.

Now $32 \times 27 = 864$; then $864 \div 4 = 216$, and $\sqrt[3]{216} = 6$ ft. *Ans.*

33. If a round melon 4 inches in diameter weighs 8 lbs., what is the weight of a similar melon 12 inches in diameter?

34. If a cube of gold whose side is 3 inches is worth $6400, what is the worth of a cube of gold whose side is 8 inches?

35. If a pyramid 60 feet high contains 12500 cu. ft., how many cu. ft. are there in a similar pyramid 30 ft. high?

36. If a conical stack of hay whose height is 12 feet contains 5 tons, what is the weight of a similar stack whose height is 20 feet?

37. If a wire $\frac{1}{8}$ of an inch in diameter and $\frac{7}{8}$ of an inch long, weighs 500 lbs., what is the weight of a suspension bridge of the same length, having 10 wire cables, each 4 in. in diameter?

38. If a cylindrical cistern 6 feet in diameter will contain 30 hogsheads of water, how much will a similar cistern contain, whose diameter is 20 feet?

367. To find the side of a cube whose solidity shall be *double*, *triple*, &c, that of a cube whose side is given.

Cube the given side, multiply it by the given proportion, and the cube root of the product will be the side of the cube required.

39. What is the side of a cubical mound, which contains 8 times as many solid feet as one whose side is 3 ft. *Ans.* 6 ft.

40. Required the side of a cubical vat, which contains 3 times as many solid feet as one whose side is 5 ft.

41. If a cube of silver whose side is 4 inches, is worth $200, what is the side of a cube of silver, worth $1600?

42. I have a cubical box whose side is 6 feet; I want another which will contain $\frac{1}{8}$ part as much: What will be the length of its side?

43. Required the side of a cubical vat which shall contain $\frac{1}{27}$ part as much as one whose side is 12 feet?

QUEST.—367. How find the side of a cube whose solidity is double, triple, &c., that of a cube whose side is given?

SECTION XV.

EQUATION OF PAYMENTS.

ART. 368. EQUATION OF PAYMENTS is the *process of finding the average time of two or more payments which are due at different times.*

OBS. The *average* time of two or more payments is often called the *mean* or *equalized* time.

369. From principles already explained, it is manifest, when the *rate* is fixed, the *interest* depends both upon the *principal* and the *time.* (Art. 241.) Thus, if a given principal produces a certain interest in a given time,

Double that principal will produce *twice* that interest;

Half that principal will produce *half* that interest; &c.

In *double* that time the same principal will produce *twice* that interest;

In *half* that time it will produce *half* that interest; &c.

370. Hence, it is evident that any given principal will produce the same interest in any given time, as

One half	that	principal	will	produce	in *double* that time;
One third	"	"	"	"	in *thrice* that time;
Twice	"	"	"	"	in *half* that time;
Thrice	"	"	"	"	in *a third* of that time, &c.

Thus, the interest of \$2 for 1 year, =\$1 for 2 years;
the interest of \$3 for 1 year, =\$1 for 3 years; &c.
the interest of \$4 for 1 month,=\$1 for 4 months;
the interest of \$5 for 1 month,=\$1 for 5 months; &c. Hence,

371. *The interest of any given principal for* 1 *year,* 1 *month, or* 1 *day, is the same as the interest of* 1 *dollar for as many years, months, or days, as there are dollars in the given principal.*

QUEST.—368. What is Equation of Payments? *Obs.* What is the average time of two or more payments called? 369. When the rate is fixed, upon what does the interest depend? 371. What is the interest of any given principal for 1 year, 1 month, or 1 day, equal to?

CASE I.—*When the several items have the same date, but different credits.*

1. If you owe a man $5 payable in 8 months, and $10 in 2 months, at what time may both payments be made without loss to either party?

Analysis.—Since the interest of $5 for 1 month is the same as the interest of $1 for 5 months, (Art. 371,) the interest of $5 for 8 months must be equal to the interest of $1 for 8 times 5 months; and 5 mo. × 8=40 mo. In like manner the interest of $10 for 2 months is equal to the interest of $1 for 2 times 10 months, or 20 months. Now 40 months and 20 months are 60 months; therefore, you are entitled to the credit of $1 for 60 months. But $1 is $\frac{1}{15}$ of $15, consequently you are entitled to the credit of $15, $\frac{1}{15}$ part of 60 months, and 60 months ÷ 15=4 mo. Therefore, both payments may be made in 4 months without loss to either party. Hence,

372. To find the *average time* of two or more payments, when the items have the *same date*, but *different credits*.

Multiply each debt by the time before it becomes due; then divide the sum of the products thus obtained by the sum of the debts, and the quotient will be the average time required.

OBS. 1. If one of the debts or items is to be *paid down*, its product will be *nothing*; but in finding the *sum* of the debts, this payment must be added with the others.

2. This rule is based upon the supposition that *discount* and *interest paid in advance*, are *equal*. But this is not exactly true; consequently, the rule, though in general use, is not strictly accurate. (Art. 261. Obs. 1.)

2. If I owe a man $30, due this day, $20, payable in 4 months, $40 in 6 months, and $60 in 3 months, at what time may I justly pay the whole at once?

Note.—Since $30 is already due, it is entitled to no credit, and if multiplied by 0, its product is 0. But the sum of the products shows how long a credit $1 must have to equal the credit to which the sum of all the debts is entitled; therefore the $30, which is now due, must be added with the other debts to form the true divisor.

$$
\begin{array}{ll}
\$30 \times 0\ m. = & 00 \\
\$20 \times 4\ m. = & 80 \\
\$40 \times 6\ m. = & 240 \\
\$60 \times 3\ m. = & 180 \\
\hline
150 & 15|0)50|0 \\
 & \textit{Ans. } 3\frac{1}{3}\text{ m.}
\end{array}
$$

3. A farmer has 3 notes; one of $50, due in 2 months; another of $100, due in 5 months; and the third of $150, due in 8 months: what is the average time of the whole?

QUEST.—372. How find the average time of two or more payments, when the items have the same date, but different credits? *Obs.* If one of the items is to be paid down, how proceed?

4. A merchant buys goods, and agrees to pay $400 down, $400 in 4 months, and $400 in 8 months: what is the average time of the whole?

5. A man borrows $600, and agrees to pay $100 in 2 months, $200 in 5 months, and the balance in 8 months: when can he justly pay the whole at once?

6. A man buys a house for $1600, and agrees to pay $400 down, and the rest in 3 equal annual instalments: what is the average credit for the whole?

7. I have $1200 owing to me, $\frac{1}{2}$ of which is now due; $\frac{1}{4}$ of it will be due in 4 months, and the remainder in 8 months: what is the average time of the whole?

8. A grocer bought goods amounting to $1500, for which he was to pay $250 down, $300 in 4 months, and $950 in 9 months: when may he pay the whole at once?

9. A young man bought a farm for $2000, and agrees to pay $500 down, and the balance in 5 equal annual instalments: what is the average time of the whole?

CASE II.—*When the charges have different dates, but the same credit.*

10. A man bought the following bills of goods on 6 months' credit: Jan. 1st, 1853, $50; Jan. 16th, $75; Jan. 28th, $25; Feb. 24th, $250; March 14th, $100: at what time must a note for the whole amount be dated, so that the buyer shall have 6 months' credit?

Suggestion.—Since the note is to be made payable in 6 months from date, it is plain that the average date of the several charges, that is, the average time from the first charge to the last, will be the date of the note.

Operation.

Jan. 1,	$50 × 0*d*.	=	0000
" 16,	75 × 15	=	1125
" 28,	25 × 27	=	675
Feb. 24,	250 × 54	=	13500
Mar. 14,	100 × 72	=	7200
	500		5|00)225|00

The average time from the first charge is 45 days.

Therefore, the note must be dated Feb. 14th, 1853. Hence,

QUEST.—373. How find the average time, when the charges have different dates, but the same credits? *Obs.* From what date do you reckon time? Do you count the day from which and the day to which you reckon? When the items have different dates and different credits also, from what do you reckon the *time*, and how proceed?

373. To find the average time, when the charges have *different dates*, but the *same credit.*

Multiply each charge by the number of days from the date of the first charge to its own date, divide the sum of the products by the sum of the charges, and the quotient will be the average time from the first charge.

OBS. 1. Any convenient date may be taken from which to reckon the time, observing that the day *from* which you reckon must *not* be counted; but the day *to* which you reckon, *must* be counted. Thus, the *average time* of the last example may be found by multiplying each charge by the *number* of *days* from the *time* the first charge is *due* to the time it is itself due, then proceeding as before.

2. When the items have *different dates* and *different credits*, the time should be reckoned from the day on which the *first payment* is to be made; then multiply each charge by the number of days from this date to the time it is itself due, and proceed according to the rule above.

3. If the average time contains a fraction *less* than *half* a day, it is disregarded; if *equal* to, or *greater* than a half, 1 day is added.

11. Bought the following amount of goods on 4 months' credit: March 10th, 1852, $200; April 15th, $160; May 1st, $440: at what time is the whole amount payable?

12. Bought the following bills on 8 months: July 5th, 1853, $620.25; Aug. 11th, $240.56; Sept. 20th, $321.64; Oct. 12th, $510.38; Nov. 1st, $308.17: when ought a note for the whole amount to be dated?

13. A merchant bought the following bills of goods: March 19th, $350 on 4 months; April 1st, $430 on 130 days; May 16th, $540 on 95 days; June 10th, $730 on 3 months: what is the average time for the payment of the whole?

14. Bought the following bills of goods on 90 days' credit: May 10th, $375.63; May 18th, $738.45; June 3d, $860.40; June 17th, $692.38; July 3d, $379.68; July 12th, $417.13: at what time will the whole be due at once?

15. A grocer sold the following amount of goods: June 3d, $380 on 90 days' credit; June 10th, $485 on 30 days; July 21st, $834 on 60 days; July 27th, $573 on 110 days; Aug. 2d, $485 on 80 days: at what time will the whole be due?

16. Sold the following bills of goods on 3 months: Sept. 5th, 1850, $1163.25; Sept. 20th, $2368.41; Oct. 7th, $3561.34; Oct. 23d, $840.90; Nov. 13th, $1307.63: at what time must a note for the whole amount be dated to give the buyer the specified credit?

CASE III.—*Partial payments made before a debt is due.*

17. Bought a bill of goods of $600 on 6 months; in 2 months I paid $150, and 2 months later I paid $200 more: what is the average time for paying the balance.

Suggestion.—The payment of $150, 4 months before it is due, entitles me to a credit on $1 for 600 months; and the payment of $200, 2 mo. before it is due, entitles me to a credit on $1 for 400 mo. Therefore, by both payments I am entitled to a credit on the balance $250 for 4 mo. after the debt is due. Hence,

Operation.

$\$150 \times 4 = 600$ mo.
$\$200 \times 2 = \underline{400}$ mo.
$600 - \overline{350} = 250)\overline{1000}$ mo.
Ans. 4 mo.

374. To find the *average* time on the balance of a debt, when partial payments have been made before the debt is due.

Multiply each partial payment by the number of days or months from the time it was made to the time it would be due; divide the sum of the products by the balance unpaid, and the quotient will be the average time for the payment of the balance.

18. A man gave his note for $1500, payable in 10 months; in 4 months he paid $350; 2 months after he paid $150; 1 month later he paid $100: how long ought the time for paying the balance to be extended?

19. A grocer bought goods amounting to $2100 on 120 days credit; 30 days after he paid $470: in 30 days more he paid $330; and in 30 more he paid $700: what time ought to be allowed him on the balance?

20. A printer bought a lot of paper for $1835 on 6 months; in 2 months he paid $325; in 1 month more he paid $370; 1 month later still he paid $530: how long time ought he to be allowed on the balance?

21. A man bought a house for $16280 on 2 years' credit; in 8 months he paid $1235; in 4 months more he paid $2017; 3 months later he paid $3269; and in 2 months after this he paid $1735: how long ought the payment of the balance to be deferred?

QUEST.—374. How find the average time on the balance of a debt when partial payments have been made before the debt is due?

CASE IV.—*Averaging Accounts Current bearing interest.*

375. *An account current* is a statement exhibiting the *debits* and *credits* of the business transactions of one person with another.

Dr. *Henry Duncan in Account with George Wells.* *Cr.*

1853.			1853.		
Jan. 1	For Merchandise.	$250.00	Jan. 21	By sundries	$130.00
Feb. 15	" "	200.00	April 1	" draft on B.	160.00
Mar. 22	" "	550.00	" 11	" cash	210.00

What is the balance due July 1, 1853, at 6 per cent. interest?

Operation.

Drs. $250.00 × 181*d.* = 45250		Crs. $130.00 × 161*d.* = 20930	
200.00 × 136*d.* = 27200		160.00 × 91*d.* = 14560	
550.00 × 101*d.* = 55550		210.00 × 81*d.* = 17010	
Int. 21.33	6|000)128|000	Int. 8.75	6|000)52|500
$1021.33 Amt.	$21.333 Int.	Amt. $508.75	Int. $8.75
508.75			

$512.58 balance due July 1st, 1853. Hence,

376. To find the *true balance* of an account current.

I. *Multiply each item of debit and credit by the number of days from its entry to the time of settlement.*

II. *Divide the sum of the products on each side by* 6000, *and the quotients will be the interest due on the respective sides at* 6 *per cent.* (Art. 247.*a.*)

III. *Finally subtract the amount of the smaller side from that of the greater, and the remainder will be the true balance.*

OBS. In some cases, interest on accounts current does not begin until a specified time after the entry. Hence, it facilitates the operation to place against each item the number of days from the time it is due to the time of settlement.

23. Put the following items into the form of an account current, and find the true balance Jan. 1st, 1854, at 7 per cent. A. B. bought of C. D., July 16th, 1853, merchandise $350; Aug. 11th. $460; Sept. 9th, $570; Sept. 14th, $840; Oct. 18th, $780. The former paid Aug. 1st, in stock $260; Sept. 30th, in grain $340; Oct. 5th, cash $500; Oct. 21st, $625.

QUEST.—376. How find the true balance of an account current?

PARTNERSHIP.

377. PARTNERSHIP *is the associating of two or more individuals together for the transaction of business.*

The *persons* thus associated are called *partners;* and the association is termed a *company, firm* or *house.*

The *money* employed is called the *capital* or *stock;* and the *profit* or *loss* to be shared among the partners, the *dividend.*

CASE I.—*When each partners' stock is employed for the same time.*

Ex. 1. A and B formed a partnership; A furnished \$300 capital, and B \$500; they gained \$200: what was each partner's share of the gain?

Suggestion.—The amount of the whole stock was \$300+\$500=\$800; A's part of it was $\frac{300}{800}=\frac{3}{8}$, and B's part was $\frac{500}{800}=\frac{5}{8}$.

Then, A must have had $\frac{3}{8}$ of the gain; and $\$200\times\frac{3}{8}=\$75$
And B " " " $\frac{5}{8}$ " " and $\$200\times\frac{5}{8}=125$

PROOF.—The sum of the shares equals the whole gain, \$200

Or, \$800 stock : \$200 gain : : \$300 A's stock : A's gain.
Now, $\$200\times300\div\$800=\$75$ A's share.

Again, \$800 stock : \$200 gain : : \$500 B's stock : B's gain.
And, $\$200\times500\div\$800=125$ B's share. Hence,

378. To find each partner's share of the gain or loss, when the *stock* of each is employed for the *same time.*

Multiply each man's stock by the whole gain or loss; divide the product by the whole stock, and the quotient will be his share of the gain or loss.

Or, make each man's stock the numerator, and the whole stock the denominator of a common fraction; multiply the whole gain or loss by these fractions, and the products will be the respective shares of the gain or loss.

QUEST.—377. What is partnership? What are the persons thus associated called? What is the association called? What is the money employed called? What the profit or loss? 378. How is each man's share of the gain or loss found, when the stock of each is employed for the same time?

OBS. 1. This rule is applicable to questions in Bankruptcy, General Average, and all other operations in which there is to be a division of property in specified proportions.

2. The preceding case is often called *Single* Fellowship. But since a *partnership* is always composed of *two* or *more* individuals, it is somewhat difficult to see the propriety of calling it *single*.

2. A, B, and C form a partnership; A furnishes \$600, B \$800, and C \$1000; they gain \$480: what is each man's share of the gain?

3. A Bankrupt owes A \$1200, B \$2300, C \$3400, and D \$4500; his whole effects are worth \$5600: how much will each creditor receive?

4. A, B, C, and D make up a purse to buy lottery tickets; A puts in \$30, B \$40, C \$60, and D \$70; they draw a prize of \$2000: what is each man's share?

CASE II.—*When the stocks are employed for unequal periods.*

5. A and B formed a partnership; A put in \$300 for 2 months, and B \$200 for 6 months; they gained \$150: what was each man's just share of the gain?

Suggestion.—The gain of each depends both upon the *capital* he furnished, and the *time* it was employed. (Art. 369.)

A's stock $\$300 \times 2 = \$600$, the same as \$600 for 1 mo.
B's " $200 \times 6 = 1200$, " " 1200 "
Sum of products, \$1800,
A's share of the gain must therefore be $\frac{600}{1800} = \frac{1}{3}$.
B's " " " " " " $\frac{1200}{1800} = \frac{2}{3}$.
Now $\$150 \times \frac{1}{3} = \$50$, A's share.
And $\$150 \times \frac{2}{3} = \$100$, B's share. Hence,

379. To find each partner's share of the gain or loss, when the *stock* of each is employed for *different periods.*

Multiply each partner's stock by the time it is employed; make each man's product the numerator, and the sum of the products the denominator of a common fraction; then multiply the whole gain or loss by each man's fractional share of the stock, and the product will be his share of the gain or loss.

OBS. This case is often called *Compound* or *Double* Fellowship.

QUEST.—*Obs.* To what is this rule applicable? What is it sometimes called? 379. When each man's stock is employed for different periods, how proceed? *Obs.* What is this case sometimes called?

6. A, B, and C enter into partnership; A puts in $500 for 4 mo., B $400 for 6 mo., and C $800 for 3 mo; they gain $340: what is each man's share of the gain?

7. A and B hire a pasture together for $60; A put in 120 sheep for 6 months, and B put in 180 sheep for 4 months: what should each pay?

8. The firm A, B, and C lost $246; A had put in $85 for 8 mo., B $250 for 6 mo., and C $500 for 4 mo.: what is each man's share of the loss?

9. A and B engaged in an adventure of $800; A continued his stock for 6 months, and received $54 gain; B continued his 4 months, and received $40 gain: what sum did each contribute.

MEDIAL ALLIGATION.

380. ALLIGATION *is the method of finding the average price of compounds, which consist of ingredients of different values; or of forming compounds which shall have a given average value.*

It is of two kinds, *Medial* and *Alternate.*

Note.—The term *alligation* is from the Latin *alligo*, to *bind* or *tie* together. It had its origin in the manner of connecting the numbers together by a curve line, in the solution of a certain class of examples.

381. *Medial Alligation* is the process of finding the *mean price* of a mixture of two or more ingredients of *different values*, when the *quantity* and *value* of the articles mixed *are given.*

Note.—The term *medial* is derived from the Latin *medius*, which signifies a *mean* or *average.*

1. A farmer mixed 8 bushels of corn which cost $.92 per bushel, with 12 bushels of oats which cost $.56 per bushel: what is the mixture worth per bushel?

Suggestion.—By adding the cost of the corn to the cost of the oats, the cost of both is $14.08.

But the mixture contains 8 bu. +12 bu.=20 bu. Now if 20 bu. cost $14.08, one bu. will cost $\frac{1}{20}$ of $14.08, which is $.704. Hence,

8	bushels at	$.92=	$7.36
12	"	.56=	6.72
20	bu.	whole cost	$14.08

20) $14.08

Ans. $.704 per bu.

QUEST.—380. What is Alligation? Of how many kinds is it? *Note.* What is the meaning of the term alligation? 381. What is medial alligation? 382. How find the mean value of a mixture, when the quantity and the price of each of the ingredients are given?

382. To find the *average value* of a mixture, when the quantity and the price of each of the ingredients are given.

Divide the whole cost of the ingredients by the whole quantity mixed, and the quotient will be the mean price of the mixture.

PROOF.—*Multiply the whole mixture by the mean price, and if the product is equal to the whole cost, the work is right.*

2. A grocer mixed 72 lbs, of sugar worth 6d. a pound, 110 lbs. worth 8d. a pound, and 95 lbs. worth 10d. a pound: what is the worth of a pound of the mixture?

3. A refiner melted 5 lbs. of gold 22 carats fine, 2 lbs. 3 oz. 20 carats fine, and 1 lb. 7 oz. 18 carats fine: what was the fineness of the mixture?

4. A drover bought 45 horses at $87 apiece; 64 oxen at $48 apiece, and 71 cows at $28 apiece: what must he sell the whole at apiece in order to make $100?

ALTERNATE ALLIGATION.

383. *Alternate Alligation* is the process of finding *what quantity* of two or more ingredients, whose prices are given, must be taken to form a mixture of a *given price.*

Note.—The term *alternate* is from the Latin *alternatus*, signifying *by turns*, and indicates the connection of the prices which are *less* than the *mean price*, with those which are *greater*. Alternate alligation embraces three Cases.

CASE I.—*When the price of each ingredient and the price of the mixture are given, to find the* QUANTITY *of each ingredient.*

5. A grocer wished to mix four kinds of tea, worth 3s. 5s. 7s. and 8s. a pound, so that the mixture may be worth 6s. a pound: what quantity of each must be taken.

	First,			*Second,*			*Third Operation.*	
	3	1 lb.		3	2 lbs.		3	2+1=3 lbs.
6	5	2 lbs.	6	5	1 lb.	6	5	2 =2 lbs.
	7	3 lbs.		7	1 lb.		7	3 =3 lbs.
	8	1 lb.		8	3 lbs.		8	3+1=4 lbs.

Suggestion.—We first connect each price, which is *less* than the price of the mixture, with one or more of the prices which

QUEST.—383. What is alternate alligation? *Note.* What is the meaning of the term alternate?

are *greater*; and each *greater* price with one or more of those that are *less*. We then place the *difference* between the *price* of each kind and that of the mixture on the right of the price with which each is connected, and the differences which stand opposite the respective prices, denote the *relative quantities* which will form the mixture required. Hence,

384. To find the *quantity* of each ingredient, when its price and the price of the required mixture are given.

I. *Write the prices of the ingredients under each other, beginning with the least; then connect, with a curve line, each price which is less than the price of the mixture with one or more of those that are greater; and each greater price with one or more of those that are less.*

II. *Write the difference between the price of each ingredient and that of the mixture, opposite the price of the ingredient with which it is connected. If only one difference stands against any price, it will denote the quantity to be taken of that price; but if more than one, their sum will be the quantity.*

PROOF.—*Find the value of all the ingredients at their given prices; if the amount is equal to the value of the whole mixture at the given price, the work is right.*

OBS. 1. The above rule is based on the principle that the *excess* of one ingredient *above* the mean price of the mixture, counterbalances the *deficiency* of another ingredient which is *below*. Thus, in the first *operation*, 1 lb. of the 1st kind of tea, lacks 3s. of the average price of the mixture; but adding 3 lbs. of the 3d kind, which is 1s. a pound above the average price, will counterbalance this deficiency. The deficiency of the 2d kind is made up by the excess of the 4th.

In the *second operation*, the deficiency of the 1st is made up by the excess of the 4th; and the deficiency of the 2d, by the excess of the 3d.

In the *third operation*, the deficiency of the 1st is partly made up by the excess of the 4th, and partly by the excess of the 3d.

2. It is immaterial in what manner we select the pairs of ingredients, provided the price of one of the ingredients is *less* and the other *greater* than the *mean* price of the mixture.

3. Examples under this rule admit of as many different answers, as there are methods of connecting the prices of the ingredients which are less than that of the mixture, with those that are greater.

6. A goldsmith has gold of 16, 18, 22, and 24 carats fine: how much of each may be taken to form a mixture 20 carats fine?

QUEST.—384. How find the quantity of each ingredient, when its price and the price of the mixture are given?

7. How much coffee at 9, 11, and 14 cents a pound, will form a mixture worth 12 cents a pound?

8. How much ginger at 15, 18, 21, and 22 cents a pound, will form a mixture worth 19 cents a pound?

9. A merchant wished to buy four kinds of silk, worth 68, 75, 83, and 85 cents per yard, so that the average price should be 80 cents: how many yards of each can he take?

CASE II.—*When the quantity of one of the ingredients and the mean price of the mixture are given.*

10. How many pounds of starch worth 11 and 15 cents a pound, must be mixed with 16 lbs. at 10 cents, so that the mixture may be worth 13 cents a pound?

Suggestion.—By connecting the prices of the ingredients as in the last case, we find the differences between them and the price of the mixture to be 2, 3, and 5. Then the *difference* of the ingredient whose quantity is given, is to each *remaining difference*, as the *quantity given* is to the *quantity required* of each of the other ingredients. That is,

2 cts. : 2 cts. : : 16 lbs. : to the lbs. at 11 cts., or 16 lbs.
2 cts. : 5 cts. : : 16 lbs. : to the lbs. at 15 cts., or 40 lbs.
Ans. 16 lbs. at 11 cts., and 40 lbs. at 15 cents. Hence,

385. When the quantity of one ingredient and the price of the mixture are given, *to find the quantity of the other ingredients.*

Find the difference between the price of each ingredient and the mean price of the required mixture, as before; then by proportion,

As the difference of that ingredient whose quantity is given, is to each particular difference, so is the quantity given to the quantity required of each ingredient.

11. How much molasses at 25 30, and 37 cents per gallon, must be mixed with 20 gals. at 20 cts., that the mixture may be worth 28 cts. per gallon?

12. How much corn at 45, 56, and 65 cents per bushel, must be mixed with 25 bu. of oats at 40 cts., that the mixture may be worth 50 cents a bushel?

QUEST.—385. When the quantity of one ingredient and the price of the mixture are given, how find the quantity required of the other ingredients?

CASE III.—*When the whole quantity mixed and the mean price of the mixture are given.*

13. A man mixed 75 baskets of strawberries at 6, 8, and 12 cents a basket; the mixture was worth 10 cents a basket: how many baskets of each kind did he mix?

Suggestion.—We first find the difference between the price of each ingredient and the mean price as before, viz: 6 cents, 2 cents, and 2 cents, the sum of which is 10 cents.

Then the *sum* of the differences, is to each *particular difference*, as the whole quantity mixed, is to the quantity of each ingredient. That is,

10 cts. : 6 cts. : : 75 bas. : to the bas. at 12 cts., or 45 bas.
10 cts. : 2 cts. : : 75 bas. : to the bas. at 8 cts., or 15 bas.
10 cts. : 2 cts. : : 75 bas. : to the bas. at 5 cts., or 15 bas.

Ans. 45 bas. at 12 cts., 15 bas. at 8 cts., and 15 bas. at 5 cts.

Hence,

386. When the quantity to be mixed and the price of the mixture *are* given, *to find the quantity of each ingredient.*

Find the difference between the price of each ingredient and the mean price of the required mixture, as before; then by proportion,

The sum of the differences, is to each particular difference, as the whole quantity to be mixed, to the quantity required of each ingredient.

14. A farmer having oats at 25 cents, corn at 50 cents, peas at 80 cents, made a mixture of 100 bushels which was worth 60 cents a bushel: how many bushels of each did he use?

15. A grocer has four kinds of spice worth 75, 80, 85, and 95 cents a pound: how much of each will form a mixture of 50 lbs., worth 87 cents a pound?

16. A man bought four kinds of wool, at 50, 62, 75, and 83 cents a pound, and made a mixture of 750 lbs., which cost him 70 cents a pound: how many pounds of each kind did he take?

QUEST.—386. When the whole quantity mixed and the mean price of the mixture are given, how find the quantity of each ingredient?

REDUCTION OF CURRENCIES.

387. The term *currency* signifies *money*, or the *circulating medium of commerce.*

388. *Reduction of currencies is the process of changing money from the denominations of one country to the denominations of another, without altering its value.*

OBS. 1. The *intrinsic value* of the coins of different nations, depends upon their *weight* and the *purity* of the metal of which they are made. (Art. 146.a. Obs. 1.) The *legal value* of foreign coins is determined by the laws of the country.

2. For the *present standard weight* and *purity* of the gold and silver coins of the United States, see Art. 146.a. Obs. 2.

389. *Foreign coins and moneys of account which have been made current in the United States, by act of Congress, with their values annexed.*

Pound sterling of Gt. Britain,	$4.84	Rix Dollar of Bremen,	$.78¾
Do. Canada, Nova Scotia, New Brunswick,	4.00	Specie Dollar of Denmark,	1.05
		Do. Sweden and Norway,	1.06
Franc of France and Belgium,	.186	Rouble, silver, of Russia,	.75
Livre Tournois of France,	.185	Florin of Austria,	.485
Florin of Netherlands,	.40	Lira of Lombardy,	.16
Do. Southern States Germany,	.40	Lira of Tuscany,	.16
Guilder of Netherlands,	.40	Do. of Sardinia,	.186
Real Vellon of Spain,	.05	Ducat of Naples,	.80
Do. Plate of Spain,	.10	Ounce of Sicily,	2.40
Milree of Portugal,	1.12	Leghorn Livre,	.16
Do. Azores,	.83⅓	Tael of China,	1.48
Marc Banco of Hamburg,	.35	Rupee, British India,	.445
Thaler, or Rix Dollar, Prussia,	.69	Pagoda of India,	1.84

Ex. 1. Reduce £20 sterling to Federal money.

Suggestion.—The legal value of £1 is $4.84, consequently £20 must be 20 times as much; and $4.84 × 20 = $96.80. *Ans.*

2. Reduce £5, 13s. 6d. to Federal money.

Suggestion.—Reduce 13s. 6d. to the decimal of a pound, and multiply $4.84 by the number of pounds; the result is the answer. Hence,

Operation.
£1 = $ 4.84
£5, 13s. 6d. = £5.675
Ans. $27.467

QUEST.—387. What is currency? 388. What is reduction of currencies?

391. To reduce *Sterling* to *Federal* money.

I. *Reduce the given shillings, pence, and farthings to the decimal of a pound, and annex it to the pounds.*

II. *Multiply the legal value of* £1 *expressed in Federal money, by the number of pounds with the decimal annexed, and point off the product as in multiplication of decimals.*

OBS. All foreign coins may be reduced to Federal currency, by multiplying the *value* of *one* expressed in Federal money by the *number* of coins.

Reduce the following to Federal money.

3. £100, 5s.
4. £275, 15s.
5. £450, 7s. 6d.
6. £368, 16s. 4d. 2 far.
7. £591, 12s. 8d. 1 far.
8. £463, 13s. 6d. 3 far.
9. £623, 17s. 9d. 2 far.
10. £708, 11s. 4d. 1 far.
11. 8763 francs.
12. 2365 roubles, Russian.
13. 9271 florins.
14. 6235 taels, Chinese.

15. Reduce $27.467 to Sterling money.

Suggestion.—We divide the given dollars by $4.84, because $4.84 make £1. Then, reducing the decimal .675 to shillings and pence, the result is £5, 18s. 6d. Hence,

Operation.

$4.84)$27.467

£5.675

Ans. £5, 18s. 6d.

392. To reduce Federal to Sterling money.

Divide the given sum by the legal value of £1 *expressed in Federal money, and point off the quotient as in division of decimals. The figures on the left of the decimal point will be pounds; those on the right, decimals of a pound, which must be reduced to shillings, pence, and farthings.* (Art. 201.)

OBS. Federal money may be reduced to *any foreign currency*, by dividing the given sum by the Federal value of the unit money of the given currency.

16. Reduce $486.42 to ster.
17. Reduce $1452.50 to ster.
18. Reduce $16256.75 to ster.
19. Reduce $20273.375 to ster.
20. Reduce $50375.625 to ster.
21. Reduce $125370.84 to ster.
22. Reduce $2564 to francs.
23. Reduce $8256 to francs.
24. Reduce $3265 to florins.
25. Reduce $7264 to roubles.

QUEST.—391. How reduce Sterling money to Federal? *Obs.* How may all foreign coins be reduced to Federal money? 392. How reduce Federal money to Sterling? *Obs.* How reduce Federal money to any foreign currency?

26. Reduce \$45 to New England currency.

Suggestion.—Since there are 6s. N. E. cur. in \$1, in \$45 there are 45 times as many, or 270s. Now reducing the shillings to pounds, the result is £13, 10s. Hence,

$$\begin{array}{r} \$45 \\ 6s. \\ \hline 2|0)27|0s. \\ \hline \textit{Ans.}\ £13,\ 10s. \end{array}$$

393. To reduce Federal money to the State currencies.

Multiply the given sum by the number of shillings which, in the required currency, make \$1, and the product will be the answer in shillings, and decimals of a shilling. Reduce the shillings to pounds, and the decimals to pence and farthings. (Art. 201.)

OBS. For Table of the different State currencies and their origin, see p. 148.

Reduce the following to the specified currencies.

27. \$378 to N. E. currency.
28. \$565.75 to N. E. currency.
29. \$465.45 to N. Y. currency.
30. \$875.50 to N. Y. cur.
31. \$640.25 to Pa. currency.
32. \$950.60 to Ga. currency.
33. \$1000 to Canada cur.
34. \$25.78 to Canada cur.

35. Reduce £15, 7s. 6d., N. Y. currency, to Federal money.

Suggestion.—Reducing the pounds to shillings, and the pence to the decimal of a shilling, we have 307.5s. Now, as 8s. N. Y. cur. make \$1, 307.5s. will make as many dollars as 8 is contained times in 307.5; and 307.5÷8=\$38.4375. Hence,

Operation.

$$\begin{array}{r} £15,\ 7s. = 307.0\ s. \\ \text{and } 6d. = 0.5\ s. \\ \hline 8)307.5\ s. \\ \hline \textit{Ans.}\ \$38.4375. \end{array}$$

394. To reduce the State currencies to Federal money.

Reduce the pounds to shillings, and the given pence and farthings to the decimal of a shilling; then divide the sum by the number of shillings which, in the given currency, make \$1, and the quotient will be the answer in dollars and cents.

Reduce the following examples to Federal money.

36. £48, 15s. 4d. N. E. cur.
37. £73, 4s. 5d. N. E. currency.
38. £100, 18s. 3d. N. Y. cur.
39. £256, 5s. $2\frac{1}{2}$d. N. Y. cur.
40. £296, 12s. Penn. cur.
41. £430, 8s. Penn. cur.
42. £568, 10s. Ga. currency.
43. £1000, 15s. 4d. Can. cur.

QUEST.—393. How reduce Federal money to the State currencies? 394. How reduce the State currencies to Federal money?

EXCHANGE.

395. EXCHANGE *in commerce, signifies the receiving or paying of money in one place, for an equal sum in another, by drafts or bills of Exchange.*

OBS. 1. A *Bill of Exchange* is a written order, addressed to a person, directing him to pay at a specified time, a certain sum of money to another person, or to his order.

2. The person who *signs* the bill is called the *drawer* or *maker;* the person in whose favor it is drawn, the *buyer* or *remitter;* the person on whom it is drawn, the *drawee*, and after he has accepted it, the *accepter;* the person to whom the money is directed to be paid, the *payee;* and the person who has legal possession of it, the *holder* or *owner.*

396. The *acceptance* of a bill or draft is a promise to pay it at *maturity* or the *specified time.* The common method of accepting a bill, is for the drawee to write the word *accepted* and his *name* under it, across the bill, either on its face or back.

OBS. If the *payee* wishes to *sell* or *transfer* a bill of exchange, it is necessary for him to *endorse* it, or write his name on the back of it.

397. The *par of exchange* is the *standard* by which the comparative worth of the currency of different countries is estimated.

OBS. 1. The *par of exchange* may be either *intrinsic*, or *nominal.*

The *intrinsic par* is the *real value* of the currency of different countries, as determined by the *weight* and *purity* of their respective *coins.*

A *nominal par* is a *conventional standard* established by commercial usage, and may be *above* or *below* the intrinsic par.

2. Exchange is seldom *stationary*, or *at par*, long at a time. It varies according to the circumstances of trade.

When the balance of trade is against a country, that is, when the exports are *less* than the imports, bills on the foreign country will be *above par*, for the reason that there will be a greater demand for them to pay the balance due abroad. On the other hand, when the balance of trade is in favor of a country, foreign bills will be *below par*, for the reason that fewer will be required.

3. The *course of exchange* is its *fluctuation above* and *below par.*

398. *Exchange* is of two kinds, *inland* or *domestic*, and *foreign. Domestic exchange* relates to monetary transactions between *different places* in the *same* country. *Foreign exchange* relates to monetary transactions between *different* countries.

QUEST.—395. What is Exchange? *Obs.* What is a bill of exchange? Who is the drawer? The drawee? The payee? The owner? 396. What is the par of exchange? Of how many kinds is it? What is the intrinsic par? The nominal par. *Obs.* What is the course of exchange? 397. Of how many kinds is exchange? What is domestic exchange? Foreign?

INLAND OR DOMESTIC EXCHANGE.

399. *Inland, or Domestic Exchange*, is reckoned at a certain per cent. on the legal currency of the country.

$5588. NEW YORK, March 27th, 1853.

1. At sight, pay to the order of Messrs. Brady & Co., Fifty-five hundred and eighty-eight dollars, value received, and charge the same to WILLIAM DWIGHT.

To Messrs. J. K. WALKER & Co.
Merchants, Mobile.

What is the value of this bill at 2 per cent. premium?

Suggestion.—Since exchange is 2 per cent. premium, the draft is worth the amount stated in it, and 2 per cent. besides. We therefore find 2 per cent. on $5288, and add it to the draft. Hence,

Operation.
$5288 draft.
.02 rate.
$105.76 premium.
$5393.76. *Ans.*

400. To calculate exchange on domestic bills or drafts,

Multiply the amount of the bill by the given per cent. expressed in decimals, and the result will be the exchange.

2. A merchant in Philadelphia wishes to remit $8278 to New Orleans: what will a draft cost at $2\frac{1}{2}$ per cent. discount?

3. A man in St. Louis wishes to buy a draft on New York for $1127.50: what will it cost him at $1\frac{1}{4}$ premium?

4. A merchant in New Orleans consigned 500 bales of cotton, each weighing 378 lbs., to his agent in Boston, which the agent sold at 8 cents a pound, and charged $2\frac{1}{2}$ per cent. commission; the merchant finally sold a draft on his agent for the sum due him, at 1 per cent. premium: how much did he receive for his cotton?

5. B of New York sent 876 pieces of cloth, containing $35\frac{1}{2}$ yards apiece, to D in Cincinnati, and agreed to pay him $2\frac{1}{2}$ per cent. commission, and 3 per cent. guarantee. B sold the cloth at $4.87\frac{1}{2}$ per yard, paid $69.50 freight, and $35.15 insurance. For what sum can B draw on D; and what will he receive for his cloth, if he sells the draft at $1\frac{1}{2}$ per cent. premium?

QUEST.—400. How do you calculate exchange on domestic bills or drafts?

FOREIGN EXCHANGE.

401. Exchange between the *United States* and *England* is commonly reckoned at a *certain per cent.* on a *nominal par*, which assumes £1 *Sterling* to be equal to $\$4\frac{4}{9}$, or $\$4.44\frac{4}{9}$. Thus, when exchange is 9 per cent. above par, $£1 = 1.09 \times \$\frac{40}{9}$; and $\$1 = £\frac{9}{40} \div 1.09$.

OBS. 1. The *nominal par* of exchange on England, is the same as the original legal value of a pound Sterling fixed by Congress in 1799 for estimating duties.

2. The reason that exchange on England is always so much *above* par, is chiefly owing to the fact that the *basis*, or *nominal par*, upon which it is calculated, is far *below* the *real value* of a pound Sterling. It assumes £1 to be worth only $\$4.44\frac{4}{9}$, whereas its intrinsic value is \$4.861, and its present legal value is \$4.84, which is 9 per cent. *more* than the *nominal par*. (Art. 147. Obs. 2.)

It is obvious, therefore, that 9 per cent. must be added to the *nominal par* of exchange, to make it equal to the *legal value* of a *pound Sterling*; and exchange reckoned on this basis ($\$4.44\frac{4}{9}$ to a pound), must be at a premium of 9 per cent. or over, before English funds are in reality at par, taking the *legal* or *intrinsic value* of a pound as the standard.

Exchange £310, 10s. NEW YORK, April 5th, 1853.

1. At thirty-days sight of this first of Exchange, (the second and third of the same date and tenor unpaid,) pay Charles Rowe, Esq., or order, Three hundred and ten pounds, ten shillings sterling, with or without farther advice.

JAMES CONRAD.

To Messrs. BARING & CO.
Brokers, London.

What is the value of this Bill of Exchange, at $8\frac{1}{2}$ per cent. premium?

Suggestion.—The given shillings (10s.) are equal to £.5, which we annex to the pounds. Now, as the rate of exchange is $8\frac{1}{2}$ per cent. above par, it is evident that $£1 = 1.085 \times \$\frac{40}{9}$, or \$4.822. We therefore multiply the given pounds by the nominal par value of £1 increased by the per cent. premium, and the result is the value of the bill. Hence,

Operation.

$$\begin{array}{r} £310,\ 10s = £310.5 \\ 4.822 \\ \hline 6210 \\ 6210 \\ 24840 \\ 12420 \\ \hline \textit{Ans.}\ \$1497.2310 \end{array}$$

QUEST.—401. How is exchange on England commonly reckoned? *Obs.* Why is exchange on England always so much above par? What does the nominal par assume to be the value of a pound Sterling? What is its intrinsic value? Its legal value? At what premium then must exchange be, before English funds are in reality at par, taking the legal value of £1 as the standard?

402. To find the value in Federal money of Bills of Exchange on England, at a given per cent. premium.

I. *Reduce the shillings, pence, and farthings to the decimal of a pound, and annex it to the given pounds.* (Art. 200.)

II. *Multiply this sum by the nominal par value of* £1, *increased by the given per cent. exchange, and the product will be the value of the bill in Federal money.*

Or, multiply the pounds and decimal of a pound by $\frac{40}{9}$ *; then find the per cent. exchange and add it to this product; the result will be the value of the bill in Federal money.*

OBS. In negotiating foreign bills, it is customary to draw *three* of the *same date* and *amount*, which are called the *First*, *Second* and *Third of Exchange;* and collectively, a *Set of Exchange.* These are sent by different ships or conveyances, and when the first is accepted or paid, the others are void.

2. What is the value of a bill of exchange on London for £1568, 5s., at 9 per cent. premium?

3. When exchange is $9\frac{1}{2}$ per cent. premium, what is the value of a bill for £2573, 15s. 8d.?

4. What is the Federal value of £3181, 7s. 6d., at 10 per cent. premium?

5. A traveler paid \$2730 for a bill of exchange on London, at 8 per cent. premium: what was the amount of the bill?

Suggestion.—Since the rate of exchange was 8 per cent., it is plain £1 $= 1.08 \times \$\frac{40}{9}$, or \$4.80. Now if \$4.80 will buy £1, \$2730 will buy as many pounds as \$4.80 are contained times in \$2730, which is £568.75. The decimal .75 is equal to 15s. Hence,

Operation.

$$\begin{array}{r} \$4.80)\$2730.00\ 00 \\ \hline £568.75 \\ 20 \\ \hline s.15.00 \end{array}$$

Ans. £568, 15s.

403. To find the value in Sterling money of any sum of Federal money, at a given per cent. exchange.

Divide the given sum by the nominal par value of £1 *in Federal money increased by the per cent. exchange, and the quotient will be the value in pounds and the decimal of a pound. Reduce the decimal to shillings, pence, and farthings, annex them to the pounds, and the result will be the answer.* (Art. 210.)

QUEST.—402. How do you find the value in Federal money of bills of exchange on England at a given per cent. premium?

6. What is the Sterling value of \$1560.75, at 9 per cent. exchange?

7. What is the Ster. value of \$4368.50, at 10 per cent. Ex.?

8. What is the Ster. value of \$5280.60, at $8\frac{1}{2}$ per cent. Ex.?

9. What is the Ster. value of \$56000, at $9\frac{1}{4}$ per ct. exchange?

10. A merchant consigned 2560 bbls. of flour to his agent in Liverpool, who sold it at £1, 8s. 6d. per barrel, and charged 2 per cent. commission: what is the net amount of the flour in Federal money, allowing 8 per cent exchange?

12. A merchant in N. Y. shipped 1000 bales of cotton weighing 360 lbs. per bale, to his agent in London, who sold it at $8\frac{1}{2}$d. per pound, paid $\frac{7}{8}$d. a pound for freight, and charged $2\frac{1}{2}$ per cent. commission. What was the net proceeds in Federal money, allowing $8\frac{1}{2}$ per cent. exchange?

404. In France, accounts are kept in *francs* and *centimes*. The *franc* is the *unit* money. It is subdivided into decimes or *tenths*, and centimes or *hundredths*, according to the *decimal notation*, in the same manner as Federal money.

10 *centimes* make 1 *decime*; 10 *decimes* make 1 *franc*.

OBS. In business matters, *decimes* are expressed in *centimes*. Thus, in quoting exchanges, instead of saying 5 francs, 2 decimes, and 3 centimes are worth a dollar, it is customary to say, exchange is 5 francs and 23 centimes per dollar; in circulars and prices current, it is written, Francs, 5.23 to 5.18, &c.

405. In negotiating Bills of Exchange on *France*, it is customary to make the *dollar* the *fixed sum* or *basis* of calculation, while the *French currency* is the *variable* sum.

406. To find the value in Federal money of Bills of Exchange on France.

Divide the given sum by the exchange value of \$1 expressed in francs and centimes, and the quotient will be the value of the bill in Federal money.

OBS. Formerly exchange on France was calculated at a certain per cent. on a nominal par of exchange, which assumed a dollar to be equal to $5\frac{1}{4}$ francs. This method, though still adhered to in books, is discarded by business men.

13. What is the value of a bill of exchange on Paris for 4515 francs, exchange at 5.25 francs per dollar? *Ans.* \$860.

14. What is the value of a bill of exchange on Havre for 10640 francs, exchange at 5.16 francs per dollar.

15. What is the value of 25265 f. exchange 5.20 f. per \$1?

16. What is the value of 68432 f. exchange $5.16\frac{1}{4}$ f. per \$1?

ARITHMETICAL PROGRESSION.

408. PROGRESSION *is continued proportion.* It is of two kinds, *arithmetical* and *geometrical.*

409. *Arithmetical progression* is a series of numbers, which increase or decrease by a common difference; as 3, 5, 7, 9, 11, 13, &c.; or 13, 11, 9, 7, 5, &c.

OBS. If the series increases, it is called *ascending*; if it decreases, *descending*. Arithmetical progression is sometimes called *equidifferent series*.

410. When *four* numbers are in arithmetical progression, *the sum of the extremes is equal to the sum of the means.*

Thus, in the series 3, 5, 7, 9, the sum $3+9=5+7$.

Again, if *three* numbers are in arithmetical progression, *the sum of the extremes is double the mean.*

Thus, in the series 9, 6, 3, the sum $9+3=6+6$.

411. In an *ascending* series, each succeeding term is found by adding the common difference to the preceding term. Thus, if the first term is 3, and the common difference 2, the series is 3, 5, 7, 9, 11, 13, &c.

In a *descending* series, each succeeding term is found by subtracting the common difference from the preceding term. Thus, if the first term is 15, and the common difference 2, the series is 15, 13, 11, 9, 7, &c.

412. In arithmetical progression, there are five parts to be considered, viz: *the first term, the last term, the number of terms, the common difference, and the sum of all the terms.* These parts have such a relation to each other, that if any *three* of them are given, the other *two* may be easily found.

413. To find the *sum* of all the terms, when the *extremes* and the *number* of terms are given.

Multiply half the sum of the extremes by the number of terms, and the product will be the sum of the given series.

QUEST.—408. What is progression? Of how many kinds is it? 409. What is arithmetical progression? *Obs.* If the series increases, what is it called? If it decreases, what? 410. When four numbers are in arithmetical progression, to what is the sum of the extremes equal? 411. In an ascending series, how is each successive term found? 413. When the extremes and number of terms are given, how find the sum of all the terms?

1. If the extremes of a series are 2 and 14, and the number of terms is 7, what is the sum of all the terms? *Ans.* 56.

2. If the extremes of a series are 36 and 4, and the number of terms 9, what is the sum of all the terms?

3. How many strokes would a clock which goes to 24 o'clock, strike in a day?

415. To find the *common difference*, when the *extremes* and the *number* of terms are given.

Divide the difference of the extremes by the number of terms less 1, *and the quotient will be the common difference required.*

4. If the extremes are 2 and 38, and the number of terms 13, what is the common difference? *Ans.* 3.

5. The extremes are 3 and 19, the number of terms 9: what is the com. dif. and the sum of the series?

416. To find the *number* of terms, when the *extremes* and *common difference* are given.

Divide the difference of the extremes by the common difference, and the quotient increased by 1 *will be the number of terms.*

6. If the extremes are 2 and 53, and the common difference 3, what is the number of terms? *Ans.* 18.

7. A man spent $3 the first holiday, $45 the last, and each day $3 more than on the preceding: how many holidays did he have, and how much did he spend?

417. When the *sum* of the series, the *number* of terms, and *one* of the extremes are given, to find the *other extreme.*

Divide twice the sum of the series by the number of terms, and from the quotient take the given extreme.

8. If one extreme is 10, the number of terms 6, and the sum of the series 42, what is the other extreme? *Ans.* 4.

9. If the sum of the series is 1924, one extreme 27, and the number of terms 26, what is the other extreme?

QUEST.—415. When the extremes and number of terms are given, how find the common difference? 416. When the extremes and common difference are given, how find the number of terms? 417. When the sum of the series, the number of terms, and one of the extremes are given, how find the other extreme?

GEOMETRICAL PROGRESSION.

418. *Geometrical progression is a series of numbers which increase by a common multiplier, or decrease by a common divisor;* as 2, 4, 8, 16, 32, &c.; or 32, 16, 8, 4, 2.

OBS. 1. If the series increases, it is called *ascending;* if it decreases, *descending.* The numbers which form the series, are called the *terms* of the progression. The *common multiplier*, or *divisor*, is called the *ratio.*

2. In an *ascending* series, each succeeding term is found by multiplying the preceding by the ratio. Thus, if the first term is 2, and the ratio 3, the series is 2, 6, 18, 54, &c.

In a *descending* series, each succeeding term is found by dividing the preceding by the ratio. If the first term is 54 and the ratio 3, the series is, 54, 18, 6, 2.

3. If the *first* term and *ratio* are the *same*, the progression is simply a series of powers; as 2; 2×2; $2\times2\times2$; $2\times2\times2\times2$, &c.

4. In *Geometrical* as well as in *Arithmetical progression*, there are five parts to be considered, viz: *the first term, the last term, the number of terms, the ratio,* and *the sum of all the terms.* These parts have such a relation to each other, that if any *three* of them are given, the other *two* may be easily found.

419. To find the *last* term, when the *first* term, the *ratio*, and the *number* of terms are given.

Multiply the first term into that power of the ratio whose index is 1 *less than the number of terms, and the product will be the last term required.*

OBS. The several *amounts* in compound interest, form a *geometrical series* of which the principal is the 1st term; the amount of $1 for 1 year the ratio, and the number of years + 1 the number of terms. Hence the *amount* of any sum at compound interest, may be found in the same way as the *last term* of a geometrical series.

1. If the first term of a geometrical progression is 4, and the ratio 3, what is the 5th term? *Ans.* 324.

2. If the first term is 48, and the ratio $\frac{1}{3}$, what is the 5th term.

3. The first term of a series is 3, the ratio 4: what is the 7th term?

QUEST.—418. What is geometrical progression? *Obs.* When the series increases, what is it called? When it decreases, what? What are the terms of the progression? In an ascending series, how is each succeeding term found? How in a descending series? If the first term and the ratio are the same, what is the series? 419. When the first term, the ratio, and the number of terms are given, how do you find the last term? *Obs.* How find the amount of any sum at compound interest by geometrical progression?

4. The first term of a series is 2, the ratio 2: what is the 23d term?

5. If a scholar receives 1 credit mark for the 1st example he solves, 2 for the 2d, 4 for the third, and so on, the number being doubled for each example, how many marks will he receive for the 12th?

6. What is the amount of $225, at 6 per cent. compound interest for 4 years?

7. What is the amount of $310.50, at 7 per cent. compound interest for 5 years?

420. To find the *sum* of the series, when the *ratio* and the *extremes* are given.

Multiply the greatest term by the ratio, from the product subtract the least term, and divide the remainder by the ratio less 1.

OBS. 1. When the *first* term, the *ratio*, and the *number* of terms are given, to find the sum of the series we must first find the last term, then proceed as above.

2. The *sum* of an *infinite series* whose terms decrease by a common divisor, may be found *by multiplying the greatest term into the ratio, and dividing the product by the ratio less* 1. The *least* term being infinitely small, is of no comparative value, and is therefore neglected.

8. If the extremes are 4 and 972, and the ratio 3, what is the sum of the series? *Ans.* 1456.

9. The first term is 3, the ratio 2, and the number of terms 9: what is the sum of the series?

10. The extremes of a series are 24 and 48144, and the ratio $1\frac{1}{4}$: what is the sum of the series?

11. What is the sum of the infinite series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c.

12. What is the sum of the infinite series $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c.

13. What is the sum of the series .1; .01; .001, &c.

14. A man bought a garden 3 rods wide and 4 rods long, and agreed to pay 1 cent for the 1st sq. rod, 4 cents for the 2d, 16 cents for the 3d, and so on, quadrupling each sq. rod: how much did his garden cost him?

15. A lady bought a dress containing 12 yards, agreeing to pay 1s. for the 1st yard, 2s. for the 2d, 4s. for the 3d, and so on: how much did her dress cost?

QUEST.—420. When the ratio and the extremes are given, how find the sum of the series? *Obs.* How find the sum of an infinite series, whose terms decrease by a common divisor?

MENSURATION.

421. MENSURATION is the art of measuring *magnitudes*.

OBS. The term *magnitude*, denotes that which has one or more of the three dimensions, *length*, *breadth*, and *thickness*.

422. A *line* is *length* without *breadth*.

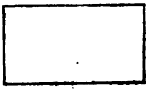
A *surface* is that which has *length* and *breadth*, without *height* or *thickness*.

423. In measuring *surfaces*, it is customary to assume a *square* as the *measuring unit*, whose side is a *linear unit* of the same name; as a square foot, a square rod, &c. (Thomson's Legendre's Geometry, IV. 4. Sch.)

Note.—For the demonstration of the following principles, see references.

424. A *square* is a figure which has *four equal* sides, and all its angles *right angles*. (Art. 153. Obs.)

Fig. 1.



A *parallelogram* is a quadrilateral figure whose *opposite* sides are *equal* and *parallel*. It may be *right-angled*, or *oblique angled*. (Figs. 1, 3.)

A *rectangle* is a *right-angled parallelogram*. (Fig. 1.)

425. To find the area of a *rectangle*, and a *square*.

Multiply the length by the breadth. (Leg. IV. 5.)

Note.—When the *area* and *one side* of a rectangle are given, *the other side* is found by dividing the *area* by the *given side*. (Art. 291. Note.)

1. How many acres are there in a field 120 rods long, and 90 rods wide?
Ans. $67\frac{1}{2}$ acres.

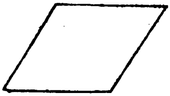
2. How many acres in a field 800 rods long, and 128 rods wide?

3. Find the area of a square field whose sides are 65 rods in length.

4. A man fenced off a rectangular field containing 3750 sq. rods, the length of which was 75 rods: what was its breadth?

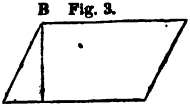
5. One side of a rectangular field is 1 mile in length, and it contains 160 acres: what is the length of the other side?

Fig. 2.



426. A *rhombus* is a quadrilateral figure whose *sides* are *equal* and its *opposite* sides *parallel*, but its *angles* not *right angles*. (Fig. 2.)

B Fig. 3.



A

A *rhomboid* is an *oblique angled parallelogram*. (Fig. 3.)

OBS. The term *altitude*, denotes perpendicular height; as A, B, Fig. 3.

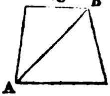
427. To find the area of a *rhombus*, and *rhomboid*.

Multiply the length by the altitude. (Leg. IV. 5.)

6. The length of a rhombus is 17 ft., and its perpendicular height 16 ft.: what is its area? *Ans.* 272 sq. ft.

7. What is the area of a rhomboid whose altitude is 25 rods, and its length 28.6 rods?

Fig. 4. B



A

428. A *trapezium* is a quadrilateral figure, having *only two* of its sides *parallel*. (Fig. 4.)

OBS. A *diagonal* is a straight line joining two opposite angles; as A B, Fig. 4.

429. To find the area of a *trapezium*. (Leg. IV. 7.)

Multiply half the sum of the parallel sides by the altitude.

8. The parallel sides of a trapezium are 15 feet and 21 feet, and its altitude 12 feet: what is its area? *Ans.* 216 feet.

9. Find the area of a trapezium whose parallel sides are 25 rods and 37 rods, and its altitude 18 rods.

430. To find the area of a *triangle*. (Art. 354. Leg. IV. 6.)

Multiply the base by half the altitude.

OBS. 1. The *base* of a triangle is found by dividing the area by *half* the *altitude*. (Art. 355.)

2. The *altitude* of a triangle is found by dividing the area by *half* the *base*.

10. What is the area of a triangle whose base is 45 feet, and its altitude 20 feet? *Ans.* 450 sq. ft.

11. What is the area of a triangle whose base is 156 feet, and its altitude 63 feet?

431. To find the area of a *triangle*, when *the three sides* are given.

From half the sum of the three sides subtract each side respectively; then multiply together half the sum and the three remainders, and extract the square root of the product.

12. What is the area of a triangle whose sides are respectively 10 feet, 12 feet, and 16 ft.? *Ans.* 59.92+ft.

13. What is the area of a triangle whose sides are each 12 yds.?

432. A *circle* is a plane figure bounded by a *curve line*, every part of which is *equally distant* from a certain point within, called the *centre*. (Fig. p. 147.)

The *circumference* is the curve line by which it is bounded.

The *diameter* is a straight line which passes through the centre, and is terminated on both sides by the circumference.

The *radius* or *semi-diameter* is a straight line drawn from the centre to the circumference.

OBS. From the definition of a circle, it follows that all the *radii* are *equal;* also, that all the *diameters* are *equal.*

433. To find the *circumference* of a circle, when the diameter is given. (Leg. V. 11. Sch.)

Multiply the given diameter by 3.14159.

14. What is the circumference of a circle whose diameter is 15 ft.? *Ans.* 47.12385 ft.

15. What is the circumference of a circle whose diameter is 100 rods?

434. To find the *diameter* of a circle, when the circumference is given.

Divide the given circumference by 3.14159.

OBS. The *diameter* of a circle may also be found by dividing the *area* by .7854, and extracting the *square root* of the quotient.

16. What is the diameter of a circle whose circumf. is 94.2477 ft.? *Ans.* 30 ft.

17. What is the diameter of a circle whose circumference is 628.318 yards?

435. To find the area of a *circle.* (Leg. V. 11.)

Multiply half the circumference by half the diameter; or, multiply the circumference by a fourth of the diameter.

OBS. The area of a circle may also be found by multiplying the *square* of its diameter by the decimal .7854.

18. What is the area of a circle whose diameter is 100 ft.? *Ans.* 7854 sq. ft.

19. What is the area of a circle whose diameter is 120 rods?

20. What is the area of a circle whose circumference is 160 yards?

21. Required the diameter of a circle containing 50.2656 square rods!

22. Required the diameter of a circle containing 201.0624 square feet?

436. A *solid* is a magnitude which has *length, breadth,* and *thickness.*

437. In measuring *solids,* it is customary to assume a *cube* as the *measuring unit,* whose sides are *squares* of the same name. The sides of a cubic inch, are square inches; of a cubic foot, are square feet, &c. (Art. 154. Obs.)

438. To find the *solidity* of bodies whose sides are *perpendicular* to each other. (Art. 164. Leg. VII. 11. Sch.)

Multiply the length, breadth, and thickness together.

OBS. When the *contents* of a solid and *two* of its *sides* are given, the *other side* is found by dividing the *contents* by the *product* of the two sides. (Art. 294.)

23. How many cubic feet are there in a stick of timber 60 feet long, $3\frac{1}{3}$ feet wide, and 2 feet thick? *Ans.* 400 cubic feet.

24. How many cubic feet in a wall 100 feet long, 15¼ feet high, and 3¼ feet thick?

25. A man wishes to make a cubical bin, which shall contain 19683 solid feet: what must be the length of its side?

26. If a stick of timber containing 400 cubic feet, is 60 feet long, and 3½ feet thick, what is its width?

439. A *prism* is a solid whose bases or ends are *similar*, *equal*, and *parallel*, and whose sides are *parallelograms*.

OBS. 1. When the bases of a prism are *triangles* it is called a *triangular* prism; when square, a *square* prism, &c.

2. All prisms whose ends are *parallelograms* are called *parallelopipedons*.

3. A *right* prism is one whose sides are perpendicular to its bases. All other prisms are called *oblique*.

440. To find the solidity of a *prism*.

Multiply the area of the base by the height. (Leg. VII. 12.)

OBS. 1. The *height* of a prism is the perpendicular distance between the planes of the bases. Hence, in a right prism, the height is equal to the length of one of the sides.

2. This rule is applicable to *all prisms*, triangular, quadrangular, &c.; also to *all parallelopipedons*,

27. What is the solidity of a prism whose base is 5 feet square, and its height 15 feet? *Ans.* 375 cubic feet.

28. What is the solidity of a triangular prism whose height is 20 feet, and the area of whose base is 460 square feet?

441. To find the *lateral surface* of a right prism.

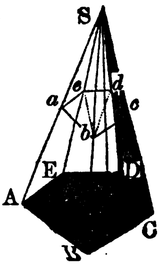
Multiply the length by the perimeter of the base.

OBS. If we add the areas of both ends to the lateral surface, the sum will be the whole surface of the prism.

29. Required the lateral surface of a triangular prism whose perimeter is 4½ inches, and its length 12 inches. *Ans.* 54 square inches.

30. Required the lateral surface of a quadrangular prism whose sides are each 2 feet, and its length 19 feet.

Fig. 5.



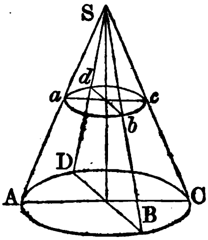
442. A *pyramid* is a solid whose base is a *triangular*, *square*, or *polygonal plane*, and whose sides terminate in a point called the *vertex*. (Fig. 5.)

A *frustum* of a *pyramid* is the part which remains after cutting off the top by a plane parallel to the base, as *a b c d e*, Fig. 5.

443. A *cone* is a solid whose base is a *circle*, and its sides terminate in a point called the *vertex*. (Fig. 6.)

A *frustum* of a *cone* is the part which remains after cutting off the top by a plane parallel to the base, as *a b c d*.

Fig. 6.



444. To find the solidity of a *pyramid*, or *cone*. (Leg. VII. 18. VIII. 4.)

Multiply the area of the base by $\frac{1}{3}$ of the altitude.

31. Required the solidity of a square pyramid, the side of whose base is 25 feet, and whose height is 60 feet. *Ans.* 12500 cubic feet.

32. Required the solidity of a cone, the diameter of whose base is 30 feet, and whose height is 90 feet.

445. To find the *lateral* or *convex surface* of a regular pyramid, or cone. (Leg. VII. 16. VIII. 3.)

Multiply the perimeter of the base by $\frac{1}{2}$ the slant-height.

OBS. The *slant-height* of a regular pyramid, is the distance from the vertex or summit to the middle of one of the sides of the base.

33. What is the lateral surface of a triangular pyramid whose slant-height is 10 feet, and each side 8 feet? *Ans.* 120 feet.

34. What is the convex surface of a cone, the perimeter of whose base is 560 yards, and slant-height 120 yards?

446. To find the solidity of a *frustum* of a pyramid, or cone. (Leg. VII. 19. Sch. VIII. 6.)

To the sum of the areas of the two ends, add the square root of the product of these areas; then multiply this sum by $\frac{1}{3}$ of the perpendicular height.

35. The areas of the ends of a frustum of a cone are 9 square feet, and 4 square feet, its height 15 feet: what is its solidity? *Ans.* 95 feet.

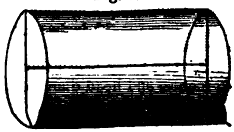
36. The two ends of a frustum of a pyramid are 4 feet and 3 feet square, its height 10 feet: what is its solidity?

447. The convex surface of a *frustum* of a pyramid, or cone, is found *by multiplying half the sum of the circumferences of the two ends by the slant-height.* (Leg. VII. 17.)

37. The circumferences of the two ends of a frustum of a pyramid are 12 feet and 8 ft., and its slant-height 7 ft.: what is its convex surface? *Ans.* 70 sq. ft.

448. A *cylinder* is a long circular body of uniform diameter, whose ends are *equal parallel circles*. (Fig. 7.)

Fig. 7.



449. To find the solidity of a *cylinder*. (Leg. VIII. 2.)

Multiply the area of the base by the height or length.

39. Required the solidity of a cylinder 6 feet in diameter, and 20 feet high. *Ans*. 565.488 cubic feet.

40. Required the solidity of a cylinder 30 feet in diameter, and 65 feet long.

450. To find the *convex surface* of a cylinder.

Multiply the circumference of the base by the height.

41. What is the convex surface of a cylinder 16 inches in circumference and 40 inches long? *Ans*. 640 square in.

42. What is the convex surface of a cylinder, whose diameter is 20 feet, and its height 65 feet?

451. A *sphere* or *globe* is a solid terminated by a curve surface, every part of which is *equally distant* from a certain point within called the *centre*.

452. To find the surface of a *sphere* or *globe*.

Multiply the circumference by the diameter. (Leg. VIII. 9.)

43. Required the surface of a globe 13 inches in diameter. *Ans*. 531 sq. in. nearly.

44. Required the surface of the earth, its diameter being 8000 miles.

453. To find the *solidity* of a sphere or globe.

Multiply the surface by $\frac{1}{6}$ of the diameter.

45. What is the solidity of a globe 12 in. in diameter? *Ans*. 904.77792 in.

46. What is the solidity of the earth, its diameter being 8000 miles.

GAUGING OF CASKS.

454. To find the contents or capacity of casks.

Multiply the square of the mean diameter into the length in inches; then this product multiplied into .0034 *will be the wine gallons, or multiplied into* .0028 *will be the beer gallons.*

OBS. The *mean* diameter of a cask is found by adding to the head diameter .7 of the difference between the head and bung diameters when the staves are *very much* curved; and by adding .5 when *very little* curved, and by adding .65 when they are of a *medium* curve.

47. How many wine gallons does a cask contain whose length is 35 inches, its bung diameter 30 inches, and its head diameter 26 inches, it being but little curved? *Ans*. 93.296 gallons.

48. How many beer gallons in a cask 54 inches long, whose bung diameter is 42 inches, and head diameter 36 inches, its staves being much curved?

MISCELLANEOUS EXAMPLES.

1. A farmer having sold $\frac{1}{2}$ and $\frac{1}{3}$ of his sheep, had 95 left: how many had he at first?

2. A man having \$15750, spent $\frac{1}{2}$ for a house, $\frac{1}{3}$ the remainder for a barn, and $\frac{1}{4}$ of the balance for a carriage: how much had he left?

3. What is the difference between $\frac{19}{20}$ of 275, and $\frac{3}{4}$ of 315.

4 What number is that, $\frac{1}{2}$ of which exceeds $\frac{1}{4}$ by 387?

5. What number is that, $\frac{2}{3}$ and $\frac{3}{5}$ of which make 255.

6. What number must be added to $1375\frac{1}{4}$ to make $81193\frac{1}{9}$?

7. What must be taken from $1137\frac{1}{5}$ that $793\frac{1}{4}$ may be left?

8. What must be added to $217\frac{1}{2}$ that the sum may be $17\frac{1}{4}$ times $19\frac{1}{2}$?

9. What number multiplied by $45\frac{1}{5}$, will produce $288\frac{1}{4}$?

10. What number divided by $37\frac{1}{2}$, will give $193\frac{1}{4}$ for the quotient?

11. Bought $\frac{7}{8}$ of a ship, and sold $\frac{4}{5}$ of it: how much was left?

12. A broker negotiated a bill of exchange of \$10360, at $1\frac{1}{2}$ per cent.: what was his commission?

13. At what per cent. must \$6376 be loaned, to draw \$135.49 interest in 3 months?

14. What sum must be loaned at 7 per cent., to gain \$455 interest in 3 months?

15. What sum must be loaned at $6\frac{1}{2}$ per cent. interest, to gain \$650 semi-annually?

16. In what time will \$8284 gain \$365, at 6 per cent. interest?

17. At 7 per cent. int., in what time will \$857.25 double itself?

18. At what per cent. interest will \$500 double itself in 10 years?

19. What is the present worth of \$1365, payable in 6 months, when money is worth 7 per cent. per annum?

20. At 6 per cent. discount, what is the present worth of \$1623.28, due in 1 year?

21. What is the bank discount on a note of \$730, payable in 4 months, at $6\frac{1}{2}$ per cent.?

22. What is the bank discount on a note of \$1575, payable in 60 days, at 7 per cent.?

23. What will 35 shares of railroad stock cost, at $10\frac{1}{2}$ per cent. advance? *Ans.* \$3867.50.

24. What cost 63 shares of bank stock, at $3\frac{1}{2}$ per cent. discount?

25. What premium must a man pay annually for insuring \$8500 on his store and goods, at $1\frac{1}{4}$ per cent.?

26. If I obtain insurance on goods, worth \$16265, at $2\frac{1}{2}$ per cent., and the goods are lost, how much shall I lose?

27. Paid \$78.75 insurance annually, which was $1\frac{1}{2}$ per cent. on the sum insured: what was the amount of my policy?

28. Paid \$24.54 insurance on \$6544: what was the per cent.?

29. Received \$862.50 dividend on \$17250 stock: what was the per cent.?

30. Bought a farm for \$5640, spent \$258 upon it, then sold it for 15 per cent. profit: how much did it sell for?

31. Bought goods for \$4390, and sold them on 6 mos. at $22\frac{1}{2}$ per cent. above cost: what was the profit, allowing 7 per cent. interest?

32. Bought 15000 gals. oil for \$8500: allowing $1\frac{1}{2}$ per cent. leakage, how must it be sold to gain 15 per cent.

33. If I buy 1675 yards of flannel for \$368.50, how must I retail it per yard to gain 25 per cent.? *Ans.* $27\frac{1}{2}$ cts.

34. A grocer bought 2500 lbs. of coffee for \$250, and sold it at 6 per cent. loss: what did he get per pound?

35. A merchant bought 1824 yds. of cloth, at \$2.50 per yard and retailed it at \$3 per yard: what per cent. was his profit, and how much did he make?

36. A shop-keeper bought 100 pieces of lace, for \$250, and sold them for \$375: what per cent. did he make, and how much?

37. If a grocer buys 3680 lbs. of cheese, at $4\frac{1}{2}$ cts. per. lb., and sells it at $6\frac{1}{2}$ cts., what per cent. and how much is his profit?

38. What is the ad valorem duty, at $33\frac{1}{3}$ per cent., on a quantity of cloths which cost \$10436?

39. What is the ad valorem duty, at $15\frac{1}{4}$ per cent., on a cargo of tea, invoiced at \$35856?

40. At $37\frac{1}{2}$ per cent., what is the duty on a quantity of silks which cost \$23265?

41. The sum of two numbers is 856, and their difference is 75: what are the numbers?

42. The sum of two numbers is 5643, and their difference is 125: what are the numbers?

43. The difference of two numbers is 63, and the smaller number is 365: what is the greater number?

44. The product of two numbers is 3750, and one of the numbers is 75: what is the other?

45. What number is that, $\frac{5}{9}$ of which is 34 times 45?

46. What number is that, $\frac{3}{5}$ of $\frac{5}{8}$ of which is 63 times $\frac{3}{4}$ of 120?

47. How long will it take a person to count a billion, if he counts 50 a minute, and works 6 hours per day, for 5 days a week, and 52 weeks a year?

48. How many dollars, each weighing $412\frac{1}{2}$ grains, can be made from 7 lbs. 1 oz. 18 pwt. 18 grs. of silver?

49. How many pounds of silk will it take to spin a thread which will reach round the earth, allowing its circumference to be 25000 miles, and $2\frac{1}{2}$ oz. to make 160 rods of thread?

50. How many times will the hind wheel of a carriage, 7 ft. 6 in. in circumference, turn round in 7 miles, 1 furlong, 30 rods?

51. How many times will the fore wheel of a carriage, 5 ft. $7\frac{1}{2}$ in, in circumference, turn round in the same distance?

52. How many oranges, at $\frac{3}{8}$ of $8\frac{1}{3}$ cts. apiece, will $\$217\frac{3}{5}$ buy?

53. Paid $\$251\frac{1}{4}$ for $\frac{2}{10}$ of $245\frac{1}{4}$ acres of land: what was that per acre?

54. What number is that, $\frac{1}{4}$ of which exceeds $\frac{1}{5}$ by 428?

55. What number is that, $\frac{1}{3}$ and $\frac{1}{4}$ of which is 510?

56. A father gave his eldest son twice as much as the second, the second three times as much as the third, who had $1573: how much did he give to all?

57. A man having 4 children, gave twice as much to the 4th as to the 3d; twice as much to the 2d as to the 4th; and to the 1st twice as much as to the 2d, which was $7860: what did he give to all?

58. A man gave $\frac{1}{3}$ of his estate to his eldest daughter; $\frac{1}{2}$ the remainder to the 2d; and $\frac{1}{4}$ of the remainder to the 3d, who received $3560: what was his estate?

59. Bought a piece of land $45\frac{3}{4}$ r. long, and $28\frac{4}{5}$ r. wide, at the rate of $75 for $2\frac{1}{4}$ acres: what did it come to?

60. What part of $\frac{3}{4}$ of £6, 12s. is $\frac{1}{2}$ of 13 shillings and 6 pence?

61. What is $45\frac{1}{2}$ times $\frac{3}{8}$ of £276, 15 shillings and 6 pence?

62. What cost 735 barrels of apples, each containing $2\frac{1}{2}$ bu., at 1s. 8d. ster. per bushel?

63. What cost 3 pieces of cloth, each containing 27 ells French, at 3s. 4d. per yard? *Ans.* £20, 5s.

64. What cost 248 pair of boots, at £.625 sterling a pair?

65. If 56 lbs. of butter cost $15.60, what will .078 ton cost?

66. If 48 ells Flemish of cloth cost $480, what will 125 ells English cost?

67. If 96 horses eat 192 tons of hay in a winter, how many tons will 150 horses eat in 6 winters?

68. If .1 cwt. of sugar cost $9\frac{3}{8}$s., what will $\frac{3}{25}$ ton cost?

69. If $\frac{1}{4}$ cwt. of veal cost $\$1\frac{3}{4}$, how much will 872 lbs. cost?

70. If .5 cwt. of ginger cost $\$7\frac{1}{7}$, how much will $\frac{23}{100}$ ton cost?

71. What cost 260 loads of wood, each containing 96 cu. ft., if 45 cords cost $\$87\frac{3}{5}$?

72. A man sold a sheep for £$1\frac{1}{4}$, a calf for £$\frac{4}{5}$, $\frac{1}{4}$ s. and a pig for $\frac{5}{8}$s. $\frac{3}{4}$d.: what did he get for all?

73. A goldsmith melted up $\frac{3}{4}$ lb. $10\frac{1}{2}$ pwts. of gold, at one time, and $3\frac{1}{2}$ oz. 10 grs. at another: how much did he melt in all?

74. A man having $2\frac{1}{4}$ oz. of silver, sold $6\frac{3}{4}$ pwts.: how much had he left?

75. A man owing £$\frac{4}{5}$, $2\frac{1}{2}$s., paid $7\frac{1}{4}$s. $2\frac{1}{2}$d.: what does he still owe?

76. How many tiles 9 in. square, will cover a hall 63 ft. by 8 ft.?

77. A man sold a house for $9265, and thereby lost 10 per cent.: what per cent. would he have gained, had he sold it for $12000?

78. A man being asked how much money he had in bank, answered $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ of it was $4260: how much had he?

79. How many acres in a farm $157\frac{1}{2}$ rods and $4\frac{1}{2}$ feet square?

80. If $25\frac{3}{5}$ Eng. ells of silk cost £10, 15s., what will $85\frac{1}{3}$ yds. cost?

81. A man having a garden $35\frac{3}{5}$ r. long, and $20\frac{3}{4}$ r. wide, took $6\frac{1}{2}$ ft. from each side for a walk: how much land was left?

82. If $15\frac{1}{2}$ yards of silk cost $$18\frac{3}{4}$, what will $56\frac{3}{4}$ yards cost?

83. A grocer used a false weight of $13\frac{1}{2}$ oz, for a pound: what was the amount of his fraud in weighing 500 pounds?

84. If $\frac{4}{7}$ bbl. of apples costs $$\frac{4}{5}$, how much will $85\frac{7}{8}$ bbls. cost?

85. If $\frac{7}{13}$ lb. of lard cost $\frac{14}{17}$s. how much will $\frac{29}{30}$ ton cost?

86. If $\frac{3}{16}$ of a ton of hay costs £$\frac{7}{8}$, what will $\frac{19}{20}$ of a cwt. cost?

87. How much will $\frac{5}{16}$ of a drum of figs come to, at the rate of $\frac{4}{5}$ of a dollar for $\frac{3}{4}$ of a drum?

88. Bought $48\frac{1}{2}$ lbs. of tea for $$27\frac{5}{8}$: how much can be bought for $125?

89. How many rolls of paper 9 yds. long and 27 in. wide, will be required to cover the walls of a room $18\frac{1}{2}$ ft. long, $16\frac{1}{4}$ ft. wide, and $12\frac{1}{5}$ ft. high?

90. A man gave $\frac{1}{4}$ of his property to his son, and $\frac{1}{6}$ to his daughter, which was $3250 less than her brother's portion: what was the amount of his property?

91. A man lost $\frac{3}{4}$ of his money by gambling, and afterwards receiving $56, he had $1644: what had he at first?

92. If a man spends $\frac{5}{12}$ of his time in labor, $\frac{1}{3}$ in sleep, $\frac{1}{16}$ in eating, and $1\frac{1}{2}$ hour each day in reading, how much time will be left?

93. If a cane 3 feet in length, cast a shadow 5 feet long, how high is a steeple whose shadow is 175 feet?

94. Bought 16 hhds. molasses for 75 firkins of butter, worth $16\frac{1}{2}$ cents a pound: what was the molasses per gallon?

95. A can dig a cellar in 45 days, B in 63 days: how long will it take both together to dig it?

96. If a cubic foot of water weighs 1000 oz., how many tons are there in a reservoir which covers 5 acres, the water being $8\frac{1}{2}$ ft. deep ?

97. If I pay \$8400 for $\frac{4}{5}$ of a ship, for what must I sell $\frac{11}{16}$ of it, that I may make 16 per cent. on it ?

98. A farmer sold 176 sheep, which was $\frac{2}{3}$ of $\frac{3}{5}$ of all he had ; the remainder he divided equally between his two sons : how many did each receive ?

99. A garrison having been besieged 108 days, found that $\frac{1}{10}$ more than half of their provisions were consumed ; how much longer would they last ?

100. A garrison of 1520 men have 416955 lbs. of flour : how long will it last them, allowing each man $\frac{3}{4}$ lb. per day ?

101. How long will $119\frac{3}{7}$ hhds. of water last a ship's company of 80 men, allowing each man $\frac{2}{5}$ gal. per day ?

103. A can saw a cord of wood in 4 hours, B in 5 h. and C in 6 h. : how long will it take all three to saw a cord ?

104. If a pole 1 rod long cast a shadow $22\frac{1}{2}$ ft., what is the height of a tree whose shadow is $216\frac{1}{2}$ feet ?

105. If 6 men can build a wall 30 feet long, 6 feet high, and 3 feet thick, in 15 days, when the days are 12 hours long, how many days will it take 30 men to build a wall 300 feet long, 8 feet high, and 6 feet thick, working 8 hours a day ?

106. A is $37\frac{1}{2}$ years old, and $\frac{3}{5}$ of his age is $\frac{3}{4}$ the age of B: how old is B ?

107. C has 465 sheep, and $\frac{4}{15}$ of his number is equal $\frac{31}{43}$ of D's : how many sheep has D ?

108. Twenty-one forty-fifths of 405 is $\frac{22}{27}$ of how many times 20 ?

109. Thirty-five sixty-fourths of 768 is $\frac{21}{84}$ of how many sixteenths of 575 ?

110. A man wishes to buy a square farm which contains $318\frac{1}{40}$ acres : what is the length of its side ?

111. The length of a rectangular farm containing 196 acres is twice its breadth : what are its length and breadth ?

112. How many acres in a rectangular field, the length of which is 128 r. and its diagonal 160 rods ?

113. What is the area of a triangle whose hypothenuse is 50 yds., and its perpendicular 30 yards ?

114. What is the area of a triangle whose hypothenuse is 100 rods, and its base 60 rods ?

115. What o'clock is it when $\frac{1}{10}$ of the time to midnight is equal to $\frac{1}{3}$ of the time since noon ?

116. Required the mean proportional between 121 and 5.76.

117. If $\frac{1}{4}$ of the time past midnight is equal to $\frac{2}{5}$ of the time to noon, what o'clock is it?

118. Required the mean proportional between $\frac{25}{36}$ and $\frac{81}{144}$.

119. A regiment containing 6912 soldiers, was so arranged that the number in rank was triple that in file: how many were there in rank and file?

120. A man and a boy together can reap an acre of wheat in 8 hours; the man can reap it in 11 h.: how long will it take the boy to reap it?

121. A cistern has two pipes; one will fill it in 48 minutes; the other will empty it in 72 minutes: how long will it take to fill the cistern when both are running?

122. A, B, and C together have 1440 acres of land; A has twice, and C three times as much as B: how many acres has each?

123. A man agreed to work for 9s. per day, and to forfeit 7s. 6d. for every day he was idle: at the end of 90 days he received £14, 2s. How many days did he work?

124. A man being asked the price of his carriage, said that his horses and carriage cost $968, and his carriage cost 3 times as much as his horses: what was the cost of his carriage?

125. How much will it cost to carpet a parlor 18 ft. square with carpeting $\frac{3}{4}$ yd. wide, which is worth $1.50 per yard?

126. A, B, and C joined in a speculation; A put in $500, B $700, and C put in the balance; they gained $1200, of which C received $480 for his share: how much did A and B receive, and how much did C put in?

127. A, B, and C gain $3600, of which A receives $6, as often as B receives $10, and C $14: what was the share of each?

128. The hour and minute hand of a clock are exactly together at noon: when will they next be together?

129. A farmer having lost $\frac{1}{4}$ of his sheep, and sold $\frac{1}{4}$ of them, had 500 left: how many had he at first?

130. If $\frac{1}{5}$ of a post stands in the mud, $\frac{1}{4}$ in the water, and 10 feet above the water, what is the length of the post?

131. Two persons start from the same place, one goes south 4 miles per hour, the other west 5 miles per hour: how far apart are they in 9 hours?

132. A messenger traveling 8 miles an hour, was sent to Mexico with despatches for the army; after he had gone 51 miles, another was sent with countermanding orders, who could go 19 miles as quick as the former could go 16; how long will it take the latter to overtake the former; and how far must he travel?

ANSWERS.

ADDITION.—Arts. 21–29.

Ex.	Ans.
Art. 21.	
1, 2.	Given.
3.	$786.
4.	8689.
5.	57757.
6.	651465.
7.	8651761.
8.	998943483.
9.	988.
10.	7673.
11.	88765.
12.	85879944.
15.	1460.
16.	23770.
17.	161524.
18.	131570.
19.	1999990.
20.	1913.
21.	16840.
22.	220083.
23.	100003.
24.	134735.
25.	104022.
Art. 29.	
1.	$64.
2.	966 lbs.
3.	96 yrs.
4.	$976.
5.	$109.
6.	106 s.
7.	$550.
8.	$3347.
9.	$860.
10.	$2378.
11.	$2164.
12.	1893 sch.
13.	$1816.
14.	143 m.
15.	582 a.
16.	$690.
17.	238 cts.
18.	$1401.
19.	$2788.
20.	$264.
21.	$846.
22.	754 sh. 365 l. 1119 b.
23.	$6821.
24.	$2824.
25.	$4900.
26.	$1444.
27.	503 ts.
28.	476 m.
29.	73 yrs.
30.	$1648.
31.	$34950.
32.	$33700.
33.	$3147.
34.	$512.
35.	611. bu. $513.
36.	$627.
37.	630 lbs.
38.	$3789.
39.	$1125.
40.	$2385 r. $554 g.
41.	$1582.
42.	$1828.
43.	525 m.
44.	$4930.
45.	2234822.
46.	4604345.
47.	5067843.
48.	4984097.
49.	178346.
50.	17069453.
51.	31231.
52.	96633.
53.	1052238.
54.	62497360.
55.	74482084-58.
56.	$3068 f. $6136 for all.
57.	$16646.
58.	$37650.
59.	A.D. 1727.
60.	$8475.
61.	———
62.	365 days.
63.	$312.
64.	156 strokes
65.	$49245.
66.	28256972.
67.	850000000.
68.	103131.
69.	120524.
70.	93649.
71.	1062086.
72.	1012939.
73.	1065910.
74.	9716789.
75.	8596013.
76.	36481665.

SUBTRACTION.—Arts. 34–40.

Ex.	Ans.
Art. 34.	
1, 2.	Given.
3.	$232.
4.	413.
5.	358 dolls.
6.	418 feet.
7.	3382 lbs.
8.	3231 qts.
9.	32352.
10.	613184.
11.	581141.
12.	3151721.
15.	248.
16.	54182.
17.	124907.
18.	66104149.
19.	Given.
20.	6121.
21.	2754087.
22.	932417.
23.	6834501.
24.	8960895.
25.	31090814.
Art. 40.	
1.	113 yds.
2.	$221.
3.	189 g.
4.	1003 bu.
5.	$3791.
6.	$1420.
7.	$382.
8.	$1079.
9.	374 bu.
10.	$1989.
11.	$479.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
12.	———	29.	$19.	46.	12520 bu.	62.	1487.
13.	$1291.	30.	146 ts.	47.	$1491.	63.	2445.
14.	53 m.	31.	$1090.	48.	$9699.	64.	1148.
15.	93 m.	32.	$3838.	49.	$21422.	65.	2336.
16.	———	33.	$5250.	50.	$8000.	66.	378.
17.	1706.	34.	$323.	51.	22225.	67.	926.
18.	67 yrs.	35.	1983 a.	52.	16014416.	68.	4794.
19.	———	36.	565 men.	53.	184815000-	69.	1187.
20.	$72320.	37.	$773.		000.	70.	16546.
21.	427721.	38.	$18053.	54.	5 times.	71.	6209408.
22.	214412.	39.	$154.	55.	627067.	72.	61484.
23.	1056109.	40.	$5491.	56.	44 yrs.	73.	26973.
24.	194099.	41.	$6749.	57.	77 yrs.	74.	34060.
25.	11763528.	42.	$1695.	58.	198 rem.	75.	$27096.
26.	100 a.	43.	$2752.	59.	———	76.	1912 B's.
27.	$986.	44.	$1913.	60.	———		4482 C's.
28.	$22.	45.	$332.	61.	8 yrs.	77.	5986.

MULTIPLICATION.—ARTS. 49–54.

ART. 49.			ART. 54.	
1, 2. Given.	18. 608240.		1. $2790.	17. 4935 s.
3. 936.	19. 76342.		2. $2552.	18. 8071 bu.
4. 484.	20. 41479110.		3. $36720.	19. 2944 qts.
5. 960 r.	21–23. Given.		4. 4056 s.	20. $22224.
6. 880 m.	24. 8915.		5. 5301 s.	21. $1482.
7. 9096.	25. 19200.		6. $37152.	22. $8991.
8. 88480.	26. 23074.		7. $42255.	23. $10584.
9. 505505.	27. 88832.		8. 11370 s.	24. $4096.
10. 18073812.	28. 175252.		9. $410400.	25. 35720 d.
11. $4698.	29. 268435125.		10. $420.	26. 16425 d.
12. $664.	30. 507166416.		11. $414.	27. 90625 lbs.
13. 1917 s.	31. 55837500-		12. $945.	28. 176175 lbs.
14. $624.	20.		13. $1792.	29. 78475 m.
15. $6153.	32. 56277323-		14. $1664.	30. $77970.
16. 5200 s.	52.		15. $2522.	31. 10101255.
17. 40030.	33. 787713314-		16. $2090.	32. 154725876.
	068.			33. 287805313.

34. 6065742688.	40. 8823403762605-	45. 351039462230.
35. 49238237975.	25.	46. 2172 s.
36. 2859019905.	41. 2015028.	47. $4632 gain.
37. 40456146766.	42. 8496120.	48. 321 s. gain.
38. 83339299596.	43. 404444040.	49. 8288 d.
39. 4223058409402.	44. 6342737821.	50. 1024 m.

CONTRACTIONS IN MULTIPLICATION.—ARTS. 57–62.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
	ART. 57.	17.	8906345700000-00.	33.	25800000000.
2.	$2295.	18.	9460305068000-00.	34.	4059360000.
3.	$888.	19.	7831206507300-00.	35.	14760000000.
4.	$684.	23.	3840000.	36.	6204000000.
5.	$4950.	24.	10940000.	37.	1672650000000.
6.	1872 s.	25.	2075994000.	38.	1075635900000.
7.	8610 m.	26.	390677500000.	41.	7767837.
8.	25760 bu.	27.	372000.	42.	34572393.
9.	16128 s.	28.	11840000.	43.	677789532.
10.	$91080.	29.	373520000.	44.	733832433.
11.	280 lbs.	30.	3603200000.	45.	839832327.
12.	35200 p.	31.	55447000000.	46.	78937735437.
13.	476000.	32.	37800000000.	47.	874319056722.
14.	534860000.			48.	960311096793.
15.	1204670800000.			49.	8759995239996.
16.	26900785000000			50.	9999989000001.

SHORT DIVISION.—ARTS. 67–73.

ART. 67.	15. 11111.	30. 48 lbs.	42. 2050.	53. 666666$\frac{6}{8}$
3. 7.	16. 1243143	31. 7615.	43. 5070.	54. 116717-$3\frac{3}{6}$.
4. 6.	20. 51.	32. 6573.	44. 5021.	55. 70340-$1\frac{6}{10}$.
5. 6.	21. 312.	33. 16334.	45. 80405.	56. 73614-$5\frac{8}{11}$.
6. 9.	22. 8231.	34. 3144.	46. 187$\frac{1}{4}$ or.	57. 69222-$7\frac{10}{12}$.
8. 123 sh.	23. 711.	35. 107 bls.	47. 62 pair.	58. 80028-$3\frac{9}{12}$.
9. 124 a.	24. 7111.	36. 6010.	48. 151$\frac{1}{5}$ b.	
10. 122 tms.	25. 811.	37. 7000.	49. 52 y.	
12. 321 yds.	26. 8111.	38. 5100.	50. 162$\frac{1}{6}$ a.	
13. 21312.	27. 911.	39. 71000.	51. 781150$\frac{3}{7}$	
14. 12212.	28. 247 a.	41. $107.	52. 584038.	

LONG DIVISION.—ARTS. 74–77.*a*.

ART. 74.	19. 1080$\frac{29}{63}$.	8. 11 t.	19. 72, & 12 r.
5. 127208$\frac{3}{8}$.	20. 901$\frac{118}{156}$.	9. 11 c.	20. 588, & 8 r.
6. 1342314.		10. 120 m.	21. 24, & 61 r.
7. 326561.	ART. 77.*a*.	11. 200 m.	22. 8, & 18 r.
8. 336568.	1. 24 h.	12. 250 m.	23. 227, & 10 r.
9. 6437612.	2. 36 yds.	13. 13$\frac{4}{32}$ mos.	24. 269, & 3 r.
10. 72225723.	3. 43 c.	14. 20 hhds.	25. 2813, & 27 r.
14. 245.	4. 108 t.	15. 11 m.	26. 34, & 34 r.
15. 1826$\frac{20}{45}$.	5. 18 m.	16. 14, & 4 r.	27. 173, & 25 r.
16. 1212$\frac{25}{67}$.	6. 20 d.	17. 42.	28. 158, & 40 r.
17. 1227$\frac{48}{81}$.	7. 10$\frac{5}{21}$ m.	18. 39, & 7 r.	29. 388, & 55 r.

SUBTRACTION OF FRACTIONS.—ART. 129.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
4. [illegible]	8. $\frac{4079}{5256}$.	12. $\frac{491}{1680}$.	17. $7\frac{4}{5}$.	22. $\frac{1}{3}$.
5. $\frac{1}{4}$.	9. $\frac{8}{35}$.	13. $\frac{1}{18}$.	18. $17\frac{23}{28}$.	23. $\frac{127}{288}$.
6. [illegible]	10. $\frac{16}{165}$.	15. $5\frac{3}{8}$.	20. $5\frac{1}{3}$.	24. 2.
7. $\frac{1}{8}$.	11. $\frac{8}{77}$	16. $5\frac{13}{14}$.	21. $39\frac{4}{5}$.	25. 0.

26. $\$15\frac{7}{8}$.	28. $38\frac{41}{80}$ yds.	30. $1579\frac{181}{1449}$ b.	32. $132\frac{47}{63}$ rods.
27. $116\frac{7}{16}$ a.	29. $409\frac{109}{216}$ lbs.	31. $124\frac{7}{12}$ m.	33. $310\frac{13381}{17600}$ t.

MULTIPLICATION OF FRACTIONS.—ARTS. 132-7.

3. 4.	11. $1\frac{1}{4}$.	21. 258.	31. 54.	46. $\frac{1}{7}$.
4. $10\frac{1}{2}$.	12. 3.	22. 389.	32. $83\frac{7}{16}$.	47. $\frac{1}{6}$.
5. 6.	13. 6.	25. 9.	35. 657.	48. $\frac{4}{21}$.
6. 6.	14. 18.	26. $16\frac{1}{5}$.	36. $916\frac{2}{3}$.	49. $\frac{3}{98}$.
7. 4.	15. $28\frac{4}{5}$.	27. 25.	39. $\frac{4}{21}$.	51. $73\frac{1}{8}$.
8. 3.	16. $86\frac{2}{5}$.	28. $32\frac{4}{5}$.	40. $\frac{35}{72}$.	52. $501\frac{11}{15}$.
9. $7\frac{1}{2}$.	17. $17\frac{1}{15}$.	29. 35.	41. $\frac{5}{24}$.	53. $1999\frac{37}{56}$.
10. $13\frac{1}{5}$.	18. $32\frac{4}{15}$.	30. $47\frac{3}{11}$.	42. $\frac{13}{28}$.	54. $4393\frac{7}{20}$.

ART. 137.*a.*	14. 235 p.	28. $1237\frac{1}{2}$ c.	42. $652\frac{2}{3}$. s.	56. 80.
1. \$4.	15. $\$16\frac{2}{3}$.	29. $781\frac{1}{4}$ cts.	43. $\$65\frac{13}{16}$	57. $5\frac{1465}{2304}$.
2. $6\frac{3}{10}$ cwt.	16. $33\frac{1}{3}$ cts.	30. $800\frac{5}{8}$ cts.	44. $\$615\frac{5}{32}$.	58. $72\frac{47}{64}$.
3. $\$9\frac{3}{16}$.	17. $61\frac{7}{8}$ s.	31. $243\frac{3}{4}$ cts.	45. $743\frac{3}{4}$ in.	59. $74\frac{131}{280}$.
4. $8\frac{16}{21}$ bbls.	18. 360 cts.	32. $\$3\frac{1}{2}$.	46. $1221\frac{117}{140}$	60. $4603\frac{341}{378}$
5. $11\frac{5}{32}$ c.	19. $292\frac{1}{2}$ cts.	33. $\$10\frac{19}{24}$.	47. 2310.	61. $868\frac{1}{2}$.
6. $27\frac{7}{36}$ a.	20. $216\frac{3}{4}$ cts.	34. $28\frac{1}{8}$ s.	48. $8\frac{67}{75}$.	62. $3794\frac{2}{5}$.
7. $44\frac{2}{5}$ s.	21. \$56.	35. $273\frac{3}{4}$ cts.	49. $61\frac{37}{68}$.	63. $\frac{170}{243}$.
8. $61\frac{1}{4}$ s.	22. $157\frac{1}{2}$ cts.	36. $61\frac{49}{64}$ s.	50. $1490\frac{5}{32}$.	64. $\frac{147}{967}$.
9. $\$96\frac{1}{4}$.	23. $\$16\frac{1}{2}$.	37. $\$41\frac{33}{40}$.	51. $16\frac{40}{57}$.	65. $156\frac{2031}{2068}$
10. $\$7\frac{1}{45}$.	24. $\$4\frac{3}{8}$.	38. $621\frac{9}{16}$ c.	52. $\frac{2}{9}$.	66. $109\frac{13}{128}$.
11. $\$12\frac{21}{32}$.	25. $\$3\frac{2}{3}$.	39. $73\frac{15}{16}$. d.	53. $32\frac{2}{5}$.	67. $\frac{522}{1225}$.
12. 136 cts.	26. $\$\frac{27}{40}$.	40. $66\frac{1}{4}$ s.	54. $72\frac{23}{48}$.	68. $6\frac{69}{160}$.
13. $112\frac{1}{2}$ cts.	27. $\$\frac{15}{28}$.	41. $391\frac{3}{8}$ ct.	55. $197\frac{5}{9}$.	

DIVISION OF FRACTIONS.—ARTS. 138-141.

4. $\frac{2}{15}$.	10. $\frac{5}{221}$.	19. $\frac{280}{361}$.	26. $3\frac{33}{136}$.	33. $75\frac{1}{5}$.
5. $\frac{2}{19}$.	11. $\frac{5}{238}$.	20-21. g.	27. $7\frac{31}{207}$.	34. $212\frac{1}{7}$.
6. $\frac{5}{21}$.	12-15. g.	22. $5\frac{1}{7}$.	28. $6\frac{18}{95}$.	35. $323\frac{3}{11}$.
7. $\frac{17}{132}$.	16. $22\frac{1}{2}$.	23. $1\frac{19}{33}$.	29. $3\frac{23}{26}$.	36. $3\frac{622}{901}$.
8. $\frac{1}{8}$.	17. $\frac{3}{7}$.	24. $\frac{75}{112}$.	30-31. g.	37. $2\frac{1435}{2176}$
9. $\frac{9}{25}$.	18. $1\frac{371}{620}$.	25. $\frac{35}{48}$.	32. $87\frac{1}{2}$.	

EXAMPLES FOR PRACTICE.—ART. 141.*a*.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
1.	130 bu.	12.	$8\frac{27}{116}$ bbls.	23.	$\frac{319}{475}$.	34.	$\frac{3}{5}$.
2.	104 a.	13.	$\$8\frac{5}{124}$.	24.	$185\frac{115}{153}$.	35.	$5\frac{29}{34}$.
3.	145 lbs.	14.	7 cts.	25.	$182\frac{803}{1031}$.	36.	$\frac{24}{851}$.
4.	172 bu.	15.	$9\frac{44}{189}$ s.	26.	$4\frac{3970}{6059}$.	37.	$\frac{1}{120}$.
5.	124 gals.	16.	$\$1\frac{73}{121}$.	27.	$1\frac{9401}{33924}$.	38.	$4\frac{11}{21}$.
6.	$87\frac{3}{11}$ yds.	17.	$6.	28.	$\frac{16}{325}$.	39.	$\frac{507}{512}$.
7.	$5\frac{19}{37}$ yds.	18.	$\$2\frac{623}{626}$.	29.	$17\frac{1}{7}$.	40.	$\frac{19}{52}$.
8.	10 m.	19.	$11\frac{39}{44}$ t.	30.	$1\frac{8}{19}$.	41.	$\frac{11}{567}$.
9.	$5\frac{21}{50}$ lbs.	20.	$87\frac{81}{137}$ s.	31.	$\frac{2}{441}$.	42.	$1\frac{77}{3875}$.
10.	$5\frac{5}{13}$ lbs.	21.	$157\frac{61}{127}$ b.	32.	$\frac{138}{1705}$.	43.	$\frac{1909}{5084}$.
11.	$10\frac{5}{13}$ c.	22.	$9\frac{19}{192}$.	33.	$\frac{128}{1575}$.	44.	$\frac{1058}{4615}$.

COMPLEX FRACTIONS.—ARTS. 142–144.

ART. 142.	10. $\frac{22}{15}$.	2. $1\frac{3235}{6579}$.	10. $\frac{5472}{9635}$.	17. $\frac{2257}{10912}$.
3. $\frac{12}{7}$.	11. $\frac{163}{206}$.	3. $1\frac{3119}{9048}$.	11. $13\frac{1}{3}$.	18. $2\frac{436}{8557}$.
4. $\frac{11}{12}$.	12. $\frac{805}{837}$.	4. $2\frac{9659}{73980}$.	12. $1\frac{183}{832}$.	19. $2\frac{302}{1323}$.
5. $\frac{9}{10}$.	13. $\frac{1255}{4}$.	5. $\frac{2161}{4270}$.	13. $\frac{8}{9}$.	20. $1\frac{46688}{55363}$.
6. $\frac{41}{54}$.	14. $\frac{5}{3332}$.	6. $\frac{3601}{8883}$.	14. $1\frac{19}{51}$.	21. $\frac{72697}{367500}$.
7. $\frac{16}{11}$.		7. $27\frac{1737}{3256}$.	15. $\frac{520}{1813}$.	22. $1\frac{123}{280}$.
8. $\frac{4}{3}$.	**ART. 144.**	8. $\frac{43003}{77254}$.	16. $58\frac{5179}{6027}$.	23. $1401\frac{411}{485}$
9. $\frac{441}{220}$.	1. $2\frac{13}{16}$.	9. $2\frac{2}{3}$.		

EXERCISES IN FRACTIONS.—ART. 144.

1. $165\frac{53}{56}$ yds.	13. $\$1\frac{1439}{3576}$.	20. $\$182\frac{21}{32}$.	31. $122\frac{12}{19}$.
2. $297\frac{5}{48}$ m.	14. $\$8\frac{1007}{1710}$.	21. $\$456\frac{37}{80}$.	32. $140\frac{7}{73}$.
3. $426\frac{19}{35}$.	15. $26\frac{137}{168}$.	22. $\$765\frac{3}{5}$.	33. $4\frac{8}{19}$.
4. $489\frac{4}{5}$.	16. $2\frac{47}{272}$.	23. $\$3576\frac{1}{2}$.	34. $3\frac{57}{94}$.
5. $91\frac{3}{20}$. acres.	17. $240\frac{13}{24}$ sum.	24. $58\frac{4271}{5000}$ m.	35. 1001.
6. $\$422\frac{3}{40}$.	$78\frac{5}{24}$ dif.	25. $144\frac{129}{134}$.	36. $85\frac{5}{7}$.
7. $\frac{11}{48}$ ship.	18. $950\frac{6}{7}$ prod.	26. $6\frac{593}{841}$.	37. $87\frac{2}{5}$.
8. $151\frac{3}{4}$ bbls.	$7\frac{7}{25}$ quot.	27. $1608\frac{736}{1323}$.	38. $\$37\frac{37}{128}$.
9. $\$241\frac{21}{32}$.	19. $339\frac{16}{45}$ sum.	28. £$1\frac{279}{2359}$.	39. $\$580\frac{43}{80}$.
10. $\$1950\frac{3}{10}$.	$187\frac{34}{45}$ dif.	29. £$8\frac{308}{1735}$.	40. $\$42\frac{5}{16}$ gain.
11. $159220\frac{7}{8}$.	$19977\frac{23}{45}$ p.	30. $10\frac{205}{392}$ pear.	41. $47\frac{143}{456}$.
12. $438\frac{7}{18}$.	$3\frac{1627}{3411}$ quot.		

REDUCTION.—ART. 162.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
7.	872 s.	52.	2472080 feet.		1 h. 46 m. 40 s.
8.	10416 far.	53.	816000 a.	95.	270000″.
9.	£4, 18s.	54.	$94\frac{194}{1089}$ sq. r.	96.	15300′.
10.	257s. 5d.	55.	$466\frac{1}{4}$ a.	97.	1296000″.
11.	648000 far.	56.	437 a 102 r.	98.	24°, 7′, 40″.
12.	81438 far.	59.	806 sq. ft.	99.	315s. 13°, 20′.
13.	£105, 4s. 8d.	60.	40 sq. yards.	100.	231 s. 14°, 26′
14.	£58. 11s. 7d. 1f.	61.	5 a. 2 r. 20 r.		40″.
15.	408462 far.	62.	24 a.	101.	£11, 5 s.
16.	98683 pwts.	63.	129600 in.	102.	41 s.
17.	348200 grs.	64.	2557440 in.	103.	63 sheep.
18.	60144 grs.	65.	562 T. 24 ft.	104.	£6, 6 s. $10\frac{1}{2}$ d.
19.	2 lbs. 1 oz. 12 p.	66.	129 C. 56 ft.	105.	$\$55\frac{1}{2}$.
20.	4 oz. 9 p. 20 grs.	67.	8320 ft.	106.	87 sp.; $\$32\frac{3}{8}$.
21.	6lbs. 1oz. 7p. 2g.	68.	8100 cu. in.	107.	$\$708\frac{36}{55}$.
22.	108858 drs.	69.	9288 in.	108.	$2251\frac{7}{43}$ E.
23.	3650 lbs.	70.	2160 ft.		$4502\frac{14}{43}$ half E.
24.	1131440 oz.	71.	112 ft.		$1125\frac{25}{43}$ double.
25.	23200 drs.	72.	$3\frac{3}{4}$ C.	109.	369062 in.
26.	54 lbs. 11 oz.	73.	$31\frac{127}{128}$ C.	110.	551546 in.
27.	15cwt. 2q. 15 lbs.	74.	144 gals. 2 qts.	111.	$\$1837\frac{1}{2}$.
28.	6 lbs. 12 oz.	75.	96 hhds. 17 gals.	112.	1579500.
29.	3c. 2q. 4lb. 8oz.	76.	7720 pts.	113.	23743200 times.
30.	7t. 12c. 3q. 10 lb.	77.	40432 gi.	114.	$481\frac{4}{7}$ firkins.
31.	333cwt.7lbs.8oz.	78.	17 bbls. 13 gals.	115.	$966\frac{2}{3}$ bottles.
32.	16320 drs.	79.	36 hhds. 6 gals.	116.	44 yds.
33.	44928 sc.	80.	5428 qts.	117.	$21\frac{5}{8}$ A.
34.	30 oz. 2 drs.	81.	887 pts.	118.	$114\frac{7}{18}$ sq. yds.
35.	13 lbs. 1 oz. 4d.	82.	17176 qts.	119:	156 sq. yds.
37.	356400 in.	83.	420352 pts.	120.	72 yds.
39.	49 r. 3 ft. 8 in.	84.	108 bu.	121.	85 yds.
40.	5 m.	85,	2675 bu.	122.	$33325.
41.	4752000 in.	86.	1318140 sec.	123.	$71750.
42.	7650722 in.	87.	525960 min.	124.	$\$49\frac{49}{512}$.
43.	11 m. $269\frac{17}{33}$ r.	88.	31556928 sec.	125.	880000.
44.	1585267200 in.	89.	11045160 min.	126.	2 w. 6 d. 20 h.
45.	30420 in.	90.	157 h. 50 m. 40 s.	127.	95000000 miles.
46.	636 na.	91.	850 w. 7 h. 36 m.		18849 w. $34\frac{2}{3}$ h.
47.	1620 na.	92.	10305 mo. 3 w.	128.	2700 bricks.
48.	140 yds. 3 qrs.		5d. 16 h.	129.	144 suits.
49.	76 F. e.	93.	6508 y. 4 m. 3 w.	130.	12000 shingles.
50.	260 E. e. 2 qrs.		5 d.	131.	144 farms.
51.	$480964\frac{1}{2}$ feet.	94.	31 y. 10m. 1w. 3d.	132.	215136 bricks.

FRACTIONAL COMPOUND NUMBERS.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
Art. 164.		10.	$\frac{57}{100}$ cwt.	20.	$\frac{497}{2880}$ d.	28.	$\frac{779}{2500}$.
3.	$\frac{13}{16}$ s.	11.	$\frac{21}{400}$ qr.	21.	$\frac{11}{180}$ hr.	29.	$\frac{1}{4}$.
4.	$\frac{29}{32}$ bu.	12.	$\frac{1501}{10560}$ m.	22.	$\frac{1607}{268800}$ wk.	30.	$\frac{1}{9}$.
5.	$\frac{11}{16}$ pk.	13.	$\frac{37}{64}$ m.	23.	$\frac{37}{6400}$ cwt.	31.	$\frac{1}{27}$.
6.	$\frac{31}{32}$ gal.	14.	$\frac{227}{320}$ l.	24.	$\frac{1}{3200}$ T.	32.	$\frac{1}{27}$.
7.	$\frac{5}{32}$ gal.	15.	$\frac{11}{16}$ yd.	25.	$\frac{41}{168}$ hhd.	33.	$\frac{1279}{4268}$.
8.	$\frac{1}{4}$ hhd.	16–18.	Given.	26.	$\frac{157}{160}$ gal.	34.	$\frac{2719}{6516}$.
9.	$\frac{11}{40}$ T.	19.	$\frac{5}{48}$ d.	27.	$\frac{3017}{21600}$.	35.	$\frac{9}{20}$.

Art. 165.

3. 2s. 6d.
4. 14s. $3\frac{3}{7}$d.
5. 5 d. 14 h. 24 m.
6. 9 h. 20 m.
7. 1 m. 1 f. 24 r.
8. 3 fur. $22\frac{2}{3}$ r.
9. 1 q. 18 lbs. 12 oz.
10. 8 cwt. 2 q. $7\frac{1}{7}$lbs.
11. 2 pks. $5\frac{1}{3}$ qts.
12. 274 A. $45\frac{5}{7}$r.
13. 80 cu. ft.
16. $\frac{49}{188}$ d.
17. $\frac{40}{37}$ r.
18. $\frac{33}{100}$ ft.
19. $\frac{160}{1787}$ na.
20. $\frac{1}{50}$ lb.
21. $\frac{27}{28}$ sec.
22. $\frac{256}{2565}$pt.
23. $\frac{3}{4}$ sq. ft.
24. $\frac{4}{5}$ cu. ft.

Art. 167.

3. £$\frac{1}{1344}$.
4. $\frac{1}{7680}$ lb.
5. $\frac{1}{51200}$ T.
6. $\frac{1}{7200}$ m.
7. $\frac{1}{54450}$ A.
8. $\frac{1}{256}$ C.
9. $\frac{1}{630}$ hhd.
10. $\frac{1}{4}$.
11. $\frac{1}{2}$.

COMPOUND ADDITION.

Art. 168.

3. £19, 9s. 5d. 3 f.
4. £53, 5s. 5d.
5. £58, 18s. 4d.
6. 45 lbs. 4 oz. 2 p. 10 g.
7. 3 lbs. 7 oz. 12 p.
8. 61 lbs. 7 oz. 9 p.
9. 19lb. 11o. 5p. 23g.
10. £70, 17s. 9d.
11. 15 c. 33 lbs. 9 oz.
12. 1267 lbs. 13 oz.
13. 10 T. 178 l. 12oz.
14. 28 yds. 3 qrs. 1 n.
15. 118 yds. 3 q. 2 n.
16. 65 bu. 1 pk.
17. 92 bu. 3 p. 2 q.
18. 99 m. 5 fur. 11 r.
19. 6 hhds. 53 g. 3 q.
20. 8 p. 59g. 2q. 1 pt.
21. 109yds. 8f. 142 i.
22. 31 a. 61 r. 48 ft.
23. 99 cu. ft. 227 in.
24. 73C. 69ft. 177in.
25. 87 lbs. 6 oz. 7drs. 2 sc. 9 grs.
26. 177m. $7\frac{3}{4}$fur. 35r. 3yds. 0ft. 10in.
27. 64 l. 0 m. $7\frac{1}{4}$ fur. 27 r. 8 ft.
28. 200 yrs. 11 mos. 0 wk. 4 ds.
29. 353A. 52sq.r. $10\frac{1}{2}$ sq. yds. $3\frac{3}{4}$ ft.
30. 30 c. 9s. 20° 9′ 16″.

Art. 168.*a*.

3. 8 oz. 3 p. $22\frac{2}{5}$ g.
4. 3 p. $15\frac{23}{40}$ grs.
5. 12 c. 75 l. 8 oz.
6. 43 l. 8oz. $10\frac{2}{21}$ d.
7. 1 mile.
8. 6 r. 10 ft. $9\frac{3}{5}$ in.
9. 1 qr. 2 n. $1\frac{7}{16}$ in.
10. 2 q. 1 n. $2\frac{3}{28}$ in.
11. 3R. 1r. $158\frac{13}{16}$ ft.
12. 17 y. 8 ft. $75\frac{6}{7}$ in.
13. 4 C. 77 cu. ft. $1166\frac{2}{5}$ cu. in.
14. 17 cu.ft. 1584 c.i.
15. 23 g. 1 q. $1\frac{2}{3}$ pt.
16. 1 bu. 2 pks. 1 q. $\frac{11}{12}$ pt.
17. 9 hrs. 37 min. $25\frac{61}{70}$ sec.
18. 1 yd. 2 q. $3\frac{3}{4}$ n.
19. $1580\frac{1}{4}$ lbs.

COMPOUND SUBTRACTION.—ARTS. 169, 170.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
3.	£8, 7s. 11d. 2 far.	12.	9 m. 18 r. 7 ft. 10 in.	21.	3 yrs. 2 m. 23 d.
4.	£36, 3s. 7d. 2 far.	13.	54 a. 149 r. 38 s.f.	22.	3 yrs. 7 m. 20 d.
5.	8T. 5 c. 2 q. 5 lbs.	14.	70 a. 0 r. 33 r.	23.	118m. 4fur. 34 r. 13½ ft. 10 in.
6.	28T. 17 cwt. 3 qr. 8 lbs.	15.	128 ft. 1652 in.	24.	96° 61½ m. 224 r. 3½ yds. 2 ft.
7.	9 gals. 1 qt. 3 gi.	16.	48C. 106ft. 58 in.	25.	162 A. 115 r. 19¼ yds. 7 ft. 95 in.
8.	58 hhds. 6 g. 2 q.	17.	8 yrs. 2 m. 5 d. 16 h. 15 min.	26.	191 A. 2 R. 22 r. 87¼ ft.
9.	6 oz. 18 p. 2 grs.	19.	6 yrs. 4 m. 25 d.		
10.	13 yds. 1 qr. 3 na.	20.	69 yrs. 1 m. 21 d.		
11.	3 yds. 2 qrs. 2 n.				

SUBTRACTION OF FRACTIONAL COMPOUND NUMBERS.

ART. **170.*a*.**

3. 10 d. 1 far.
4. 6 d. 3 far.
5. 13 cwt. 81 lbs. $14\frac{10}{21}$ oz.
6. 88 lbs. $13\frac{28}{45}$ oz.
7. 199 r. $1\frac{5}{14}$ ft.
8. $2\frac{74}{77}$ in.
9. 3 R. $23\frac{9}{10}$ r.
10. 157 $\frac{7}{26}$ sq. ft.
11. 75 cu. ft. $1468\frac{4}{5}$ cu. in.
12. 24cu.ft. $1295\frac{3}{8}$c.i.
13. 43 gals. $1\frac{7}{23}$ qts.
14. 3 qts. $\frac{1}{3}$ pt.
15. 25 gals. 3q. $1\frac{1}{5}$ p.
16. $924\frac{9}{10}$ lbs.
17. 3 d. 19 h. 6 min.
18. 11 yds. $2\frac{5}{6}$ na.
19. $26\frac{1}{4}$ A. $26\frac{1}{20}$ r.
20. 10g. 3qts. 1p. 3g.

COMPOUND MULTIPLICATION.

ART. **171.**

1, 2. Given.
3. £127, 12s. 6d.
4. £187, 14s.
5. £8, 9s. 3d. 3 f.
6. £56, 6s. 3d.
7. £44, 4s.
8. 7670 d. 2 h. 4 m. 48 sec.
9. 3lbs. 1o. 14p. 4g.
10. 5 lbs. 2 oz. 8 p.
11. 8 T. 7 cwt. 9 lbs.
12. 5 T. 18 cwt. 2 q. 2 lbs. 8 oz.
13. 101 c. 15 lb. 7 oz.
14. 604 gals. 1 q. 2 g.
15. 53 m. 3 fur. 20 r.
16. 319 m. 1 f. 30 r.
17. 328 yds. 2 qrs.
18. 96 a. 90 sq. r.
19. 693 sq. yds.
20. 18 C. 111 ft.
21. 1512 ft. 1064 in.
22. 48° 23′ 20″.
23. 1814° 29′ 52″.
24. 807 gals.
25. 2452 gals. 2 qts.
26. £391, 14s. 5d.
27. 3365 bu. 2 p. 4 q.
28. 4937 yds. 2 qrs.
29. 35309 T. 2 cwt. 74 lbs.
30. 4687 bu. 2 pks.

COMPOUND DIVISION.

ART. **173.**

4. £2, 9s. 4d. $2\frac{2}{3}$f.
5. £5, 18s. 5d. $1\frac{5}{6}$f.
6. £5, 7s. 1d. $3\frac{3}{4}$ f.
7. £4, 15s. 4d.
8. £6, 5s. 3d. $1\frac{1}{5}$ f.
9. £2, 2s. 6d. $2\frac{1}{12}$f.
10. 5 oz. 8 p. 8 g.
11. 1 *l.* 3o. 13 p. $9\frac{3}{4}$ g.
12. 10 lbs. $11\frac{1}{2}$ oz.
13. 9 lb. $8\frac{4}{5}$ oz.
14. 9 yds. 2 q. $1\frac{3}{8}$ n.
15. 4 m. 4 f. $17\frac{5}{11}$ r.
18. 15 bu. $7\frac{5}{9}$ qts.
19. £1, 1s. $3\frac{76}{81}$ far.

APPLICATION OF THE COMPOUND RULES.

Ex. Ans.	Ex. Ans.	Ex. Ans.
Art. 175.	5. £117, 7s. 3d.	10. £6813, 12s. 9d.
1. £47, 3s. $2\frac{1}{4}$ d.	6. £9429, 0s. 5d.	11. £11, 3s. $7\frac{43}{54}$ d.
2. £400, 9s. $5\frac{7}{12}$d.	7. £67, 9s. $8\frac{9}{40}$ d.	12. £86, 9s. 7d. gain.
3. £121, 1s.	8. £3, 7s. $7\frac{1}{16}$d.	13. £846, 0s. 1d.
4. £14, 11s. 2d.	9. £324, 1s. 3d.	14. £351, 2s. 1d.

ADDITION OF DECIMALS.—ART. 187.

3. 320.67.	8. 19.57605.	13. 330.967.	18. 0.7186423.
4. 2986.0501.	9. 760.573.	14. 10.709341.	19. 744.8785.
5. 81.271.	10. 1310.9902.	15. 2.0728.	20. 127.00462-93.
6. 111.9925.	11. 177.998.	16. 0.408763.	
7. 8.5284508.	12. 33.4013.	17. 0.607677.	

SUBTRACTION OF DECIMALS.—ART. 189.

3. 250.3905.	12. 0.999999.	17. 0.005994.	24. 40.177.
4. 14.544.	13. 130.84106-99.	18. 0.3222.	25. 262.901099.
5. 13.25.	14. 8897.8195-07.	19. 538.978.	26. 28999.971.
6. 144.96063.	15. 55999.999-001.	20. 7855.9997-64.	27. .405.
7. 0.875.	16. 0.675.	21. .45.	28. .568431.
8. 10.69995.		22. .0053.	29. .881 A.
9. 0.23578.		23. .000061.	30. .785 hhd.
10. 1.1011.			31. 39.163 yds.
11. 1.400091.			

MULTIPLICATION OF DECIMALS.—ARTS. 191, 192.

1. 231.41 yds.	14. 0.00187440781.	27. .006420512.
2. 259.875 gals.	15. 0.0024048072.	28. .000015230.
3. 589.875 ft.	16. 0.000058175003.	29. 371.0846634.
4. 371.25 C.	17. 0.0004000751.	30. .00194944563.
5. 519.675 r.	18. 0.08568931.	31. 56.44306340.
6. 474.6875 m.	19. 0.00031275.	32. .0037503660.
7. 65365 lbs.	20. 0.0000022780402	33. .174441456.
8. 44.3955 bbls.	21. 0.0000025.	34. 189.344142242.
9. 0.50005.	22. 0.00042.	35. 172.922687063.
10. 50.1565195.	23. 0.001825.	36. .0000004.
11. 460.51.	24. 0.00064125.	37. .000000305.
12. 2650.1.	25. 0.00071014734.	38. .00000012624.
13. 5678.	26. .01899018.	39. .0000000000015092

DIVISION OF DECIMALS.—ARTS. 194, 195.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
1.	6 coats.	10.	14.3.	21.	24.578075.	27.	.2826+.
2.	9 loads.	13.	0.8.	22.	.8576034.	28.	35.4216+.
3.	12.3 days.	14.	0.001777+.	23.	910000000-	29.	5.999898+.
4.	23.9139 A.	15.	2.4.		000.	30.	20.99993+.
5.	4.5 rods.	16.	10000.	24.	990000000-	31.	.92887+.
6.	3.15 bbls.	17.	5000000.		000.	32.	1.551948+.
7.	24.3936 d.	18.	17.6.	25.	.000078435.	33.	.38+.
8.	6.9 days.	19.	0.62.	26.	.00000090-	34.	104.034.
9.	15 boxes.	20.	31.7199+.		34.	35.	.0101.

REDUCTION OF DECIMALS.—ART. 196.

2. $\frac{1}{8}$.	7. 0.8; .8333-	17. .025.	6. 2.625s.
3. $\frac{33}{50}$.	+; .1.	18. .23125.	7. .089285+h.
4. $\frac{3}{4}$.	8. 0.16; .4;	19. .8125.	8. 0.75625d.
5. $\frac{3}{8}$.	.04.	20. .026875.	9. .2825 T.
6. $\frac{21}{40}$.	9. 0.625; .4;	21. .04761947,	10. 0.833+yd.
7. $\frac{1}{40}$.	.05.	&c.	11. .84375 m.
8. $\frac{7}{8}$.	10. 0.025; .00-	22. .071428571,	12. 0.84375lbs.
9. $\frac{1}{16}$.	28+.	&c.	13. .8125 s.
	11. 0.025; .003.		14. £.25104+.
ART. 197.	14. .03703, &c.	**ART. 200.**	15. £31.278125
4. 0.75; .8.	15. .142857142,	3. £.775.	16. .00079365-
5. 0.15; .28.	&c.	4. £.626.	+ hhd.
6. 0.375; .2; .6.	16. .0769237 &c.	5. £.0375.	17. .000375 T.

ART. 201.	6. 15 hrs. 34.56 sec.	10. 6 fur. 23 r. 3 yds.
3. 7d. 2 far.	7. 3q. 10*l*. 9o. 9.6d.	7.632 in.
4. 9s. 3d.	8. 13 c. 3 qrs. 14 lbs.	11. 1 R. 33.0928 r.
5. 3 qts. & .048 p.	9. 3pks. & .5248pt.	12. 3 qr. & .10096 n.

EXERCISES IN DECIMAL COMPOUND NUMBERS.

ART. 202.	12. £194, 10s. 7d. 2f.	23. 162 m. 72 r.
2. 2s. 10$\frac{3}{4}$d.	13. $377.5.	24. £155, 12s. 6d.
3. 12s. 9d. 3 far.	14. 36.3+d.	25. £1174, 10s.
4. 5 cwt. 58 lbs.	15. $476.82.	26. $9949.7574.
5. 96 r. 108.9 ft.	16. $1161.09.	27. 1s. 2d. 3.5 far.
6. 6s. 6d.	17. £386, 16s. 6d. 2f.	28. 11d. 1 far.
7. 10d. 1 far.	18. £14, 5s. 3 far.	29. 6.095232+d.
8. 4 cwt. 87 lbs.	19. £18, 16s. 10d. 2f.	30. $9.15.
9. 173 r.	20. $1.42857.	31. 5s. 2d. 3 far.
10. £.20833+.	21. £1, 8s.	32. 6d.
11. £.796875.	22. $6.666+.	33. 3$\frac{51}{63}$d.

ADDITION OF FEDERAL MONEY.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
ART. 211.	4. $363.433.	8. $1022.529.	12. $10.545.
1. Given.	5. $270.279.	9. $76.121.	13. $2377.69¼.
2. $12.13.	6. $281.033.	10. $216.723.	14. $1409.8978.
3. $45.805.	7. $196.51.	11. $317.207.	15. $114.7775.

SUBTRACTION OF FEDERAL MONEY.—ART. 212.

3. $10.36.	6. $339.67.	9. $0.174.	12. $900.055.
4. $81.33.	7. $156.87.	10. $54.422.	13. $.05625.
5. $41.60.	8. $0.004.	11. $100.088.	14. $213.125.

MULTIPLICATION OF FEDERAL MONEY.

ART. 215.	10. $9.140625.	17. $57.09375.	24. $1071.60.
1–4. Given.	11. Given.	18. $24.18.	25. $577.746.
5. $1.47.	12. $5.56875.	19. $38.1875.	26. $26.705.
6. $1.4725.	13. $5.625.	20. $18.375.	27. $125.75088
7. $4.6875.	14. $13.786875	21. $142.50.	28. $36.2175.
8. $6.62625.	15. $77.46875.	22. $13.005.	29. $1071.
9. $4.1875.	16. $38.85425.	23. $127.50.	30. $8970.

DIVISION OF FEDERAL MONEY.

ART. 219.	12. $3.50.	18. $1.3698+.	24. $68.493+.
7. 16 qts.	13. $1.8673+.	19. $0.02.	25. 9 yds.
8. 25.5 lbs.	14. 66 cords.	20. $1.25.	26. 252 bbls.
9. 24 melons.	15. 1.7894+b.	21. 465.55+b.	27. $.075.
10. 36 pen-ks.	16. $0.07.	22. $10.2816.	28. $0.125+,
11. 25.142+q.	17. 52 weeks.	23. $10.914.	29. 600 tons.

APPLICATIONS OF FEDERAL MONEY.

ART. 220.	4. $123.07.	7. $263.59.	11. $429.8825.
1. $17.770.	5. $1478.75.	8. $5320.1875	12. $372.755.
2. $12.95.	6. $2305.625 n.	9. $2659.275.	13. $1058.05.
3. $21.485.	$4805.625 a.	10. $1067.65.	

PERCENTAGE.

ART. 225.	11. $0.96474.	14. $300.0756.	17. $3.36.
9. $0.9021.	12. $0.1809.	15. $450.168.	18. $11.565.
10. $2 069075.	13. $60.0451.	16. $13.952.	19. $58.8875.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
20.	$21.	26.	$468.75 lo.	30.	$793.75.	36.	$2152.50.
21.	$1398.		$1031.25 l.	31.	$43.375.	37.	$3100.
22.	129.75 lost.	27.	$1285.35.	32.	$4 former.	38.	$172.125.
	735.25 left.	28.	$37.50.	33.	$120.	39.	$588.6718-
23.	$63.333+.		$951.5625.	34.	0.		75.
24.	$106.8431.	29.	$168.	35.	$1720.	40.	1780 sheep.
25.	$1.3332+.						

COMMISSION, BROKERAGE, AND STOCKS.

Art. 232.		9.	$47.62 com	16.	$3753.915.	26.	$6750.
2.	$16.0068.		$952.38 cot	17.	$1759.308.	27.	$5989.
3.	$21.51125.	10.	$52.13 b.	18.	$477.65.	28.	$456.
4.	$5.97 Agt.;		10426 s.	19.	$7526.	29.	$735.8738.
	$259.38 Ow.	11.	$350.	20.	$5000.	30.	$60946.34-
5.	$15.426+.	12.	$454.575.	22.	$527.50.		146+.
6.	$15.60.	13.	$521.93.	23.	$1275.	31.	$572.1679.
7.	$10.226+.	14.	$29.27.	24.	$3364.	32.	$45.9627+
8.	$70.993.	15.	$406.437.	25.	$450.	33.	$304.6214.

INTEREST.—Art. 241.

1–4.	Given.	15.	$1080.	28.	$0.522.	42.	$4662.248.
5.	$5; $6; $4;	18.	$60 int.;	29.	$65.166.	43.	$134.776.
	$7.		$260 amt.	30.	$7.437.	44.	$20.326.
6.	$2.118.	19.	$175 int.;	31.	$9.26.	45.	$232.7312.
7.	$3.507.		$425 amt.	32.	$2.638.	46.	$599.86 in.
8.	$3.465.	20.	$81.72 int.;	33.	$7.526.		$9808.81 a.
9.	$6.153.		$422.22 a.	34.	$29.048.	47.	$1980.058i.
11.	$9.817 int.	23.	$5.833.	35.	$18.235.		$17186.90.
	$150.067 a.	24.	$5.629.	36.	$206.58.	48.	$1527.844.
12.	$13.072 i.;	25.	$2.80 int.;	37.	$4.19375.		$11578.534
	$176.472 a.		$62.80 a.	38.	$2.916.	49.	$2864.238.
13.	$24 int.;	26.	$4.80 int.;	39.	$5.		$14472.096
	$424 amt.		$100.80 a.	40.	$108.515.	50.	$13876.676
14.	$535.	27.	$0.10.	41.	$8541.376.		$55237.856

Art. 246.		9.	$26.60.	16.	$26.072.	24.	$812.876.
3.	$12.5253.	10.	$32.437.	17.	$25.045.	25.	$17.1829.
4.	$5.434.	11.	$4.93.	18.	$28.082.	26.	$691.469.
5.	$5.924.	12.	$61.893.	20.	$5.549.	27.	$722.742.
6.	$14.883.	13.	$8.05.	21.	$31.693.	28.	$1665.248.
7.	$2.961.	14.	$10.148.	22.	$37.044.	29.	$1698.9259
8.	$4.675.	15.	Given.	23.	$786.373.	30.	$2324.811.

EXAMPLES FOR PRACTICE.—ARTS. 247–250.

Ex. Ans.	Ex. Ans.	Ex. Ans.	Ex. Ans.
1. $1.81.	14. $2.815.	27. $20.709.	40. $1382.333.
2. $5.021.	15. $1022.25	28. $20.693.	41. $4.00.
3. $1.642.	16. $1500.	29. $39.013.	42. $0.36.
4. $0.916.	17. $3960 144.	30. $27.713.	43. Given.
5. $13.904.	18. $5125.	31. $13.774.	44. $366.66.
6. $242.316.	19. $1147.50.	32. $315.091.	45. $426.499.
7. $391.613.	20. $14.734.	33. $400.251.	46. $780.07.
8. $42.	21. $167.022.	34. $637.796.	49. £6, 12s. 4d.
9. $224.193.	22. $8635.505.	35. $612.964.	50. £10, 18s. 1f
10. $675.863.	23. $16269.325	36. $753.452.	51. £111, 13s.
11. $898.88.	24. $5.265.	37. $204.185.	4d.
12. $1260.994.	25. $12.296.	38. $150.078.	52. £467. 9s.
13. $108.616.	26. $7.746.	39. $114.912.	8d.

PROBLEMS IN INTEREST.—ARTS. 253–255.

3. 6 per cent.	7. $7\frac{1}{2}$ per ct.	13. $10000.	18. $3\frac{1}{3}$ years.
4. 12 per ct.	8. 5 per cent.	14. 11666.66\frac{2}{3}$	19. $16\frac{2}{3}$ years.
5. $8\frac{10}{27}$ per ct.	9. 9 per cent.	15. $17142.85-	20. $14\frac{2}{7}$ years.
6. $7\frac{1}{2}$ per ct.	10. 5 per cent.	$7\frac{1}{7}$.	21. 10 years.

COMPOUND INTEREST.—ARTS. 257, 258.

2. $91.866.	6. $1351.791.	11. $3195.818.	15. $780.687 a.
3. $348.207.	7. $927.755.	12. $26878.32.	$261.687in.
4. $335.024.	8. $2143.099.	13. $4964.817.	16. $1524.468.
5. $1126.162.	10. Given.	14. $560.361.	17. $4297.963.

DISCOUNT.—ARTS. 260–262.

1, 2. Given.	9. $28.4312+	16. $8.75 .	23. $1264.617.
3. $443.925.	10. $27.8122+	17. $1142.02.	24. $1[illegible].1323+
4. $153.508+	11. $4950.495.	18. $736.009+	25. $17.6593+
5. $980.392+	12. $1.698.	19. $41.9888.	26. $59.5833+
6. $18.293+.	13. Given.	20. $1276.173.	27. $69.231.
7. $1674.418.	14. $5.979+.	21. $4985.	28. $457.944.
8. $1092.95+	15. $2.0625.	22. $14985.	

INSURANCE.—ARTS. 264–265.a.

2. $6.5625.	7. $206.25.	13. $573.75.	20. $\frac{4}{5}$ per cent.
3. $12.50.	8. $202.666.	14. $2390.	21. $\frac{1}{2}$ per cent.
4. $143.375.	10. $45.18.	15. $4205.	22. 7 per cent.
5. $27.30.	11. $58.80.	17. $16666.666	24. $6798.478.
6. $81.25.	12. $1950.	18. $54545.455	25. $11842.105

PROFIT AND LOSS.—Arts. 267, 268.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
2.	$6.	12.	$343.75.	20.	$33\frac{1}{3}$ per ct.	27.	$23\frac{31}{56}$ per ct.
3.	$5.79.	13.	$312.06.	21.	$33\frac{1}{3}$ per ct.		$2362.
4.	$13.	14.	$1163.75.	22.	$6\frac{62}{73}$ per ct.	30.	$20.
5.	$56.25.	15.	$29250.	23.	$4\frac{11248}{15565}$ p. c.	31.	$94.44\frac{4}{9}$.
6.	$250.	16.	Given.	24.	$4\frac{2}{7}$ per ct.	32.	$108.696.
9.	$26.1625.	17.	$18\frac{2}{11}$.	25.	$33\frac{1}{3}$ per ct.	33.	$178.57\frac{1}{7}$.
10.	$44.25.	18.	50 per ct.	26.	$16\frac{8}{47}$ per ct.	34.	$373.33\frac{1}{3}$.
11.	$71.464.	19.	25 per ct.		$190.	35.	$1714.285.

EXAMPLES FOR PRACTICE.—Art. 270.

1.	$1.98.	7.	$5.60.		$37.50 g.	19.	$21\frac{133}{327}$ p. ct.
2.	$16\frac{2}{3}$ per ct.;	8.	$.1395.	13.	$450.	20.	$.9375 p. a.
	40 c. gain.	9.	30 cts pr g.	14.	$56\frac{1}{4}$ cents.		$3125 lost.
3.	20 p. c.; $3.		$37.80 prof.	15.	12 c. pr yd.	21.	$5.0555 p.b.
4.	$19\frac{1}{21}$ per ct.	10.	$1.062.		2 cts. prof.		$586.438.
	$6 gained.	11.	$1383.75 l.	16.	$1.008.	22.	12.3 per ct.
5.	$2.30.		$9.3275 b.	17.	45 cents.	23.	$.90 pr. yd.
6.	$4.79\frac{1}{6}$.	12.	100 per ct.	18.	$14\frac{2}{7}$ per ct.		$43.344 g.

DUTIES.—Arts. 272a.–274.

2.	1841 lbs.	9.	42 bot. bkg.	6.	$8640.	16.	$1695.
3.	6396 lbs.		881 bt. net.	8.	$75.	17.	$1504.80.
4.	6804 lbs.	10.	52 bot. bkg.	9.	$71.224.	18.	$2898.
5.	1720 lbs.		988 bt. net.	10.	$131.6525.	19.	$3592.75.
6.	4037 lbs.			11.	$90.03.	20.	$4500.375.
7.	53 gal. lkg.		Art. 273.	12.	$69.0375.	21.	$2819.125.
	2575 g. net.	3.	$87.75.	13.	$186.4625.	22.	$197.375.
8.	80 gal. lkg.	4.	$202.50.	14.	$850.	23.	$4254.94.
	2208 g. net.	5.	$941.60.	15.	$1000.	24.	$882.453.

ASSESSMENT OF TAXES.—Arts. 278, 279.

3.	$12.25.	7.	$71.20 D.	15.	$510.50 F.	21.	$612.25 M.
4.	5 c. on $1.	8.	$309 G.	16.	$405.25 G.	22.	$.0075.
	$57.50. t.	11.	$30.22 B.	17.	$307.70 H.		$22.6875 A.
5.	$76.50.	12.	$117.51 C.	18.	$661 J.	23.	$24.7625 B.
6.	2 c. on $1.	13.	$159.47 D.	19.	$300.51 K.	24.	$21.8575 C.
	$102.40 C.	14.	$285.22 E.	20.	$90.75 L.	25.	$26.6425 D.

GENERAL PROBLEMS.—ARTS. 286–294.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
1.	Given.	7.	Given.	12.	\$36 tea;	18.	1350 ap.
2.	27 cows.	8.	24 years.		\$27 molas.	20.	12 sailors.
3.	185 acres.	9.	1050 fem.	14.	10 yrs.	21.	8 flocks.
5.	\$36.	11.	10 yrs.;	15.	8 rods.	23.	7 years.
6.	387 sheep.		15 yrs.	17.	\$825.	24.	12 marb.

ANALYSIS.—ART. 296–303.

1.	Given.	37.	\$85.50.	74.	1053.045 c.		$33\frac{1}{3}$ C.
2.	\$565.41.	38.	\$75.74+.	75.	1018.154 p.	90.	10 per ct.
3.	\$22.20.	39.	\$1.00.	77.	\$183.33$\frac{1}{3}$A.		\$1500 A.
4.	\$630.	41.	\$31.50.		\$166.66$\frac{2}{3}$B.	91.	.15$\frac{3205}{20453}$.
5.	\$364.61.	42.	\$2.85.	78.	$107\frac{1}{7}$ A.		\$644.15 M.
6.	\$204.	44.	£1.		$85\frac{5}{7}$ B.	95.	6 shil.
7.	\$30.	45.	\$53333$\frac{1}{3}$.		$57\frac{1}{7}$ C.	96.	4 cts.
8.	\$166.40.	46.	\$93$\frac{1}{3}$.	79.	\$600 A.	97.	\$22.50.
9.	\$47.50.	47.	\$23.125.		\$375 B.		$40\frac{10}{11}$ cts.
10.	\$97.50.	48.	60 ds.		\$525 C.	98.	$65\frac{20}{29}$ cts.
11.	\$4.50.	49.	100 ds.	80.	$666\frac{2}{3}$ A.	99.	$9\frac{39}{437}$ cts.
12.	\$12.12.	50.	50 days.		800 B.	100.	$17\frac{1}{11}$ c.
13.	\$4.16$\frac{2}{3}$.	52.	624 bu.		1000 C.	102.	20 men.
14.	\$3.44.	53.	2 cords.		$533\frac{1}{3}$ D.	103.	$7\frac{1}{2}$ days.
15.	\$2.65.	54.	228 pair.	81.	\$315 A.	104.	$2\frac{1}{4}$ mo.
16.	$108\frac{1}{2}$ m.	55.	$199\frac{2}{7}$ lbs.		\$525 B.	105.	720 m.
17.	$112\frac{1}{2}$ bu.	56.	$98\frac{6}{13}$ lbs.		\$420 C.	106.	224 bu.
18.	806 m.	57.	$69\frac{9}{14}$ t.	82.	\$1250 X.	108.	240 s.
19.	$80\frac{5}{8}$ m.	58.	$7\frac{28}{221}$ cts.		\$1750 Y.	109.	\$288.
21.	48 ds.	59.	125.8 bu.		\$2000 Z.	110.	1440 m.
22.	192 ds.	60.	50 cts.	83.	$66\frac{2}{3}$ cts.	111.	144.
23.	54 ds.	61.	748.5 yds.		\$200 1st.	112.	60.
24.	$32\frac{1}{7}$ mo.	62.	2489 lbs.		\$266.66$\frac{2}{3}$.	113.	24 feet.
25.	288 ds.	63.	48.585 cwt.		\$338.38$\frac{1}{3}$.	114.	\$136.
26.	70 cents.	64.	4738.65 lb.	84.	80 cents.	115.	\$14400.
27.	\$4.20.	65.	1192.02 y.	85.	\$64.138 A.	116.	72 yrs.
28.	36 cents.	66.	4145.732 c.		\$105.1132.	117.	48.
29.	\$5.22.	67.	54.912 b.		\$147.749C.	118.	72 sch.
31.	\$0.96$\frac{2}{7}$.	68.	46.5 yards.	86.	10 cents.	119.	\$15600.
32.	$47\frac{1}{4}$ s.	69.	\$6.75 pr. b.		\$500 B.	120.	60 trees.
33.	\$25.60.	70.	9 yards.	87.	\$.042+.	124.	\$317.
34.	\$3.	71.	128 eggs.	88.	\$.11$\frac{19}{28}$.	125.	\$30.
35.	20 cents.	72.	325 lbs.	89.	100 A.	126.	\$250.
36.	\$18.04.	73.	319 boys.		$66\frac{2}{3}$ B.	127.	\$240.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
128.	\$586.50.	144.	\$909.56$\frac{1}{4}$.	160.	\$904.357.		2$\frac{17}{48}$d.
129.	\$1218.75.	145.	\$867.75.	161.	\$11861.13.	172.	£576, 18s.
130.	\$1814.40.	146.	\$986$\frac{2}{3}$.	162.	\$61.05375		2$\frac{41}{42}$d.
132.	£183, 15s.	148.	\$420.	163.	\$666.5625	173.	£1522,13s
133.	£472, 10s.	149.	\$340.	164.	\$1104.125		11$\frac{191}{252}$d.
134.	£11250.	150.	\$749$\frac{1}{3}$.	165.	\$2762.957	174.	£135, 15s.
136.	\$453$\frac{1}{3}$.	151.	\$729.	166.	\$2122$\frac{2}{3}$.		7$\frac{27}{32}$d.
137.	\$220.	152.	\$1200.50.	167.	\$30654.	175.	\$351.0547
138.	\$240.75.	154.	\$44.95.	168.	\$5412.	176.	\$12644.-
139.	\$318.	155.	\$99.11.	169.	£149, 1s.		8359.
140.	\$1250.	156.	\$119.756.		6$\frac{3}{5}$d.	177.	\$27.4375.
141.	\$1622.25.	157.	\$367.22.	170.	£365, 4s.	178.	\$572.3625
142.	\$1092.375	158.	\$779.688.		11$\frac{17}{20}$d.	179.	\$319.625.
143.	\$278$\frac{5}{6}$.	159.	\$270.078.	171.	£117, 2s.		

SIMPLE PROPORTION.—ARTS. 327, 328.

4.	\$100.	26.	18$\frac{2}{3}$ bbls.	43.	£3, 12s. 6d.	57.	6d. gain.
5.	\$75.	27.	\$60.	44.	£41,12s.6d.	58.	\$958.609.
6.	\$30.75.	28.	75 feet.	45.	3$\frac{3}{5}$ hours.	59.	244 m. 0 f.
7.	1140 bu.	29.	100 days.	46.	5 min.		27$\frac{1}{22}$ r.
8.	240 m.	30.	105 a.	47.	12 hours.	60.	\$$\frac{2}{5}$.
11.	12 days.	31.	\$595.	48.	£313, 0s.	61.	£$\frac{377}{665}$.
12.	\$59.50.	32.	130$\frac{2}{3}$ cwt.		10. 2d.	62.	30 per ct.
13.	13$\frac{1}{3}$ mo.	33.	133$\frac{1}{3}$ sp.	49.	£4, 5s. 6d.	63.	\$8400.15$\frac{5}{8}$.
14.	\$7.	34.	\$1500.	50.	£227, 12s.	64.	\$126.3281-
15.	\$6.13$\frac{1}{2}$.	35.	\$251$\frac{1}{4}$.		1d.		25.
17.	\$784.	36.	362 days.	51.	£51,3s.1$\frac{3}{4}$d.	65.	\$195.1385.
18.	\$216.	37.	80 cents.	52.	£358,7s. 3f.	66.	\$22364.28-
19.	\$515.	38.	65$\frac{1}{3}$ wks.	53.	£40.		57.
20.	£22, 10s.	39.	£56,13s.4d.	54.	£37, 19s.	67.	\$192617.-
21.	1440 m.	40.	£186, 2s.		3$\frac{1}{4}$d.		0212.
22.	£2, 5s.		4$\frac{1}{2}$d.	55.	£50, 15s.	68.	\$3333$\frac{1}{3}$ c.
23.	\$1.70+.	41.	3s. 9$\frac{9}{16}$d.		6$\frac{8}{9}$d.		\$3833$\frac{1}{3}$.
24.	\$3.75.	42.	£585, 1s.	56.	£10, 7s.	69.	14$\frac{10}{13}$ in.
25.	\$5000.		4$\frac{1}{2}$d.		8$\frac{4}{13}$d.	70.	145$\frac{5}{8}$ yds.

COMPOUND PROPORTION.—ART. 331.

1.	Given.	7.	6 days.	12.	9 months.	17.	60 horses.
2.	36 men.	8.	7$\frac{1}{5}$ days.	13.	\$18.	18.	18 days.
3, 4.	Given.	9.	170$\frac{2}{3}$ bu.	14.	£384.	19.	37$\frac{7}{10}$ d.
5.	96 men.	10.	80 days.	15.	90 days.	20.	792 pair.
6.	10 men.	11.	6 men.	16.	25 lbs.		

DUODECIMALS.—ARTS. 333–336.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
1.	Given.	6.	892 ft. 1 in.	11.	119 ft. 4 in. 6″.
2.	78 feet 5 inches.	8.	41 ft. 9 in. 9″.		17 ft. 4 in. 6″.
3.	193 ft. 1 in. 2″.	9.	11 ft. 5 in. 6″.	12.	229 ft. 5 in. 9″.
4.	247 ft. 7 in. 2″.	10.	71 ft. 5 in. sum.		21 ft. 11 in. 1″.
5.	252 ft. 8 in. 7″.		14 ft. 7 in. dif.		
ART. 336.		8.	187 ft. 2 in. 8″.	15.	6375 feet.
2.	46 ft. 10 in. 6″.	9.	85 ft. 6 in. 8″. 6‴.	16.	472 ft. 6 in.
3.	13 ft. 7 in. 2″.	10.	38 ft. 2 in. 4″.	17.	484 ft. 1 in. 9″.
4.	82 ft. 9 in. 4″.	11.	82 ft. 5 in. 8″. 4‴.		4‴.
5.	210 ft. 4 in. 6″.	12.	86 feet.	18.	8100 bricks.
6.	1364 ft. 3 in.	13.	210 ft. 4 in. 6″.	19.	$22.50.
7.	149 ft. 5 in. 6″.	14.	2200 feet.	20.	3.555\frac{5}{9}$.

SQUARE ROOT.—ARTS. 351–362.

3. 25.	21. 13.228.	38. $\frac{32}{5}=6\frac{2}{5}$.	54. 50 ft.	69. 8.
4. 30.	22. 842.	39. $\frac{32}{3}=10\frac{2}{3}$	51.923f.	70. 15.
5. 35.	23. 8212.	40. .0231.	56. 952 sol.	71. 80.
6. 42.	24. $\frac{6}{7}$.	41. .0195.	57. 783.836.	72. 56.
7. 54.	25. $\frac{12}{24}=\frac{1}{2}$.	42. $4\frac{44}{45}$.	58. 391.918.	73. 72.
8. 69.	26. $\frac{5}{2}=2\frac{1}{2}$.	43. $14\frac{2}{77}$.	59. 1395yd.	74. 38.8844.
9. 93.	27. $\frac{29}{4}=7\frac{1}{4}$.	44. 1.7320-	60. 24 ft.	75. 65.7267.
10. 111.	28. 4735.	508+.	61. 160 r. l.	76. .08.
11. 232.	29. 2401.	45. 3.4641-	80 r. w.	77. 2.
12. 729.	30. 4096.	0161+.	62. 320 r. l.	78. .18.
14. 1.4.	31. 6561.	47. 10 yds.	80 r. w.	79. $\frac{1}{8}$.
15. 5.4.	32. 59049.	48. 50 m.	63. 24 rods.	80. $\frac{8}{15}$.
16. 15.3.	33. 32.768.	49. 200 m.	64. 15 min.	81. $\frac{35}{54}$.
17. .35.	34. $\frac{4}{9}$.	50. 60 feet.	65. 18 in. d.	82. $\frac{24}{80}$, or $\frac{3}{10}$
18. .881.	35. .3628+.	51. 24.97 ft.	66. 30 in.	83. $\frac{63}{132}$, or
19. 1.4142.	36. $\frac{8}{27}$.	52. 42.426r.	67. 24.4948.	$\frac{21}{44}$.
20. 4.123+.	37. $\frac{32}{64}=\frac{1}{2}$.	53. 56.568r.		

CUBE ROOT.—ARTS. 363–367.

3. 12.	11. 4.5.	19. $3\frac{1}{3}$.	27. 8 ft.	36. 23.148-
4. 24.	12. $\frac{3}{4}$.	20. 103.	28. 102.44 i.	14 T.
5. 72.	13. $\frac{5}{6}$.	21. 3002.	29. 113.32 i.	37. 81920 t.
6. 83.	14. $\frac{3}{5}$.	22. 5.48.	30. 58.806ft	38. $1111\frac{1}{9}$.
7. 125.	15. $\frac{2}{3}$.	23. 49.68.	33. 216 lbs.	40. 7.211 ft.
8. 1.25+.	16. $\frac{9}{16}$.	24. 73 in.	34. $12136-	41. 8 in.
9. 1331.	17. $\frac{25}{36}$.	25. 364 ft.	2.9629.	42. $\frac{3}{4}$ feet.
10. 2.3.	18. 2.3908.	26. 108 ft.	35. 1562.5.	43. 4 feet.

EQUATION OF PAYMENTS.—ARTS. 372–376.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
3.	6 months.	9.	$2\frac{1}{4}$ years.		Due A. 19.		Date O. 5.
4.	4 months.	11.	36 d. Av. t.	14.	29 d. Av. t.	18.	$3\frac{1}{3}$ month.
5.	6 months.		Due A. 15.		Due S. 6th.	19.	139 days.
6.	$1\frac{1}{2}$ years.	12.	60 d. Av. t.	15.	73d. f. J.10.	20.	5 m. 20 d.
7.	8 months.		Date S. 8d.		Due S. 21.	21.	10 m. 19 d.
8.	$6\frac{1}{2}$ months.	13.	31d. f. J.19.	16.	30 d. Av. t.	23.	$1309.916.

PARTNERSHIP.—ARTS. 378, 379.

1.	Given.		2210.526\frac{36}{114}$ D.		$120 C's.
2.	$120 A's.	4.	$300 A's.	7.	$30 apiece.
	$160 B's.		$400 B's.	8.	40.019\frac{29}{209}$ A.
	$200 C's.		$600 C's.		88.277\frac{107}{209}$ B.
3.	589.473\frac{78}{114}$ A.		$700 D's.		117.703\frac{73}{209}$ C.
	1129.824\frac{64}{114}$ B.	6.	$100 A's.	9.	$378.95 A.
	1670.175\frac{50}{114}$ C.		$120 B's.		$421.05 B.

ALLIGATION.—ARTS. 381–386.

2.	$8\frac{46}{377}$ d.		2 lbs. 18 cts.		$6\frac{2}{3}$ g. 30 ct.	15.	$8\frac{8}{9}$ lbs. 75c.
3.	$20\frac{41}{53}$ car.		1 lb. 21 cts.		$17\frac{7}{9}$ g. 37 c.		$8\frac{8}{9}$ lbs. 80c.
4.	50.41\frac{2}{3}$.		4 lbs. 22 cts.	12.	$62\frac{1}{2}$ b. 45 c.		$8\frac{8}{9}$ lbs. 85c.
6.	4, 2, 2, 4.	9.	5 yds. 68 cts.		$41\frac{2}{3}$ b. 56 c.		$23\frac{1}{3}$ l. 95 c.
7.	2 lbs. 9 cts.		3 yds. 75 cts.		$20\frac{5}{6}$ b. 65 c.	16.	$81\frac{12}{23}$ l. 50 c.
	2 lbs. 11 cts.		5 yds. 83 cts.	14.	$23\frac{9}{17}$b. 25c.		$211\frac{22}{23}$l. 62c.
	4 lbs. 14 cts.		12yds. 85cts.		$23\frac{9}{17}$b. 50c.		$326\frac{2}{23}$l. 75c.
8.	8 lbs. 15 cts.	11.	$4\frac{4}{9}$ g. 25 ct.		$52\frac{16}{17}$b. 80c.		$130\frac{16}{23}$l. 83c.

REDUCTION OF CURRENCIES.—ARTS. 391–394.

3.	$485.21.	16.	£100, 10s.	24.	8162.5 flor.	33.	£250.
4.	$1334.63.	17.	£300.	25.	$9685\frac{1}{3}$ rou.	34.	£644, 10s.
5.	$2179.815.	18.	£3358, 16s.	26.	Given.	35.	Given.
6.	$1785.083.		$7\frac{3}{4}$d.	27.	£113, 8s.	36.	162.55\frac{5}{9}$.
7.	$2863.5103	19.	£4188, 14s.	28.	£169, 14s.	37.	244.069\frac{4}{9}$.
8.	$2244.202.		$3\frac{1}{4}$d.		6d.	38.	252.28\frac{1}{8}$.
9.	$3019.626.	20.	£10408, 3s.	29.	£186, 3s.	39.	$640.651.
10.	$3405.2677		$8\frac{3}{4}$d.		7.2d.	40.	790.93\frac{1}{3}$.
11.	$1629.918.	21.	£25903, 1s.	30.	£350, 4s.	41.	1147.73\frac{1}{3}$.
12.	$1773.75.		$3\frac{3}{4}$d.	31.	£240.	42.	$2436.428[illegible]
13.	$4496.435.	22.	13784.946 f.	32.	£221, 16s.	43.	$4008.06[illegible]
14.	$9227.80.	23.	44387.096f.		1.6d.		

EXCHANGE.—ARTS. 399–407.

Ex. Ans.	Ex. Ans.	Ex. Ans.
1. Given.	4. 15553.388\frac{3}{9}$.	13. Given.
2. $8071.05.	5. Given.	14. $2062.0155.
3. $1141.5937.	6. £322, 3s. 6$\frac{1}{4}$d.	15. $4858.6538.
4. $14889.42.	7. £893, 11s. 5$\frac{1}{3}$d.	16. $13249.177.
5. $143159.948 dft.	8. £1095, 1s. 2$\frac{1}{4}$d.	17. Given.
$145307.347 recd.	9. £11583, 6s. 2$\frac{2}{3}$d.	18. 27430.9 fr.
	10. $17160.192.	20. $74.545.
ART. 401.	11. Given.	21. $1146.25.
2. $7597.30.	12. $53617.0833.	22. $2583.36.
3. $12525.7439.		

ARITHMETICAL PROGRESSION.—ARTS. 413–417.

1. Given.	3. 300.	5. 2 and 99.	8. Given.
2. 180.	4. Given.	7. 15d. $360.	9. 121.

GEOMETRICAL PROGRESSION.—ARTS. 419, 420.

1. Given.	5. 2048 m.	9. 1533.	13. $\frac{1}{9}$.
2. $\frac{48}{81}$, or $\frac{16}{27}$.	6. $284.057.	10. 240624.	14. $55924.05.
3. 12288.	7. $435.492.	11. 1.	15. £204, 15s.
4. 8388608.	8. Given.	12. $\frac{1}{2}$.	

MENSURATION.—ARTS. 425–454.

1. Given.	17. 200 yards.	32. 21205.8 feet.
2. 640 A.	18. Given.	33. Given.
3. 26 A. 65 r.	19. 11309.76 sq. r.	34. 30000 feet.
4. 50 rods.	20. 2037.18496 yds.	35. Given.
5. 80 rods.	21. 8 rods.	36. 123$\frac{1}{3}$ feet.
6. Given.	22. 16 feet.	37–39. Given.
7. 4 A. 75 r.	23. Given.	40. 45945.9+cu. ft.
8. Given.	24. 5425 cu. feet.	41. Given.
9. 558 r.	25. 27 feet.	42. 4084.067 feet.
10. Given.	26. 2 feet.	43. Given.
11. 4914 square ft.	27. Given.	44. 201061760 sq.m.
12. Given.	28. 9200 cu. feet.	45. Given.
13. 62.35+yards.	29. Given.	46. 268082346666$\frac{2}{3}$.
14. Given.	30. 152 square feet.	47. Given.
15. 314.159 r.	31. Given.	48. 244.845 + gals.
16. Given.		

MISCELLANEOUS EXAMPLES.

Ex.	Ans.	Ex.	Ans.	Ex.	Ans.	Ex.	Ans.
1.	570 sheep.	37.	$44\frac{4}{9}$ per ct.	72.	£2,6s.$11\frac{1}{4}$d.	106.	40 yrs.
2.	$3445.8125	38.	$3478.667.	73.	1 lb. 1 oz.	107.	172 sheep
3.	25 dif.	39.	$5557.68.		22 grs.	108.	$11\frac{263}{440}$ t.
4.	1548.	40.	8724.375.	74.	1 oz. $18\frac{1}{4}$ p.	109.	$46\frac{86}{113}$ six-
5.	$201\frac{6}{19}$.	41.	$390\frac{1}{2}$; $465\frac{1}{2}$.	75.	11s. $\frac{1}{2}$d.		teenths.
6.	$79817\frac{7}{9}$.	42.	2759; 2884.	76.	896 tiles.	110.	225.585 r.
7.	$343\frac{19}{20}$.	43.	428.	77.	16 per ct.	111.	250.438 r.
8.	$118\frac{7}{8}$.	44.	50.	78.	$$2086\frac{26}{49}$.		125.219 r.
9.	$6\frac{341}{904}$.	45.	2754.	79.	155 A. 39 r.	112.	76.8 A.
10.	$7246\frac{7}{8}$.	46.	15120.		$182\frac{1}{2}$ ft.	113.	600 yds.
11.	$\frac{7}{40}$.	47.	$213\frac{79}{117}$ yrs.	80.	£28, 18s.	114.	2400 sq. r.
12.	$142.45.	48.	100.		$11\frac{91}{127}$d.	115.	5 h. 20 m.
13.	$8\frac{1}{2}$ per ct.	49.	7812.5 lbs.	81.	690.931 r.	116.	26.4.
14.	$26000.	50.	5082.	82.	$68.649.	117.	4h. $10\frac{10}{23}$m
15.	$20000.	51.	6776.	83.	78 lbs. 2 oz.	118.	$\frac{45}{72}$.
16.	8 mo. 24 d.	52.	$38\frac{34}{85}$.	84.	$240.45.	119.	144 r. 48 f.
17.	14 y. 3 m.	53.	$1.138.	85.	£170, $11\frac{13}{17}$s	120.	$29\frac{1}{3}$ hrs.
	12 d.	54.	2568.	86.	$4\frac{13}{30}$s.	121.	144 min.
18.	10 per ct.	55.	360.	87.	$33\frac{1}{3}$ cts.	122.	480 a. A's.
19.	$1318.84.	56.	$15730.	88.	$220\frac{460}{657}$ lbs.		240 a. B's.
20.	$1531.396.	57.	$29475.	89.	13.9572 r.		720 a. C's.
21.	$16.212.	58.	$42720.	90.	$39000.	123.	58 days.
22.	$19.294.	59.	$274.50.	91.	$6352.	124.	$726.
24.	$6090.	60.	$\frac{3}{44}$d.	92.	3 hours.	125.	$72.
25.	$106.25.	61.	£8395, 10s.	93.	105 feet.	126.	$300 A's.
26.	$406.625.		2d.	94.	$.6875.		$420 B's.
27.	$5250.	62.	£153, 2s.	95.	$26\frac{1}{4}$ days.		$800 C's.
28.	$\frac{375}{1000}$ per ct.		6d.	96.	$57853\frac{1}{8}$ T.	127.	$720 A's.
29.	5 per cent.	64.	£155.	97.	$4186.875.		$1200 B's.
30.	$6782.70.	65.	$$43.45\frac{5}{7}$.	98.	132 sheep.		$1680 C's.
31.	$884.10.	66.	$$2083\frac{1}{3}$.	99.	72 days.	128.	1 h. $5\frac{5}{11}$m.
32.	$.662 per g.	67.	1800 T.	100.	$313\frac{1}{2}$ days.	129.	1200 sh.
34.	$0.094.	68.	£11, 5s.	101.	627 days.	130.	$18\frac{2}{11}$ feet.
35.	20 per ct.;	69.	$61.04.	103.	$1\frac{23}{37}$ hrs.	131.	57.628 m.
	$912.	70.	$$65.71\frac{3}{7}$.	104.	$9\frac{127}{178}$ rods.	132.	34 hours;
36.	50 per ct.	71.	$879.60.	105.	120 days.		823 m.

THE END.